EPJ manuscript No.
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Exploring mSUGRA in the top quark sector at the LHC with a multi-variate hypothesis test, the S2-method

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Received: date / Revised version: date

Abstract. The sensitivity to exclude mSUGRA models during the early running of the Large Hadron Collider at 7 TeV centre-of-mass energy is explored in the event topology reflecting the production of top quark pairs. The S2-method, a novel multi-variate hypothesis test on the reconstructed kinematics of the collision events is developed wherein systematic uncertainties can be added. For an integrated luminosity of 100 pb^{-1} the parameters space xxx can potentially be excluded.

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1 Introduction

Within the proton collisions provided by CERN's Large Hadron Collider new physics phenomena are searched for. The potential to exclude part of the mSUGRA parameter space is investigated for the early data taking period at a centre-of-mass energy of 7 TeV. A hypothesis test variable is defined based on the observable kinematic properties of top quark pair events and applied on pseudo-data to test the predictions of the Standard Model compared to specific mSUGRA models. The potential exclusion limits in the mSUGRA parameter space are obtained for $100\mathrm{pb}^{-1}$ of integrated luminosity using the observable top quark pair topology of at least four jets and exactly one muon in the reconstructed final state.

2 Simulating LHC collisions

In this paper the S2-method is applied to test the consistency of mSUGRA models in a typical top quark sector topology where 4 jets, an isolated muon and missing transverse energy arise from the top quark pair decay.

The Standard Model process $\mathrm{pp} \to \mathrm{t\bar{t}} + \mathrm{Njets}$ at a centre-of-mass energy of 7 TeV is generated up to $\mathrm{N} = 3$ using MadGraph/MadEvent [1] resulting in a Leading-Order cross section of $\sigma(\mathrm{t\bar{t}}) = \mathrm{xxxpb}$ for a top quark mass of $\mathrm{m_t} = 172\mathrm{GeV}$. Only the decay

$$\mathrm{t\bar{t}} \to \mathrm{bW^+\bar{b}W^-} \to \mathrm{b}\mu\nu_\mu\mathrm{\bar{b}q\bar{q}} \quad (1)$$

with a branching ratio of xxx is denoted hereafter as the signal topology $\mathrm{pp} \to \mathrm{t\bar{t}}(\mu)$, the other decay channels are denoted as $\mathrm{pp} \to \mathrm{t\bar{t}}(\mathrm{other})$. The Standard Model single-top and W+jets (up to xx extra jets) processes are generated using the same MadGraph/MadEvent tool, respectively with Leading-Order cross sections xxx pb and xxx pb. For the single-top processes both the $2 \to 2$ and the $2 \to 3$ diagrams are considered.

The phenomenology of mSUGRA models is defined by five parameter [2] of which we will fix three of them, $\mathrm{A_0} = \mathrm{xxx}$, $\mathrm{sign}(\mu)$ is taken to be positive and $\mathrm{tg}(\beta) = \mathrm{xxx}$. The remaining parameter space spanned by the two parameters $\mathrm{m_0}$ and $\mathrm{m_{\frac{1}{2}}}$, is simulated point per point using MadGraph/MadEvent [3]. In steps of 20 GeV and for all points $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ in the two dimensional grid between $100\mathrm{GeV} \le \mathrm{m_0} \le 1000\mathrm{GeV}$ and $100\mathrm{GeV} \le \mathrm{m_{\frac{1}{2}}} \le 1000\mathrm{GeV}$ an adequate sample of events is generated. The obtained cross sections at a centre-of-mass energy of 7 TeV are shown in Figure 1. The $\mathrm{pp} \to \tilde{\mathrm{g}}\tilde{\mathrm{g}}$, $\mathrm{pp} \to \tilde{\mathrm{g}}\tilde{\mathrm{q}}$ and $\mathrm{pp} \to \tilde{\mathrm{q}}\tilde{\mathrm{q}}$ processes are considered to constitute the mSUGRA phenomenology.

The parton shower and fragmentation is provided by PYTHIA [4]. The DELPHES package [5] is used to simulate the interaction of the particles with the detector, for which the geometry of the CMS detector is taken.

3 Selection of the top quark pair topology

A typical event selection is applied on the reconstructed objects in the collision event to select the top quark pair topology [6,7], namely on the hadronic jets arising from the partons and the lepton from the W boson decay. The jets are reconstructed with the Anti-Kt algorithm implemented in DELPHES using $\Delta\mathrm{R} = 0.5$ and calorimeter clusters as input. At least four jets with a transverse momentum above 30 GeV (and $|\eta| \le 2.4$) are requested together

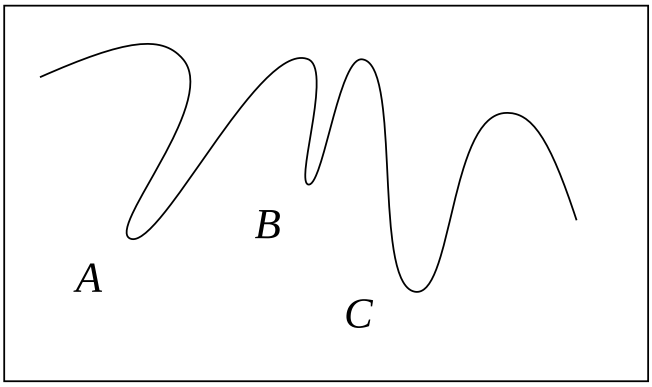

Fig. 1. The Leading-Order cross section of the mSUGRA processes considered at centre-of-mass energies of 7 TeV.

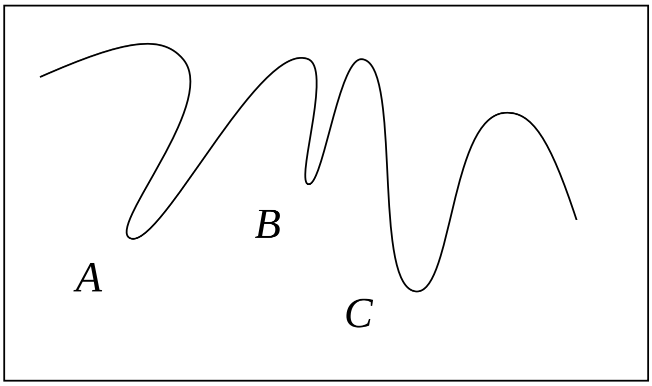

Fig. 2. The selection efficiency for mSUGRA events at a centre-of-mass energy of 7 TeV.

with exactly one isolated muon with a transverse momentum above 10 GeV (and $|\eta| \leq 2.1$). Both on the calorimeter and tracker level the isolation requirement uses a cone with $\Delta\mathrm{R} = 0.3$ around the direction of the muon at the interaction point to obtain a total isolation variable relative to the transverse momentum of the muon. A veto is applied on the presence of a second isolated lepton (**which flavor ?**) with a transverse momentum above xx GeV. These criteria result in an efficiency of xx % for the Standard Model $\mathrm{t\bar{t}}$ events decaying in a muon+jet final state. The efficiencies for the different mSUGRA parameters points $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ are shown in Figure 2.

Here needs some explanation on the pheno of the efficiency plot...

As detailed in Table 1 for an integrated luminosity of 100 pb^{-1} this results in xx selected $\mathrm{t\bar{t}}$ event in the signal topology and a total of xx selected events in the other $\mathrm{t\bar{t}}$ decay topologies, single-top and W+jets events. This has to be compared with the expected amount of additional events when the mSUGRA phenomenology is present with according to the parameter points $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ as shown in Figure 3.

Table 1. Event selection details for the considered Standard Model processes at 7 TeV centre-of-mass energy and for an integrated luminosity of 100 pb^{-1}.

process	σ_{LO}	sel. eff.	# events
$\mathrm{pp} \rightarrow \mathrm{t\bar{t}}(\mu)$	xx	xx	xx
$\mathrm{pp} \rightarrow \mathrm{t\bar{t}}(\mathrm{other})$	xx	xx	xx
single-top	xx	xx	xx
$\mathrm{pp} \rightarrow \mathrm{W} + \mathrm{Njets}$	xx	xx	xx

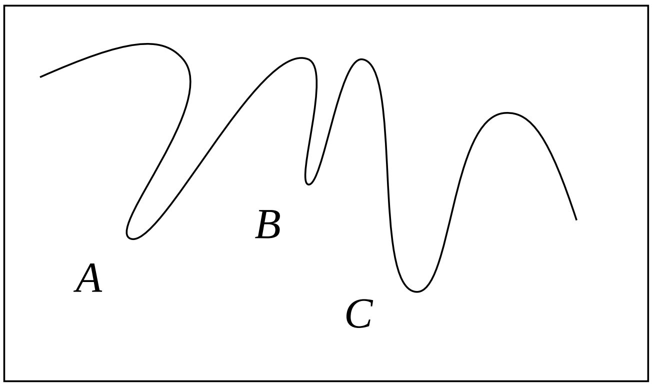

Fig. 3. The expected amount of selected mSUGRA events at a centre-of-mass energy of 7 TeV for an integrated luminosity of 100 pb^{-1}.

4 Ranking kinematic variables for sensitivity

The distributions of the expected kinematic properties of the reconstructed objects in the selected Standard Model $\mathrm{t\bar{t}}$ events is different when mSUGRA phenomenology is present. The expected difference depends strongly on the parameters of the mSUGRA model. In order to test the predictions of the Standard Model with respect to an alternative model, from a list of M kinematic variables $\mathrm{x_j}$ with $\mathrm{j} \in \{1, ..., \mathrm{M}\}$ those variables with the largest difference are identified using the overlap of the probability density distributions of both models obtained after the event selection. The distributions of the Standard Model and the model including mSUGRA obtained from the simulated events are denoted respectively by $\mathrm{P_j^{SM}(x_j)}$ and $\mathrm{P_j^{SM\oplus NP}(x_j)}$. The overlap $\mathrm{O_j}$ is defined from the binned distributions as $\mathrm{O_j} = \sum_{\mathrm{i=1}}^{\mathrm{k}} \mathrm{P_{ij}^{SM}(x) P_{ij}^{SM\oplus NP}(x)}$ where the data is divided in k bins defined to have an equal expected population of selected Standard Model $t\bar{t}$ events. The definition of the binning of the histrograms of the variables is an important part in this procedure. The binning is defined after the event selection in order to obtain and equal bin content of about 40 expected Standard Model events in the signal topology $pp \to t\bar{t}(\mu)$ in all bins.

As an example the expected distributions for four typical kinematic variables relevant in the search for new physics phenomena in the top quark sector are shown in Figure 4 for both the Standard Model contributions and a mSUGRA model of $(m_0, m_{\frac{1}{2}}) = (150\mathrm{GeV}, 150\mathrm{GeV})$. **Describe the pheno in these plots.** The overlap O_j for these variables depends on the mSUGRA parameters $(m_0, m_{\frac{1}{2}})$ and is shown in Figure 5. **Describe the pheno in these plots.**

In total XX variables are considered reflecting different aspects of the event topology. They can be classified in mainly four categories: ... . For each considered couple of parameters $(m_0, m_{\frac{1}{2}})$ the variables are ranked according to their overlap obtaining a ranked list $\{X_1, X_2, ..., X_M\}$ corresponding to overlap values $O_{X_1} \leq O_{X_2} \leq ... \leq O_{X_M}$. The variable with the smallest overlap will in general have a largest statistical power to differentiate between the Standard Model hypothesis and the alternative mSUGRA model at the $(m_0, m_{\frac{1}{2}})$ point considered. This variable is kept on the shortlist of variables for the hypothesis testing analysis to be performed in point $(m_0, m_{\frac{1}{2}})$. According to the following criteria more variables can be added to this shortlist. The variable X_j is added when its linear correlation with all variables $\{X_1, X_2, ..., X_{j-1}\}$ already on the shortlist and calculated with the selected Standard Model events is less than ρ_c and its overlap O_{X_j} is not larger than $\epsilon_c O_{X_1}$, hence ϵ_c times the overlap of the best ranked variable. The value of the parameter ρ_c is choosen to be XX in order not to take duplicated information in the hypothesis testing analysis. The value of the parameter $\epsilon_c =$ XX is chosen to optimize the result of the hypothesis testing analysis presented in Section5. With this procedure a shortlist of usually about XX variables is obtained for each mSUGRA point $(m_0, m_{\frac{1}{2}})$.

Some explainations on the observed phenomenology based on these plots...

5 Multi-variate test: the S2-method

sentence to introduce this section

The Standard Model predictions for the expected amount of selected events in each bin i of variable j can be taken from the simulated events, $F^{SM}_{i,j}$, or in a data-driven approach from a control event sample dominated by the main Standard Model processes expected in the kinematic region where the hypothesis test will be performed, $\hat{F}^{SM}_{i,j}$. The expectations of the alternative model are obtained from simulation, $F^{SM\oplus NP}_{i,j}$, but can be rescaled to include the deviations observed between the simulation and data-driven approach for the Standard Model prediction with the correction factor

$$\epsilon_{i,j} = \frac{\hat{F}^{SM}_{i,j}}{F^{SM}_{i,j}} \tag{2}$$

to obtain $\hat{F}^{SM\oplus NP}_{i,j} = \epsilon_{i,j} \cdot F^{SM\oplus NP}_{i,j}$. These stochastic variables $F_{i,j}$ follow a Poissonian distribution which converges to a Gaussian distribution when $F^{SM}_{i,j}$ exceeds about 30. Based on the principle of a likelihood ratio test between the Standard Model expectation and the alternative model, the squared-significance with respect to the observed data is

$$s^2_{i,j} = \left(\frac{\hat{F}^{SM}_{i,j} - \hat{F}^{data}_{i,j}}{\sqrt{\sigma^2(\hat{F}^{SM}_{i,j}) + \sigma^2(\hat{F}^{data}_{i,j})}} \right)^2 \tag{3}$$

reflects the deviation of the observed data in bin i of variable j with the expected Standard Model bin content. The uncertainty on the expected bin content for the Standard Model, $\sigma(\hat{F}^{SM}_{i,j})$, can include both the statistical and the systematical uncertainty arizing from the simulation or data-driven approach.

In the selected data the events with the highest probability to arise from the new physics phenomena are selected to construct a hypothesis test variable for the event sample. For each point $(m_0, m_{\frac{1}{2}})$ in the mSUGRA parameter space the squared-significances $s^2_{i,j}$ are determined for the variables appearing in the shortlist defined in Section 4. For each selected event a in the data sample the total squared-significance, S_a, is determined as the product of the squared-significances over the populated bins by this event.

The selected data events $a \in \{1, 2, ..., N\}$ are ordered according to a decreasing value of S_a obtained at each point $(m_0, m_{\frac{1}{2}})$. Only with the first fraction κ of the N events, the test variable V for the hypothesis test is calculated as $V = \sum_{a=1}^{\kappa \cdot N} S_a$. The potential to exclude the mSUGRA model with parameters $(m_0, m_{\frac{1}{2}})$ is obtained from pseudo-experiments which include either only the Standard Model processes or alternatively also the processes with the mSUGRA phenomenology beyond the Standard Model. In Figure 6 the distribution of the test variable V is shown for the case where only Standard Model processes are present in a data sample with an integrated luminosity of 100 pb^{-1} or additionally also mSUGRA processes with $(m_0, m_{\frac{1}{2}}) = (150\mathrm{GeV}, 150\mathrm{GeV})$. The probability density distributions for the V test variable are denoted respectively by $f(V|SM)$ and $f(V|SM \oplus NP)$.

The null hypothesis H_0 assumes that the data is compatible with the mSUGRA model $(m_0, m_{\frac{1}{2}})$, while the alternative hypothesis H_1 assumes the data is compatible with the Standard Model. When fixing the significance level α the value of V_α is obtained from the general expression $\alpha = \int_0^{V_\alpha} f(V|SM \oplus NP)dV$. The power of the test $1 - \beta$ is defined as $1 - \beta = \int_0^{V_\alpha} f(V|SM)dV$. To exclude the null hypothesis when no new physics is present in the data and at a confidence level of $1 - \alpha$, we consider

$1-\beta = 0.5$ as a boundary. When $1-\beta > 0.5$ the alternative hypothesis will on average be excluded, $\mathrm{V_{data}} < \mathrm{V}_\alpha$. When $1-\beta < 0.5$ the alternative hypothesis will on average not be excluded by the data, $\mathrm{V_{data}} > \mathrm{V}_\alpha$.

6 Results for the mSUGRA plane

The S2-method described in previous section is applied for each point in the mSUGRA plane $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ resulting in a $(1-\beta)$ value for a datasample of 100 $\mathrm{pb^{-1}}$ at a centre-of-mass energy of 7 TeV. Using the parameter values $\kappa = \mathrm{xxx}$ and $\epsilon = \mathrm{xxx}$ in the S2-method, this results in an expected limit shown in Figure 9 to exclude the mSUGRA model at a confidence level of 0.1. **mention some pheno and compare with other limits** The result can be optimized by tuning the values of $\kappa = \mathrm{xxx}$ and $\epsilon = \mathrm{xxx}$ in the S2-method. Figures 7 and 8 illustrate the dependency of the power of the test $(1-\beta)$ for four different mSUGRA models with respectively $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (150\mathrm{GeV}, 150\mathrm{GeV})$, $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (150\mathrm{GeV}, 300\mathrm{GeV})$, $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (300\mathrm{GeV}, 150\mathrm{GeV})$ and $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (300\mathrm{GeV}, 300\mathrm{GeV})$. Althought these four mSUGRA models contain very different phenomenology, the most optimal value for the parameters κ and ϵ which result in the highest power $(1-\beta)$ are similar for each of these mSUGRA models.

compare with the simple cut method... significance after the cuts versus significance when using the κ percent highest events

Mention something that it is better to have the 1-beta band as narrow as possible in the plane.

7 Conclusion

Results should be connected to the title, hence first mention the mSUGRA results, then the statistical method. Summarize the good points of the method and its generality (this was just an example topology and an example alternative model).

Thanks to funding agencies and DELPHES, MadGraph authors for their help.

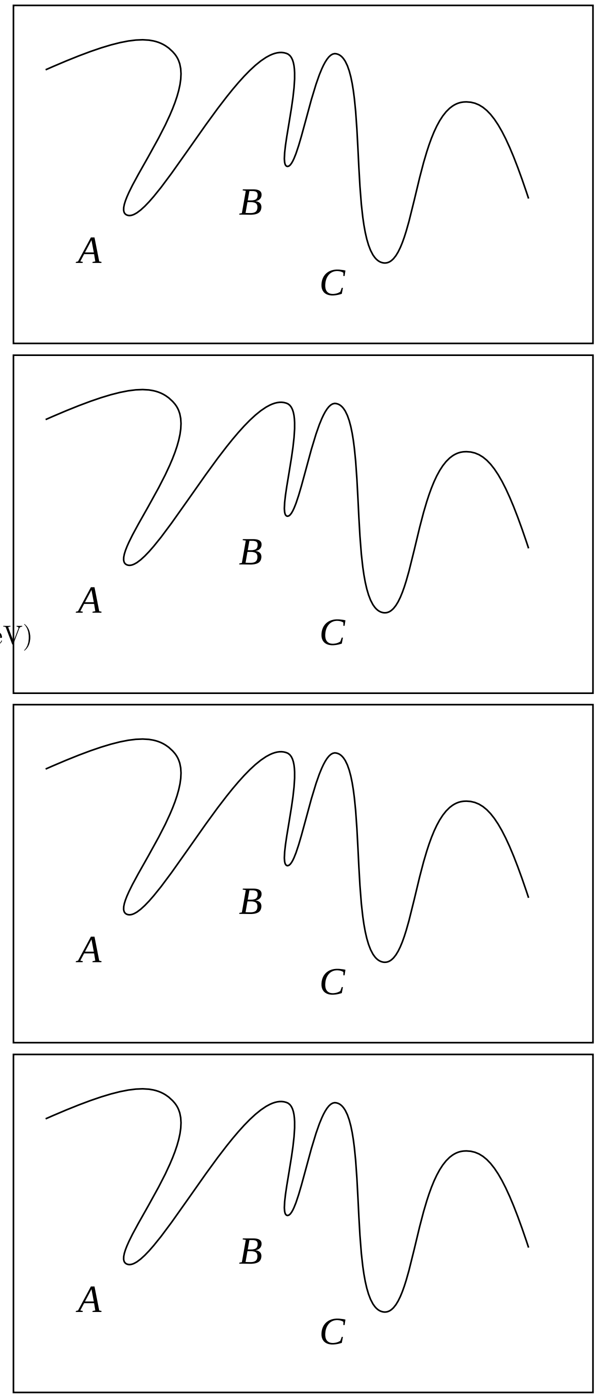

Fig. 4. The expected distributions for the missing transverse energy, the $\mathrm{H_T}$, the $\mathrm{p_T^\mu}$ and the $\mathrm{E_T^{(4)}}/\mathrm{E_T^{(1)}}$ for Standard Model and mSUGRA ($(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (150\mathrm{GeV}, 150\mathrm{GeV})$) contributions in a sample with an integrated luminosity of 100 $\mathrm{pb^{-1}}$. Pseudo-data are added in the hypothesis that data contains the mSUGRA contribution.

References

1. Main madgraph/madevent ref
2. some mSUGRA refs
3. Specific madgraph/madevent ref for mSUGRA ?
4. Main pythia ref
5. Main delphes ref
6. some CMS ref to a top quark selection
7. some ATLAS ref to a top quark selection
   Author, Journal **Volume**, (year) page numbers.
8. Author, *Book title* (Publisher, place year) page numbers

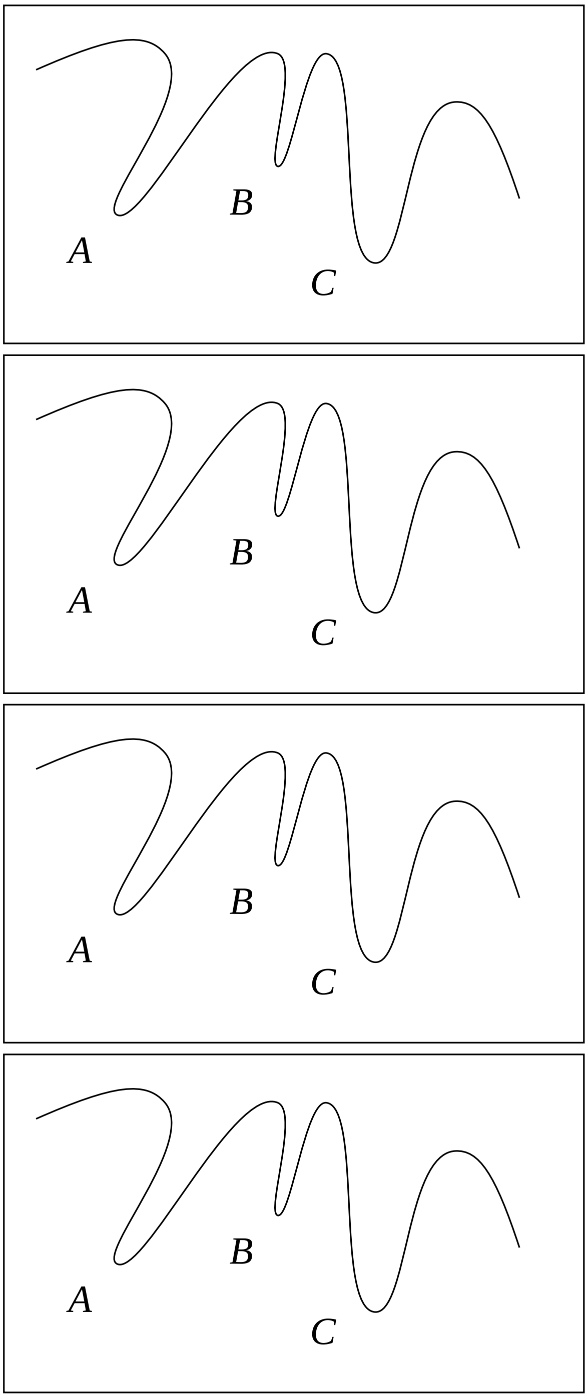

Fig. 5. The overlap for the missing transverse energy distribution, the $\mathrm{H_T}$ distribution, the $\mathrm{p_T^{\mu}}$ distribution and the $\mathrm{E_T^{(4)}/E_T^{(1)}}$ distribution between the Standard Model and the mSUGRA model.

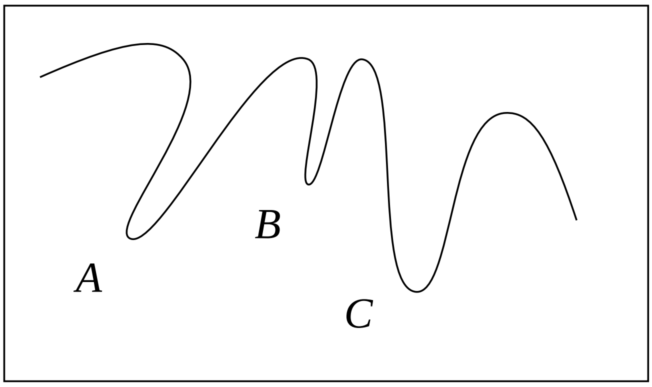

Fig. 6. The distribution of the test variable V for pseudo-experiments using a dataset of 100 pb^{-1} at 7 TeV centre-of-mass energy with only Standard Model processes or with additional mSUGRA processes at $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}}) = (150\mathrm{GeV}, 150\mathrm{GeV})$.

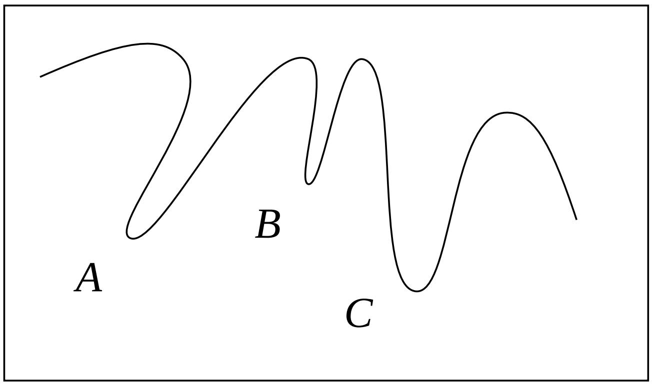

Fig. 7. The power of the test $(1-\beta)$ for four mSUGRA models with divers parameters $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ versus the parameter κ in the S2-method.

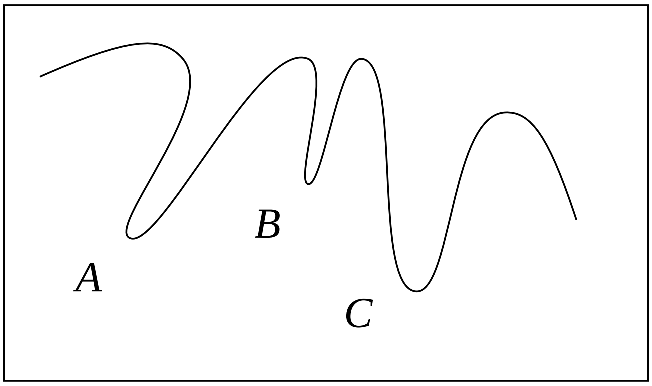

Fig. 8. The power of the test $(1-\beta)$ for four mSUGRA models with divers parameters $(\mathrm{m_0}, \mathrm{m_{\frac{1}{2}}})$ versus the parameter ϵ in the S2-method.

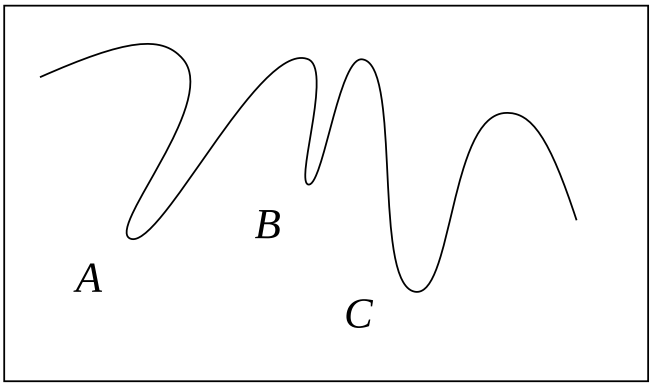

Fig. 9. The boundary where the power of the test $(1-\beta)$ equals 0.5 in the plane spand by the mSUGRA parameters $(\mathrm{m}_0, \mathrm{m}_{\frac{1}{2}})$ using an integrated luminosity of 100 pb^{-1} at a centre-of-mass energy of 7 TeV.