The expanding universe

Lecture 1
The early universe

 chapters 5 to 8

 Particle Astrophysics , D. Perkins, 2\textsuperscript{nd} edition, Oxford

5. The expanding universe
6. Nucleosynthesis and baryogenesis
7. Dark matter and dark energy components
8. Development of structure in early universe
Expanding universe : content

• part 1 : ΛCDM model ingredients: Hubble flow, cosmological principle, geometry of universe
• part 2 : ΛCDM model ingredients: dynamics of expansion, energy density components in universe
• Part 3 : observation data – redshifts, SN Ia, CMB, LSS, light element abundances - ΛCDM parameter fits
• Part 4: radiation density, CMB
• Part 5: Particle physics in the early universe, neutrino density
• Part 6: matter-radiation decoupling
• Part 7: Big Bang Nucleosynthesis
• Part 8: Matter and antimatter
The ΛCDM cosmological model

• Concordance model of cosmology – in agreement with all observations = Standard Model of Big Bang cosmology

• ingredients:
  ✓ Universe = homogeneous and isotropic on large scales
  ✓ Universe is expanding with time dependent rate
  ✓ Started from hot Big Bang, followed by short inflation period
  ✓ Is essentially flat over large distances
  ✓ Made up of baryons, cold dark matter and a constant dark energy + small amount of photons and neutrinos
  ✓ Is presently accelerating
Expanding Universe

2.7 K today

4.1 K transition to accelerated expansion

0.26 eV recombination

0.76 eV transition to matter dominated expansion

80 keV Big Bang Nucleosynthesis

1 MeV neutrino decoupling

2.5 MeV QCD transition

200 MeV electroweak transition

100 GeV hot Universe

post-inflationary reheating (?)

inflationary stage (?)

generation of baryon asymmetry

generation of dark matter

© Rubakov
Part 1
ΛCDM ingredients

Hubble expansion – redshift
Cosmological principle
Geometry of the universe – Robertson-Walker metric
Some distances

- Earth-Sun = AU = 150 x 10^9 m = 150 x M km
- Lightyear = Ly = 0.946 x 10^{16} m
- 1 year = 31.5 x 10^6 s
- parsec = pc = 3.3 Ly
- Mpc = 3.3 MLy
- Radius Milky Way ≈ 15 kpc
- Sun to centre MW ≈ 8 kpc
- Andromeda galaxy (M31) to earth ≈ 800 kpc
- Width Local Group of ≈ 30 galaxies ≈ 2 Mpc
- Average inter-galactic distance ≈ Mpc
- highest redshift observed: dwarf galaxy at z ≈ 11
Hubble law 1

- Hubble (1929): spectral lines of distant galaxies are redshifted ⇒ galaxies move away from Earth
- Receding velocity increases with distance: Amount of shift $\Delta \lambda$ depends on apparent brightness ($\sim$distance D) of galaxy

Original Hubble plot

[Graph showing velocity vs. distance for SuperNovae Ia and II and Cepeids]
Hubble law 2

• Interpret redshift $z$ as Doppler effect

• For relativistic objects

$$ z = \frac{\Delta \lambda}{\lambda} = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \quad \lambda_{\text{obs}} = \lambda_{\text{em}} \sqrt{\frac{1 + \beta}{1 - \beta}} \xrightarrow{\beta \ll 1} \lambda_{\text{em}} (1 + \beta) $$

• For close-by objects, in non-relativistic limit

$$ 1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} \approx 1 + \beta \quad \rightarrow z \approx \beta = \frac{v}{c} $$

• Hubble law becomes linear relation between velocity and distance

$$ z \approx \frac{v}{c} = H_0 D $$

• Confirms theories of Friedmann and Lemaître
Hubble law 3

• $H_0 = \text{Hubble constant today} = \text{present value (PDG 2012)}$
  
  $$H_0 = H(t_0) = (70.2 \pm 1.4) \text{ (km/sec)/Mpc}$$
  
  $t_0 = today$

  $$H_0 = 100 \text{ km/sec/Mpc} \cdot h \rightarrow h = 0.702 \pm 0.014$$

• If $H(t)$ is same at all times → constant and uniform expansion

• Observations show that expansion was not constant and that Hubble ‘constant’ depends on time

• Expansion rate evolves as function of changing energy-matter density of universe (see Friedman equations)

$$H^2(t) = \frac{8\pi G_N}{3} \rho_{\text{tot}}(t) - \frac{kc^2}{(R(t))^2}$$
Sources of redshift

• Motion of object with respect to observer – **Doppler effect** (red and blue shifts): e.g. rotation of stars in galaxies – see galaxy rotation curves (dark matter)

• **Gravitational redshift**: stretching of wavelength close to heavy object - local effect – negligible over long distances

• **Cosmological redshift**: stretching of wavelength due to expansion of universe – dominant at high redshifts

\[
1 + z = \frac{\lambda_{\text{obs}}(t_0)}{\lambda_{\text{em}}(t)} = \frac{R(t_0)}{R(t)}
\]

> R(t)=Scale of universe at time t
Cosmological redshift - time

• Redshift is the measured quantity – it is related to a given time during the expansion

• Large $z$ means small $t$, or early during the expansion

$$t = 0 \quad z = \infty$$

$$t = t_0 \quad z = 0$$

today

• Actual (proper) distance $D$ from Earth to distant galaxy at time $t$

$$D(t) = r \cdot R(t)$$

• $D$ can only be measured for nearby objects

• $r$ = co-moving coordinate distance – in reference frame co-moving with expansion - in which 2 objects are at rest
Cosmological principle - large scales

- universe is *isotropic and homogeneous* at large scales
- There is no preferred position or direction
- Expansion is identical for all observers
- Is true at scale of inter-galactic distances: $O(\text{Mpc})$

2dF quasar survey
- Two slices in declination and right ascension
- Plot redshift vs right ascension
- Each dot = galaxy
Cosmic Microwave Background photons

WMAP 5 years data

Uniform temperature up to $\Delta T/T \sim 10^{-5}$
Structures - small scales

- Over short distances universe is clumpy
- Galaxy clusters $O(10 \text{ Mpc})$ diameter
- Galaxies, eg Milky Way: 15 kpc

**COMA cluster:** 1000 galaxies
20Mpc diameter
100 Mpc (330 Mly) from Earth
Optical + IR

**Milky Way**
COBE - radio
Olbers’ paradox

• Olber (~1800): Why is the sky not bright at night?
• Suppose that universe is unlimited & filled uniformly with light sources (stars) - total flux expected is $\propto r_{\text{max}}$

➢ The sky does not look dark because:
• Observable universe has finite age – for constant expansion: light can reach us from maximum $r_{\text{max}} = O(ct_0)$
• Stars emit light during finite time $\Delta t$ – flux reduced by factor $\Delta t/t_0$
• Light is redshifted due to expansion - eg red light $\rightarrow$ IR – finally undetectable flows
• CMB fills universe – dark matter does not light up
Robertson-Walker metric 1

- Assume universe = homogeneous and isotropic fluid
  - Distance between 2 ‘events’ in 4-dimensional space-time
    \[ ds^2 = c^2 dt^2 - \left( dx^2 + dy^2 + dz^2 \right) \]

- Universe is expanding
  - Scale factor \( R(t) \), identical at all locations, only dependent on time
    \[ ds^2 = c^2 dt^2 - \left( R(t) \right)^2 \left[ dx^2 + dy^2 + dz^2 \right] \]
  - Proper distance \( D \) from Earth to distant galaxy at time \( t \)
    \[ D(t) = r \cdot R(t) \]
  - \( r \) = co-moving coordinate distance
Stretching of the universe scale

Example: closed universe

\[ R(t_1) \quad R(t_2) \]

© L. Bergström
Robertson-Walker metric 2

- space is not necessarily flat

- Introduce $k =$ spatial curvature of universe
- Distance between 2 events in 4-dim space-time

$$ ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] $$

- $k :$ curvature of space $\rightarrow$ geometry $-$ obtained from observations $-$ can be -1,0,1
- $R(t) :$ dynamics of expansion $\rightarrow$ Friedman-Lemaître equations $-$ model dependent
Curvature $k$ in FLRW model

- $k = +1, 0, -1$ depends on space geometry
- Curvature is independent of time
- Curvature is related to *global mass-energy density* of universe
  \[ \Omega_{\text{energy-matter}}(t) - \frac{kc^2}{H^2(t)R^2(t)} = 1 \]
- It affects the evolution of density fluctuations in CMB radiation – path of photon is different
- Therefore it affects pattern of the CMB radiation today
- For universe as a whole, over large distances, space-time seems to be flat: \( k=0 \text{ from CMB observations} \)
  \[ \rho_{\text{tot}}(t) = \rho_{\text{crit}}(t) \]
Global energy density

- Low density \( \rho / \rho_c < 1 \)
- Critical density \( \rho / \rho_c = 1 \)
- High density \( \rho / \rho_c > 1 \)

Cosmic destiny

- \( k < 0 \) expands forever
- \( k = 0 \) velocity asymptotically zero
- \( k > 0 \) big crunch

Expanding Universe

2012-13
Exercise

- D. Perkins, chapter 5
- Oplossing meebrengen op het examen

Example 5.1  Show that for a curvature term with $k = +1$, the Big Bang would be followed by a Big Crunch at time $t = \frac{2\pi GM}{c^3}$ where $M$ is the (assumed conserved) mass of the universe.
Curvature of universe

CMB photons

$\text{t} = 380.000 \text{ y}$

movie

$k<0$

$k=0$

$k>0$

$z = 1100$

model

$z = 0$

observe

k from CMB observations

2012-13 Expanding Universe
Hubble time – Hubble distance

• Expansion parameter $H(t)$ has dimension of $(\text{time})^{-1}$

• Hubble time = expansion time today for constant expansion

\[
t = \frac{1}{H(t)} \Rightarrow t_0 = \frac{1}{H_0} = 13.9 \times 10^9 \text{ year}
\]

• Horizon distance today in flat static universe (Hubble distance)

\[
D_H(t_0) = c t_0 = 4.2 \text{ Gpc}
\]

• But! Look back time and distance in expanding universe depend on dynamics in $H(t)$ (see part 2)

\[
H(t)^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}}(t) - \frac{k c^2}{(R(t))^2}
\]
Matter dominated flat universe

• Integration of Friedman equation with $k=0$ and only non-relativistic matter

\[
R(t) = \left(\frac{9GM}{2} \right)^{\frac{1}{3}} t^{\frac{2}{3}}
\]

\[
\frac{1}{H_0} = \frac{R(t_0)}{\dot{R}(t_0)} = \frac{3t_0}{2}
\]

\[
t_0 = \frac{2}{3} \frac{1}{H_0} = 9.1 \times 10^9 \text{ yr}
\]

• Dating of earth crust and old stars shows that age of universe is of order $14 \times 10^9$ yr

• → Need other energy components (see part 2)
Part 2
ΛCDM ingredients

Dynamics of the expansion – Friedman Lemaître equations

Energy density of universe
Dynamics of the expansion

• Einstein field equations of general relativity

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu} \]

\[ = 8\pi G_N \left( T_{\mu\nu} - \frac{\Lambda g_{\mu\nu}}{8\pi G_N} \right) \]

Geometry of space-time
Robertson-Walker metric

\[ G_N = 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \]

Energy-momentum
Function of energy content of universe
Is model dependent

• Friedmann & Lemaître (1922) : Solutions for uniform and homogeneous universe behaving as perfect frictionless fluid
Friedmann-Lemaître equation

- Time dependent evolution of universe = Friedmann-Lemaître equation

\[ H(t)^2 \equiv \left( \frac{\dot{R}(t)}{R(t)} \right)^2 = \frac{8\pi G_N}{3} \rho_{tot}(t) - \frac{kc^2}{(R(t))^2} \]

- \( \frac{kc^2}{R^2} \) = curvature term

- \( \rho_{tot}(t) \) = energy density of universe globally, 'at large'

- \( H(t) \) = expansion rate = Hubble 'constant' at time t

\[ H_0 = H(t_0) = (70.2 \pm 1.4) \left( \frac{\text{km/s}}{\text{Mpc}} \right) = \text{Hubble constant today} \]

- energy density \( \rho \) is model dependent - \( \Lambda \text{CDM model} \):

\[ \rho_{tot}(t) = \rho_{\text{baryon}}(t) + \rho_{\text{coldDM}}(t) + \rho_{\text{rad}}(t) + \rho_{\text{DarkEn}} \]
Second Friedman equation

- Consider universe = perfect fluid with energy density $\rho c^2$
- Conservation of energy in volume element $dV$; $P =$ pressure of fluid

\[ dE = -PdV \quad \rightarrow \quad d\left( \rho c^2 \cdot R^3 \right) = -Pd\left( R^3 \right) \quad \Rightarrow \quad \dot{\rho} = -3 \left( P + \rho c^2 \right) \left( \frac{\dot{R}}{Rc^2} \right) \]

- Differentiate 1st Friedman equation & substitute

\[ \ddot{R} = \frac{d\dot{R}}{dt} = -\left( \frac{4\pi G_N R}{3} \right) \left( \rho + \frac{3P}{c^2} \right) \]

- $\Lambda$CDM model: *Energy density consists of 3 components*: matter, radiation (incl. relativistic particles), constant vacuum energy

\[ \rho = \rho_{\text{matter}} + \rho_{\text{radiation}} + \rho_{\text{vacuum}} \quad \text{with} \quad \rho_{\text{vac}} = \frac{\Lambda}{8\pi G_N} = \rho_{\Lambda} \]
Equations of state 1

1. Matter component: ‘dust’ of non-relativistic particles (v<<c) – matter pressure from kinetic energy

\[ P_{\text{non-rel}} = \left( \frac{2}{3} \right) \frac{E_k}{V} = \left( \frac{2}{3} \right) \rho c^2 \times \left( \frac{v^2}{c^2} \right) \quad v \ll c \rightarrow P_{\text{non-rel}} \approx 0 \]

2. Radiation component: radiation & relativistic particles – radiation pressure of ideal gas

\[ P_{\text{rel}} = \frac{1}{3} \frac{E}{V} = \frac{\rho c^2}{3} \]

3. Vacuum energy – negative pressure equivalent to gravitational repulsion

\[ P_{\text{vac}} = -\rho c^2 = c \text{st} \]
Equation of state 2

• Relation between pressure and energy - introduce $w$

\[
\frac{P}{\rho c^2} = w
\]

\[
\rho = -3 (P + \rho c^2) \left( \frac{\dot{R}}{Rc^2} \right)
\]

\[
\dot{\rho} = -3 \rho c^2 (1 + w) \left( \frac{\dot{R}}{Rc^2} \right)
\]

\[
\rho = \text{const} \ R^{-3(1+w)} \propto (1 + z)^{3(1+w)}
\]

• For small $t$ assume that

\[
R \sim t^\beta
\]

\[
R(t) \sim t^{\frac{2}{3(1+w)}}
\]

- radiation $w = \frac{1}{3}$
- matter $w = 0$
- vacuum $w = -1$ & $\rho = \text{const}$
\[ \rho \propto (1 + z)^{3(1+w)} \]

\[ w_{\text{DarkEnergy}} = -1 \pm 0.2 \]
Critical density

• flat universe \( k=0 \) = equilibrium between open and close universe

• Friedmann equation becomes

\[
H^2(t) \equiv \frac{\dot{R}^2(t)}{R^2(t)} = \frac{8\pi \rho_c G_N}{3} \quad \Rightarrow \quad \rho_c = \frac{3H^2}{8\pi G_N}
\]

• density \( \rho_c \) is called **critical density**

• **Value today** is

\[
\rho_c(t_0) = \frac{3H_0^2}{8\pi G_N} = 9.6 \times 10^{-27} \text{ kg m}^{-3}
\]

\[
\rho_c c^2 = 5.4 \text{ GeV/m}^3 \approx 5 \text{ protons at rest per m}^3
\]

• Observations → universe is geometrically flat, so present average density of universe, away from galaxies, is a few protons per m\(^3\)
**closure parameter**

- Define **closure parameter** $\Omega$

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$$

- For general geometry with $k \neq 0$, Friedmann equation becomes

$$\frac{8\pi G_n}{3} = \frac{H^2}{\rho_c} \quad \Rightarrow \quad H^2 = \left(\frac{H^2}{\rho_c}\right) \rho - \frac{k c^2}{R^2}$$

$$1 = \frac{\rho(t)}{\rho_c(t)} - \frac{k c^2}{H^2 R^2} \quad \Rightarrow \quad 1 = \Omega(t) - \frac{k c^2}{H^2(t) R^2(t)}$$

- Define curvature term

$$\Omega_k = \frac{\rho_k}{\rho_c} = -\frac{k c^2}{H^2 R^2}$$

$$\Omega(t) + \Omega_k(t) = 1$$

Observations: flat universe and $\Omega=1$ at all times
pauze
Observed energy densities

- Energy density is composed of: dust/matter (non-rel), radiation (incl. rel particles), vacuum energy ($\Lambda$)

$$\Omega + \Omega_k = (\Omega_m + \Omega_r + \Omega_\Lambda) + \Omega_k = 1$$

- Relative contributions influence dynamics of expansion

- Present-day observations:
  1. Radiation (from CMB energy density)

$$\Omega_{rad}(t_0) = \frac{\rho_{rad}(t_0)}{\rho_c(t_0)} \approx 5 \times 10^{-5}$$

  2. Luminous baryonic matter: p, n, nuclei in stars, gas, dust

$$\rho_{lum} \approx 9 \times 10^{-29} \text{ kg m}^{-3} \quad \Rightarrow \quad \Omega_{lum} = \frac{\rho_{lum}}{\rho_c} \approx 0.01$$
Observed energy densities 2

3. Total baryon density – from fit of the model of primordial nucleosynthesis to measurements of light element abundances (chapter 6)

\[
\begin{align*}
    n_b &\approx 0.26 \text{ baryons m}^{-3} \\
    \rho_b &= 4.0 \times 10^{-28} \text{ kg m}^{-3} \quad \Rightarrow \quad \Omega_b = 0.042 \pm 0.004
\end{align*}
\]

4. Total matter density - from galactic rotation curves, gravitational lensing, etc (chapter 7) - is clumpy in \( \Lambda \)CDM model this is composed of baryons and (mostly cold) dark matter

\[
\rho_m = 2.2 \times 10^{-27} \text{ kg m}^{-3} \quad \Rightarrow \quad \Omega_m = \Omega_B + \Omega_{DM} = 0.24 \pm 0.03
\]

5. Neutrino density – relics of early universe \( \Omega_\nu < 0.004 - 0.02 \)
5. total density: fit of $\Lambda$CDM cosmological model to large set of observation data (CMB, SN, ...) — PDG 2010

$$\Omega_{total} = 1 - \Omega_k = 1.006 \pm 0.006$$

$$k \approx 0$$

6. By subtraction we obtain the vacuum energy density non clumpy – homogeneous effect — PDG 2010

$$\Omega_\Lambda = 0.726 \pm 0.015$$

- Conclusions:
  - 95% of energy content of universe is of unknown type
  - Today most of it is dark vacuum energy
  - See further in chapter 7
Evolution of energy components

- Cosmological redshift is related to the expansion rate

\[ 1 + z = \frac{R(t_0)}{R(t)} \quad z(t_0) = 0 \]

\[ \rho \propto (1 + z)^{3(1+w)} \]

1. Radiation = photons and relativistic particles

\[ \rho c^2 = \frac{E}{V} \sim \frac{h\nu}{R^3} \sim \frac{1}{\lambda R^3} \quad \Rightarrow \quad \left( \rho c^2 \right)_{\text{rad}} \sim \frac{1}{R^4} \]

\[ \frac{\rho c^2(t)}{\rho c^2(t_0)} = \frac{R^4(t_0)}{R^4(t)} \sim (1 + z)^4 \quad \Rightarrow \quad \left( \rho c^2 \right)_{\text{rad}} \sim (1 + z)^4 \]

2. Non-relativistic matter

\[ \rho c^2 = \frac{E}{V} \sim \frac{m}{R^3} \sim \frac{1}{R^3} \quad \Rightarrow \quad \left( \rho c^2 \right)_{\text{m}} \sim \frac{1}{R^3} \sim (1 + z)^3 \]
Evolution of energy components 2

3. Vacuum energy is constant in our model
\[ (\rho c^2)_\Lambda = \text{cst} \]

4. Curvature term
\[ \rho c^2 = \frac{-kc^2}{H^2 R^2} \Rightarrow (\rho c^2)_k \sim \frac{1}{R^2} \sim (1+z)^2 \]

- Dynamics of expansion can be rewritten
\[
H^2 (t) = \frac{8\pi G}{3} \left[ \rho_m (t) + \rho_r (t) + \rho_\Lambda (t) + \rho_k (t) \right] \\
= H_0^2 \left[ \Omega_m (t) + \Omega_r (t) + \Omega_\Lambda (t) + \Omega_k (t) \right] \\
= H_0^2 \left[ \Omega_m (t_0) (1+z)^3 + \Omega_r (t_0) (1+z)^4 + \Omega_\Lambda (t_0) + \Omega_k (t_0) (1+z)^2 \right]
\]
## Summary

<table>
<thead>
<tr>
<th>Dominant regime</th>
<th>Equation of state</th>
<th>Energy density</th>
<th>Scale parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation</td>
<td>$P = \frac{\rho c^2}{3}$</td>
<td>$\rho \propto R^{-4} \propto t^{-2}$</td>
<td>$R \propto t^{1/2}$</td>
</tr>
<tr>
<td>Matter</td>
<td>$P = \left(\frac{2}{3}\right) \rho c^2 \times \left(\frac{v^2}{c^2}\right)$</td>
<td>$\rho \propto R^{-3} \propto t^{-2}$</td>
<td>$R \propto t^{2/3}$</td>
</tr>
<tr>
<td>Vacuum</td>
<td>$P = -\rho c^2$</td>
<td>$\rho = \text{constant}$</td>
<td>$R \propto \exp(\alpha t)$</td>
</tr>
</tbody>
</table>

**Equations:**

- **radiation**: $\rho c^2 \sim (1+z)^4$
- **matter**: $\rho c^2 \sim (1+z)^3$
- **vacuum**: $\rho c^2 \sim \text{cst}$
- **curvature**: $\rho c^2 \sim (1+z)^2$
Lookback time

\[
H = \frac{1}{R} \frac{dR}{dt} \quad \Rightarrow \quad dt = \frac{1}{R} \frac{dR}{H} = \frac{(1+z)}{R(t_0)} \frac{dR}{H}
\]

\[
R(t) = \frac{R(t_0)}{1+z} \quad \Rightarrow \quad dR = R(t_0) \frac{-1}{(1+z)^2} dz
\]

- Time elapsed between emission of a photon at expansion time \(t_E\) by object at redshift \(z_E\) and today

\[
 t_0 - t_E = \int_{t_E}^{t_0} dt = -\int_{z_E}^{0} \frac{dz}{[(1+z)H(z)]}
\]
age of the universe

- Age of universe from integration
  From Big Bang time (\( t=0 \) and \( z=\infty \)) to present time (\( t=t_0 \) and \( z=0 \))

\[
 t_0 - 0 = \int_0^{t_0} dt = -\int_{\infty}^{0} \frac{dz}{(1+z)H(t)}
\]

- Example: flat universe (\( \Omega_k=0 \)) with \( \Omega_m=0.24 \), \( \Omega_\Lambda=0.76 \)

\[
 t_0 = (13.95 \pm 0.4) \text{ Gyr}
\]

Example 5.3: Estimate the age of a flat universe (\( k = 0 \)) if radiation is neglected and it is presently made up of matter with \( \Omega_m=0.24 \) and vacuum energy with \( \Omega_\Lambda = 0.76 \).

- Uitwerking meebrengen op examen
Age of the universe

\[
(\Omega_m + \Omega_r + \Omega_\Lambda) + \Omega_k = 1
\]

\begin{align*}
\Omega_r &= 0 \\
\Omega_k &= 0 \\
\Omega_m &= 0.24 \\
\Omega_\Lambda &= 1 - \Omega_m = 0.76
\end{align*}

\[
H_0t_0 = 1.026 \\
t_0 = (13.95 \pm 0.4) \text{ Gyr}
\]

Only matter
• Extent of region accessible via light signals to comoving observer at given time $t$

• In static flat universe

$$D_H^{static-flat}(t_0) = ct_0 = 4.2 \text{Gpc}$$

• In an expanding FLRW universe, for a photon following a line with fixed $\theta$ and $\phi$

$$ds^2 = 0$$

$$c^2 dt^2 = R^2(t) \left[ \frac{dr^2}{1 - kr^2} \right]$$

• Proper distance travelled by photon emitted at time $t_E$ and comoving position $r_E$

$$D(t_0) = r_E \cdot R(t_0)$$

• Observed at time $t_0$ and position $r=0$
Horizon 2

- Particle horizon for observer at time $t_0$: $t_E \to 0$

\[
D_H(t_0) = \left[ \int_0^{r_E} \frac{dr}{\sqrt{1 - kr^2}} \right] \cdot R(t_0) = R(t_0) \int_0^{t_0} \frac{cdt}{R(t)}
\]

- Expanding, flat, radiation dominated universe

\[
\frac{R(t)}{R(t_0)} = \left( \frac{t}{t_0} \right)^{1/2} \quad D_H(t_0) = 2ct_0
\]

- Expanding, flat, matter dominated universe

\[
\frac{R(t)}{R(t_0)} = \left( \frac{t}{t_0} \right)^{2/3} \quad D_H(t_0) = 3ct_0
\]
Horizon 3

• For a more complex cosmological model

\[
R(t_0) \int_0^r \frac{dr}{\sqrt{1-kr^2}} = -\int \frac{cdz}{H} = \frac{cI(z)}{H_0}
\]

\[
I(z) = \int_0^z \frac{dz}{\left[ \Omega_m(t_0)(1+z)^3 + \Omega_r(t_0)(1+z)^4 + \Omega_\Lambda(t_0) + \Omega_K(t_0)(1+z)^2 \right]^{1/2}}
\]

• For \( \Omega_k = 0, \Omega_m = 0.24, \Omega_\Lambda = 0.76 \) and integration to \( z = \infty \)

\[
D_H^{BB\text{expansion}} \sim 3.3 \frac{c}{H_0} = 3.3 \, ct_0 \sim 14 \, Gpc
\]
Deceleration parameter 1

- Taylor expansion of expansion parameter around $t_0$ (present time)

\[
R(t) = R(t_0) + \dot{R}(t_0)(t-t_0) + \left(\frac{1}{2}\right)\ddot{R}(t_0)(t-t_0)^2 + \cdots
\]

\[
\frac{R(t)}{R(t_0)} = 1 + H(t_0)(t-t_0) - \left(\frac{1}{2}\right)\left[ -\frac{\dddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)} \right] H^2(t_0)(t-t_0)^2 + \cdots
\]

\[
\frac{R(t)}{R(t_0)} = 1 + H_0(t-t_0) - \left(\frac{1}{2}\right)q_0 H_0^2(t-t_0)^2 + \cdots
\]

- Deceleration parameter $q(t)$: acceleration or deceleration of expansion at time $t$
Deceleration parameter 2

• Deceleration parameter \( q(t) \) is related to energy density at time \( t \) – for flat universe \((k=0)\)

\[
q(t) = - \frac{\ddot{R} R}{\dot{R}^2} = - \left( \frac{\ddot{R}}{\dot{R}} \right) \left( \frac{R}{\dot{R}} \right)^2
\]

\[
q(t) = - \left[ - \frac{4\pi G}{3c^2} \left( \rho c^2 + 3P \right) \right] \left( \frac{1}{H^2} \right)
\]

\( q(t) > 0 \Rightarrow \ddot{R} < 0 \Rightarrow \text{deceleration} \)

\( q(t) < 0 \Rightarrow \ddot{R} > 0 \Rightarrow \text{acceleration} \)

• Energy-pressure relation: equations of state for
  – Dust/matter
  – Radiation/relativistic particles
  – Vacuum energy

\[
q(t) = \frac{\Omega_m(t)}{2} + \Omega_r(t) - \Omega_\Lambda(t)
\]
Acceleration or deceleration?

\[ q(t) = \frac{\Omega_m(t)}{2} + \Omega_r(t) - \Omega_\Lambda(t) \]

• A universe dominated by matter decelerates due to gravitational collapse:
  \[ q(t) \sim \Omega_m(t) > 0 \implies \ddot{R} < 0 \]

• A universe dominated by vacuum energy accelerates
  \[ q(t) \approx -\Omega_\Lambda(t) < 0 \implies \ddot{R} > 0 \]

• Observations show that today
  \[ \begin{align*}
  \Omega_r &\ll \Omega_m \\
  \Omega_\Lambda &> \Omega_m
  \end{align*} \implies q(t_0) < 0 \implies \ddot{R}(t_0) > 0 \]
Deceleration in past & acceleration now

Physics Nobel prize 2011
Part 3
Observation data

Redshift measurements
Luminosity distance
SuperNovae Ia
Cosmic Microwave Background
Large galaxy surveys
Light element abundances
Measuring redshift

- Atoms emit light at certain wavelengths – is measured in laboratory (atom at rest)
- Atoms in quasars emit same spectrum but light is redshifted when reaching Earth
- Comparison of observed spectrum with lab spectrum gives redshift $z$
1 + z = \frac{\lambda_{\text{obs}}(t_0)}{\lambda_{\text{em}}(t)}

Examples

Restframe Wavelength (Å)

2000 4000 6000

Relative Observed Flux

z=2.410
z=2.330
z=2.226
z=1.820
z=1.560
z=1.281
z=0.980
z=0.663
z=0.311
z=0.107

2QZ Survey S326_269 z=2.410

Lya  NV  SiIV  CIV  HeII  CII]
Magnitude and distance

• Luminosity distance $D_L$ to star with intrinsic luminosity $L$

$$ F = \frac{L}{4\pi D_L^2} $$

• F = power measured at Earth

• Absolute magnitude $M = $ magnitude at 10pc distance

• Effective magnitude $m$ of star with absolute magnitude $M$

$$ M = -2.5 \log L + \text{cst} $$

$$ m(z) - M = 5\log_{10} \left( \frac{D_L}{1\text{Mpc}} \right) + 25 $$

• Large negative magnitude = bright star: e.g. $m(\text{Sun}) = -26$ ; $m(\text{Sirius, brightest star}) = -1.7$

• Far away object is dimmer and has larger (=smaller negative) $m$
SN Ia as standard candles

- Supernovae Ia are very bright – light from very distant SN can be observed – up to $z=1.7$
- Are hydrogen poor
- All have roughly the same luminosity curve

M101 at 20 Mly
SN2011fe
Flashed on 18 december 2011
SN Ia as standard candles

- 1995 (Hamuy, Philips): relation between peak luminosity and decline rate

\[ \Delta m_{15} \]

Prieto et al.
SN Ia – Nobel Prize 2011

• S. Perlmutter - Supernova Cosmology Project (SCP)  
  http://supernova.lbl.gov/

• A. Riess & B. Schmidt – High-z Supernova Search Team (HZSNS)  
  http://www.cfa.harvard.edu/supernova//HighZ.html

• High statistics SN monitoring  
  – Distance: observed magnitude  
    + absolute magnitude from lightcurve  
  – Redshift from spectra

• 1998: discover accelerated cosmic expansion

18 December 1998  
Breakthrough of the year
Deceleration in past & acceleration now

Perlmutter, 2003

0% matter

100% matter

Lage dichtheid
Versnelde expansie

now

Redshift z

past

$$\text{grote dichtheid}$$

$$\text{Vertraagde expansie}$$

$$R(z = 1) = 0.5 \times R(t_0)$$
CMB in Big Bang model

- early hot universe: radiation (relativistic particles) dominates over matter (dust, non-relativistic particles) & vacuum energy is negligible
- When cooled down below $kT \approx \text{few MeV}$:
  - Free $p, n, e, \gamma$ (+neutrino’s)
  - Plasma of $H^+$ and $He^{++}$ nuclei
  - Matter-radiation thermal equilibrium
    \[ e^- + p \leftrightarrow H + \gamma \]
- When $E < eV$ (ionisation potentials): recombination to atoms
  \[ e^- + p \rightarrow H + \gamma \]
\[ (1+z)_{dec} \approx 1100 \quad \text{and} \quad t_{dec} \approx 380,000 \text{yr} \quad \text{and} \quad T_{dec} \approx 3000K \]
- Matter-radiation decoupling
- Expect to see $\gamma$’s today as uniform Cosmic Microwave Background
- Observed 3K background photon radiation with black body spectrum
Structure formation in early universe?

Anisotropies in atom distribution are ‘frozen’ in CMB

**Last Scattering Epoch**

CMB photons are released

\[ T > 3000K \quad \text{and} \quad T < 3000K \]

© Univ Oregon
What can we learn from CMB

- Density (pressure) anisotropies in photon-baryon fluid at time of decoupling should leave imprint in distribution of \(\gamma\)'s today
- Observe temperature anisotropies \(O(10^{-5})\)

- Growth of anisotropies depends on dark matter & dark energy during expansion
- Angular distance between 2 photon directions depends on curvature of space

- CMB observations probe universe at \(z \sim 1100\)
Large Scale Structures

• Redshift measurements of large samples of galaxies
• Sloan Digital Sky Survey (SDSS 2000-2014): million of galaxies being recorded
• On average uniform population
• Galaxies = baryonic matter
• Density (pressure) anisotropies in photon-baryon fluid at time of decoupling are expected to leave imprint in distribution of baryons today
• Baryon Acoustic Oscillations (BAO) observations probe universe at small $z$ – are sensitive to dark matter
2dF quasar redshift survey 2003

SDSS survey of local sky
Low z – recent universe

Baryon Acoustic Oscillations
Abundances of light elements

• Model of *Big Bang Nucleosynthesis* predicts abundances of light nuclei: H, D, He, Li, ..
• They were formed at t \( \sim 3\) min when \( kT \sim 3\) MeV
• Abundances are measured today – probe expansion model before decoupling
• Allow to measure baryon density \( \Omega_b = 0.042 \pm 0.004 \)
Fit $\Lambda$CDM model to observations

- Fits to observations using SN Ia distance-redshift surveys
- CMB anisotropies survey by WMAP
- Anisotropies in galaxy surveys
Fit $\Lambda$CDM model to observations

<table>
<thead>
<tr>
<th></th>
<th>WMAP5 alone</th>
<th>WMAP5 + BAO + SN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>$0.0227 \pm 0.0006$</td>
<td>$0.0227 \pm 0.0006$</td>
</tr>
<tr>
<td>$\Omega_{cdm} h^2$</td>
<td>$0.110 \pm 0.006$</td>
<td>$0.113 \pm 0.003$</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
<td>$0.74 \pm 0.03$</td>
<td>$0.726 \pm 0.015$</td>
</tr>
<tr>
<td>$n$</td>
<td>$0.963^{+0.014}_{-0.015}$</td>
<td>$0.960 \pm 0.013$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.087 \pm 0.017$</td>
<td>$0.084 \pm 0.016$</td>
</tr>
<tr>
<td>$\Delta _R^2 \times 10^9$</td>
<td>$2.41 \pm 0.11$</td>
<td>$2.44 \pm 0.10$</td>
</tr>
<tr>
<td>$h$</td>
<td>$0.72 \pm 0.03$</td>
<td>$0.705 \pm 0.013$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>$0.80 \pm 0.04$</td>
<td>$0.81 \pm 0.03$</td>
</tr>
<tr>
<td>$\Omega_{m} h^2$</td>
<td>$0.133 \pm 0.006$</td>
<td>$0.136 \pm 0.004$</td>
</tr>
</tbody>
</table>
Expanding universe: content

- **Part 1:** ΛCDM model ingredients: Hubble flow, cosmological principle, geometry of universe
- **Part 2:** ΛCDM model ingredients: dynamics of expansion, energy density components in universe
- **Part 3:** Observation data – redshifts, SN Ia, CMB, LSS, light element abundances - ΛCDM parameter fits
- **Part 4:** Radiation density, CMB
- **Part 5:** Particle physics in the early universe, neutrino density
- **Part 6:** Matter-radiation decoupling
- **Part 7:** Big Bang Nucleosynthesis
- **Part 8:** Matter and antimatter