Structures in the early Universe

Particle Astrophysics chapter 8
Lecture 4
overview

• problems in Standard Model of Cosmology: horizon and flatness problems – presence of structures

• Need for an exponential expansion in the early universe
• Inflation scenarios – scalar inflaton field

• Primordial fluctuations in inflaton field
• Growth of density fluctuations during radiation and matter dominated eras

• Observations related to primordial fluctuations: CMB and LSS
• Galactic and intergalactic magnetic fields as source of structures?
• Problems in Standard Model of Cosmology related to initial conditions required: horizon and flatness problems

PART 1:
PROBLEMS IN STANDARD COSMOLOGICAL MODEL
Galactic and intergalactic B fields

- **Cosmology model assumes uniform & homogeneous medium.**
- Non-uniformity observed in CMB and galaxy distribution.

- ***Is clustering of matter in the universe due to galactic and intergalactic magnetic fields?***
  - At small = stellar scales: intense magnetic fields are only important in later stages of star evolution.
  - At galactic scale: magnetic field in Milky Way $\approx 3\mu G$.
  - At intergalactic scale: average fields of $10^{-7} - 10^{-11}$ G.
  - $\to$ intergalactic magnetic fields probably have very small role in development of large scale structures.

- **Development of large scale structures mainly due to gravity.**
Horizon in static universe

- **Particle horizon** = distance over which one can observe a particle by exchange of a light signal
- Particle and observer are *causally connected*
- In flat universe with $k=0$: on very large scale light travels nearly in straight line

- In a *static universe* of age $t$ photon emitted at time $t=0$ would travel
  \[
  D_{H}^{\text{STATIC}} = ct \quad \text{for} \quad t < 14 \text{ Gyr}
  \]
  
- Would give as horizon distance today the *Hubble radius*
  \[
  D_{H}^{\text{STATIC}}(t_0) = ct_0 = 4.2 \text{ Gpc}
  \]
Horizon in expanding universe

- In *expanding universe*, particle horizon for observer at $t_0$
  \[ D_H(t_0) = \int_0^{t_0} \frac{R(t_0)}{R(t)} c dt \]

- Expanding, flat, radiation dominated universe
  \[ \frac{R(t)}{R(t_0)} = \left( \frac{t}{t_0} \right)^{1/2} \]
  \[ D_H(t_0) = 2ct_0 \]

- Expanding, flat, matter dominated universe
  \[ \frac{R(t)}{R(t_0)} = \left( \frac{t}{t_0} \right)^{2/3} \]
  \[ D_H(t_0) = 3ct_0 \]

- expanding, flat, with $\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$
  \[ D_H(t_0) = 3.3ct_0 \]
  \[ \sim 14\text{Gpc} \]
## Summary lecture 1

<table>
<thead>
<tr>
<th>Dominant regime</th>
<th>Equation of state</th>
<th>Energy density</th>
<th>Scale parameter</th>
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<tbody>
<tr>
<td>Radiation</td>
<td>$P = \frac{\rho c^2}{3}$</td>
<td>$\rho \propto R^{-4} \propto t^{-2}$</td>
<td>$R \propto t^{1/2}$</td>
</tr>
<tr>
<td>Matter</td>
<td>$P = \left(\frac{2}{3}\right) \rho c^2 \times \left(\frac{v^2}{c^2}\right)$</td>
<td>$\rho \propto R^{-3} \propto t^{-2}$</td>
<td>$R \propto t^{2/3}$</td>
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<tr>
<td>Vacuum</td>
<td>$P = -\rho c^2$</td>
<td>$\rho = \text{constant}$</td>
<td>$R \propto \exp(\alpha t)$</td>
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<tr>
<td>$\text{vacuum}$</td>
<td>$\rho c^2 \sim \text{cst}$</td>
</tr>
<tr>
<td>$\text{curvature}$</td>
<td>$\rho c^2 \sim (1 + z)^2$</td>
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Horizon problem

• At time of radiation-matter decoupling the particle horizon was much smaller than now
• Causal connection between photons was only possible within this horizon
• We expect that there was thermal equilibrium only within this horizon
• Angle subtended today by horizon size at decoupling is about 1°
  • Why is CMB uniform (up to factor $10^{-5}$) over much larger angles, i.e. over full sky?
• Answer: short exponential inflation before decoupling
Horizon at time of decoupling

- Age of universe at matter-radiation decoupling
  \[ t_{\text{dec}} = 4 \times 10^5 \text{y} \quad z_{\text{dec}} \approx 1100 \]

- Optical horizon at decoupling
  \[ D_{\gamma\gamma}^{\text{static}}(t_{\text{dec}}) = ct_{\text{dec}} \]
  \[ D_{\gamma\gamma}(t_0) = ct_{\text{dec}} (1 + z_{\text{dec}}) \]

- This distance measured today

- Angle subtended today in flat matter dominated universe
  \[ \theta_{\text{dec}} \sim \frac{D_{\gamma\gamma}(t_0)}{D_H(t_0)} \sim \frac{ct_{\text{dec}} (1 + z_{\text{dec}})}{3c(t_0 - t_{\text{dec}})} \sim 1^\circ \]
Flatness problem 1

- universe is flat today, but has been much flatter in early universe
- *How come that curvature was so finely tuned to $\Omega=1$ in very early universe?*
- Friedman equation rewritten
  \[ |\Omega(t) - 1| = \frac{|k| c^2}{R(t)^2 H(t)^2} \]
  
  - radiation domination: $R(t) \sim t^n$
  - matter domination: $|\Omega(t) - 1| \sim t^{2/3}$

- At time of decoupling: $|\Omega - 1| \sim 10^{-16}$
- At GUT time ($10^{-34}$s): $|\Omega - 1| \sim 10^{-52}$
Flatness problem 2

• How could $\Omega$ have been so closely tuned to 1 in early universe?

• Standard Cosmology Model does not contain a mechanism to explain the extreme flatness

• *The solution is INFLATION*

\[
\begin{align*}
\text{Big Bang} & \quad \text{Planck scale} \\
\text{GUT scale} & \quad \text{today}
\end{align*}
\]

\[
\begin{align*}
t &= 10^{-44} \text{s} \\
t &= 10^{-34} \text{s}
\end{align*}
\]
The solution is INFLATION
The need for an exponential expansion in the very early universe

PART 2

EXPONENTIAL EXPANSION
Accelerated expansion domination of Dark Energy

Steady state expansion Radiation and matter dominate
Described by SM of Cosmo
Constant energy density = exponential expansion

• Assume that at Planck scale the energy density is given by a cosmological constant

$$\rho_{tot} = \rho_\Lambda = \frac{\Lambda}{8\pi G}$$

• A constant energy density causes an exponential expansion

• For instance between $t_1$ and $t_2$ in flat universe

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \rho_\Lambda}{3} = \frac{\Lambda}{3}$$

$$\frac{R_2}{R_1} = e^{H(t_2-t_1)}$$

• $H = \text{constant}$

$$H = H_0 \left(\frac{\rho_\Lambda}{\rho_c}\right)^{1/2}$$
When does inflation happen?

Inflation period

Quantum Gravity

Super Unification

Theories:
- Strings?
- Relativistic/Quantum
- Classical
When does inflation happen?

- **Grand Unification** at GUT energy scale $$(kT)_{\text{GUT}} \sim 10^{16} \text{ GeV}$$
- GUT *time* $t_{\text{GUT}} \sim 10^{-34} \text{ sec}$
- couplings of
  - Electromagnetic
  - Weak
  - Strong

  interactions are the same
- At end of GUT era: *spontaneous symmetry breaking* into electroweak and strong interactions
- Reheating and production of particles and radiation
- Big Bang expansion with radiation domination
How much inflation is needed?

- Calculate backwards size of universe at GUT scale
- after GUT time universe is dominated by radiation

\[ R(t) \sim t^{\frac{1}{2}} \]

\[ \frac{R(t_{\text{GUT}})}{R(t_0)} \sim \left( \frac{t_{\text{GUT}}}{t_0} \right)^{\frac{1}{2}} \]

- If today

\[ R(t_0) \sim c t_0 = c \times \left( 4 \times 10^{17} \text{ s} \right) \sim 4 \times 10^{26} \text{ m} \]

- Then we expect

\[ R(t_{\text{GUT}}) \sim R(t_0) \times \left( \frac{10^{-34} \text{ s}}{4 \times 10^{17} \text{ s}} \right)^{\frac{1}{2}} \]

\[ R^{\text{EXPECTED}} (t_{\text{GUT}}) \sim 1 \text{ m} \]

- On the other hand horizon distance at GUT time was

\[ D_H(t_{\text{GUT}}) \sim c t_{\text{GUT}} \sim 10^{-26} \text{ m} \]
How large is exponential expansion?

- It is postulated that *before inflation the universe radius was smaller than the horizon distance*
- Size of universe before inflation $R_1 \leq 10^{-26} m$
- Inflation needs to increase size of universe by factor
  \[
  \frac{R_2(GUT)}{R_1(\text{start inflation})} \approx \frac{1 m}{10^{-26} m} = 10^{26} \approx e^{60}
  \]

$H(t_2 - t_1) > 60$

At least 60 e-folds are needed
horizon problem is solved

- Whole space was in causal contact and thermal equilibrium before inflation
- Some points got disconnected during exponential expansion
- entered into our horizon again at later stages

• → horizon problem solved
Flatness problem is solved

- If curvature term at start of inflation is roughly 1

\[
\frac{k c^2}{R(t_1)^2 H(t_1)^2} = O(1)
\]

- Then at the end of inflation the curvature is even closer to 1

\[
\frac{k c^2}{R(t_2)^2 H^2} = \left(\frac{1}{R_2} \frac{1}{R_1}\right)^2 = e^{2H(t_1-t_2)} \approx 10^{-52}
\]

\[
\Omega = 1 \pm 10^{-52} \text{ at } t = 10^{-34} \text{ s}
\]
Conclusion so far

- Constant energy density yields exponential expansion
- This solves horizon and flatness problems
- What causes the exponential inflation?
Scalar inflaton field

PART 2
INFLATION SCENARIOS
Introduce a scalar inflaton field

• Physical mechanism underlying inflation is unknown

• Postulate by Guth (1981): inflation is caused by an unstable scalar field in the very early universe
• dominates during GUT era \((kT \approx 10^{16} \text{ GeV})\) – spatially homogeneous – depends only on time : \(\phi(t)\)
• ‘inflaton field’ \(\phi\) describes a particle with mass \(m\)
• Scalar field causes negative pressure needed for exponential growth
• Potential \(V(\phi)\) due to inflaton field
Chaotic inflation scenario (Linde, 1982)

- Inflation starts at different field values in different locations
- Inflaton field is displaced from true minimum by some arbitrary mechanism, probably quantum fluctuations
- Potential $V(\phi)$ is slowly varying with $\phi$ – slow roll approximation
- Inflation ends when field $\phi$ reaches minimum of potential
- Transformation of potential energy to kinetic energy produces particles = reheating

- Different models can fulfill this inflation scenario
Start from quantum fluctuations

Field starting in B needs more time to reach minimum than field starting in A

Potential should vary slowly

Quantum fluctuations set randomly starting value in A or B
Equation of motion of scalar field

- Lagrangian energy of inflaton field

\[ L(\phi) = T - V = R^3 \left[ \frac{\dot{\phi}^2}{2} - V(\phi) \right] \]

\[ \dot{\phi} = \frac{d\phi}{dt} \]

- Mechanics (Euler-Lagrange): \textit{equation of motion of inflaton field}

\[ \ddot{\phi} + 3H(t)\dot{\phi} + \frac{dV}{d\phi} = 0 \]

\textit{Frictional force due to expansion}

- = oscillator/pendulum motion with damping
- field oscilllates around minimum
- stops oscillating = reheating
Slow roll $\rightarrow$ exponential expansion

• At beginning of inflation kinetic energy $T$ is small compared to potential energy $V$ – field varies slowly
  \[
  \ddot{\phi} = 0 \quad \dot{\phi} \text{ small} \quad T \ll V \quad \phi \approx \phi_0 = \text{cst}
  \]

• Total energy in universe = potential energy of field
  \[
  \rho_\phi c^2 = \frac{T + V}{R^3} = \frac{\dot{\phi}^2}{2} + V(\phi) \quad \rho_\phi c^2 \sim V(\phi_0)
  \]

• Friedman equation for flat universe becomes
  \[
  H^2 = \left(\frac{\dot{R}}{R}\right)^2 = 8\pi G \rho_\phi = \frac{8\pi V(\phi_0)}{3M_{PL}^2} \quad H^2 \sim \text{cst}
  \]
• When the field reaches the minimum, potential energy is transformed in kinetic energy
  • This is reheating phase
  • Relativistic particles are created
  • Expansion is now radiation dominated
  • Hot Big Bang evolution starts
Quantum fluctuations in early universe as seed for present-day structures

PART 3
PRIMORDIAL FLUCTUATIONS
Quantum fluctuations yield anisotropies

Field starting in B needs more time to reach minimum than field starting in A

Potential should vary slowly

Reaches minimum $\Delta t$ later

Quantum fluctuations set randomly starting value in A or B
Quantum fluctuations as seed

- quantum fluctuations in inflaton field
- → fluctuations in inflation end time $\Delta t$
- Energy fluctuations $\Delta E$ are possible within

- Exponential inflation: *particles and anti-particles are inflated apart outside horizon*
- Quantum fluctuations are ‘frozen-in’ → can be treated as classical perturbations
- → *Superposition of waves*
Wavelength is much larger than horizon at end of inflation
inflaton field as cosmic fluid

• density fluctuations *modify the energy-momentum tensor*
• and therefore also the curvature of space
• The modification is seen over scale $\lambda$
• Scale can be any size within $R(t)$, even larger than horizon
• *Change in curvature of space-time yields change in gravitation potential*
• Particles created during reheating will ‘fall’ in gravitational potential wells

![Diagram showing density fluctuations and gravitational potential wells](image)
Gravitational potential fluctuations

- Gravitational potential $\Phi$ due to energy density $\rho$

  Spherical symmetric case

  $$\Phi(r) = \frac{2\pi G \rho r^2}{3}$$

- 2 cases
  - Global ‘background’ potential from ‘average’ field $\phi$
  - Potential change due to fluctuation $\Delta \rho$ over arbitrary scale $\lambda$

  $$\Phi = \frac{2\pi G \rho}{3H^2}$$

  $$\Delta \Phi = 2\pi G (\Delta \rho) \frac{\lambda^2}{3}$$

- Fractional perturbation in gravitational potential at certain location

  $$\frac{\Delta \Phi}{\Phi} = H^2 \lambda^2 \frac{\Delta \rho}{\rho}$$
Spectrum of fluctuations

- During inflation Universe is effectively in stationary state
- Fluctuation amplitudes will not depend on space and time

\[ \frac{\Delta \Phi}{\Phi} \sim \text{cst} \quad H^2 \sim \text{cst} \]

- Define *rms amplitude of fluctuation*

\[ \sqrt{\langle \delta^2 \rangle} = \left( \frac{\Delta \rho}{\rho} \right)_{\text{rms}} \]

\[ \frac{\Delta \Phi}{\Phi} = H^2 \lambda^2 \frac{\Delta \rho}{\rho} \]

\[ \sqrt{\langle \delta^2 \rangle} \sim \frac{1}{\lambda^2} \]

- Amplitude is independent of scale \( R(t) \) at which perturbation enters horizon
Clustering of particles at end of inflation – role of CDM
Evolution of fluctuations during radiation dominated era
Evolution of fluctuations during matter dominated era
Damping effects

PART 4
GROWTH OF STRUCTURE
Introduction

• We assume that the structures observed today in the CMB originate from tiny fluctuations in the cosmic fluid during the inflationary phase.
• These perturbations grow in the expanding universe.
• Will the primordial fluctuations of matter density survive the expansion?
Jeans length = critical dimension

- After reheating *matter condensates around primordial density fluctuations*

- universe = gas of relativistic particles

- Consider a cloud of gas around fluctuation $\Delta \rho$

- When will this lead to condensation?

- *Compare cloud size $L$ to Jeans length $\lambda_J$*

  $\lambda_J = v_s \left( \frac{\pi}{G \rho} \right)^{1/2}$

  Depends on density $\rho$ and sound speed $v_s$

- *Sound wave* in gas = pressure wave

- *Sound velocity* in an ideal gas

  $v_s = \sqrt{\frac{\gamma p}{\rho}}$

  $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$
Jeans length = critical dimension

• Compare cloud size $L$ with Jeans length

• when $L \ll \lambda_j$: sound wave travel time < gravitational collapse time – no condensation
  \[ t = \frac{L}{v_s} \]

• When $L \gg \lambda_j$: sound cannot travel fast enough from one side to the other
  • cloud condenses around the density perturbations
  • Start of structure formation
Need cold dark matter

• Start from upward density fluctuation $\Delta \rho$

• *Density contrast evolution* after decoupling in flat matter dominated universe

$$
\delta = \frac{\Delta \rho}{\rho} = -\frac{3 \Delta R}{R} \sim \left( \frac{3}{10} \frac{c^2}{GM} \right) R(t)
$$

$$
\frac{\delta(t_0)}{\delta(z)} \propto (1 + z)
$$

• Assume photons and baryons have same fluctuations at decoupling (adiabatic fluctuations)

• $z(\text{dec}) = 1100$ and density contrast at decoupling $= 10^{-5}$

• Expect density contrast in matter today $\delta(t_0) \sim 10^{-2}$

• Observe from galaxy distributions: much larger value

• *Need CDM in addition to baryons*
Evolution of fluctuations in radiation era

- After inflation: photons and relativistic particles
- Radiation dominated till decoupling at $z \approx 1100$
- Velocity of sound is relativistic

$$p = \frac{\rho c^2}{3} \quad v_s = \sqrt{\frac{5}{3}} \frac{p}{\rho} \sim \sqrt{\frac{p}{\rho}} \quad \Rightarrow \quad v_s \sim \frac{c}{\sqrt{3}}$$

- Horizon distance $\sim$ Jeans length: no condensation of matter

$$D_H(t) = ct \quad \lambda_J = v_s \left(\frac{\pi}{G\rho}\right)^{1/2} = ct \left(\frac{32\pi}{9}\right)^{1/2}$$

- Density fluctuations inside horizon survive – fluctuations continue to grow as $R(t)$
- But: photon and neutrino components have damping effect
Evolution of fluctuations in matter era

• After decoupling: dominated by non-relativistic matter
• Neutral atoms (H) are formed – sound velocity drops

\[ v_s = \sqrt{\frac{5}{3} \frac{p}{\rho}} \text{ ideal gas} = \sqrt{\frac{5}{3} \frac{kT}{m}} \]

\[ \left( \frac{v_s}{c} \right)_{\text{mat}} = \left( \frac{5kT}{3M_Hc^2} \right)^{\frac{1}{2}} \approx 2 \times 10^{-5} \]

• Jeans length becomes smaller
• therefore faster gravitational collapse
• Matter structures can grow

• Cold dark matter plays vital role in growth of structures
Damping by photons

• *Fluctuations are most probably adiabatic*: baryon and photon densities fluctuate together
• Photons have nearly light velocity - have higher energy density than baryons and dark matter
• If time to stream away from denser region of size $\lambda$ is shorter than life of universe $\rightarrow$ move to regions of low density
• Result: reduction of density contrast $\rightarrow$ *diffusion damping* (*Silk damping*)
Damping by neutrinos

- Neutrinos = relativistic hot dark matter
- In early universe: same number of neutrinos as photons
- Have only weak interactions – no interaction with matter as soon as $kT$ below 3 MeV
- Stream away from denser regions = collisionless damping
- Velocity close to $c$: stream up to horizon – new perturbations coming into horizon are not able to grow – ‘iron out’ fluctuations
Observations

The 2dF Quasar Redshift Survey

Billions of light years

Redshift

\[ \left( \frac{\Delta \rho}{\rho} \right)_{rms} \]

Lumpy

Smooth

\[ \text{Large scales} \]

\[ \lambda (\text{Mpc}) \]

2011-12 Structures in early Universe
From density contrast to temperature anisotropy
Angular power spectrum of anisotropies
Experimental observations

PART 5
OBSERVATIONS OF STRUCTURES
CMB temperature map

Contrast enhanced to make $10^{-5}$ variations visible!
Small angle anisotropies

- *Adiabatic fluctuations at end of radiation domination*: photons & matter fluctuate together
- Photons keep imprint of density contrast
  \[
  \left( \frac{\Delta T}{T} \right)_{\text{Adiab}} = \left( \frac{1}{3} \right) \frac{\Delta \rho}{\rho}
  \]
- Horizon at decoupling is seen now within an angle of $1^\circ$
- Temperature correlations at angles $< 1^\circ$ are due to primordial density fluctuations
- Seen as *acoustic peaks* in CMB angular power distribution
- Peaks should all have same amplitude – but: *Silk damping* by photons
\[ \sqrt{\left< \delta_{\lambda}^2 \right>} = \left( \frac{\Delta \rho}{\rho} \right)_{rms} \sim \frac{1}{\lambda^2} \]

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Angular spectrum of anisotropies

- CMB map = map of temperatures after subtraction of dipole and galactic emission → extra-galactic photons
- *Statistical analysis of the temperature variations* in CMB map – multipole analysis
- In given direction $\mathbf{n}$ measure difference with average of 2.7K

$$\Delta T(\vec{n}) = T(\vec{n}) - T_0$$

$C(\theta) = \text{correlation between temperature fluctuations in 2 directions (n,m) separated by angle } \theta$

average over all pairs with given $\theta$

$$C(\theta) = \left< \frac{\Delta T(\vec{n})}{T_o} \frac{\Delta T(\vec{m})}{T_o} \right>$$
Angular spectrum of anisotropies

• Expand in Legendre polynomials, integrate over $\phi$

$$C(\theta) = \frac{1}{4\pi} \sum_l (2l + 1) C_l P_l(\cos \theta)$$

• $C_l$ describes amplitude of correlations

• Temperature fluctuations $\Delta T$ are superposition of fluctuations at different scales ($\lambda$)

• Decompose in *spherical harmonics*

$$C_l = \frac{1}{2l+1} \sum_m (-1)^m |a_{lm}|^2$$

$$\Delta T(\mathbf{n}) = T_0 \sum_l \sum_m a_{lm} Y_{lm}$$

$$C(\theta) = \langle \frac{\Delta T(\mathbf{n})}{T_0} \frac{\Delta T(\mathbf{m})}{T_0} \rangle$$
Angular spectrum of CMB anisotropies

- Coefficients $C_\ell = \text{angular power spectrum} - \text{measure level of structure found at angular separation of}$

\[
\theta \approx \frac{\pi}{\ell} \approx \frac{200}{\ell} \text{ degrees}
\]

- Large $\ell$ means small angles
- $\ell = 200$ corresponds to $1^{\circ} = \text{horizon at decoupling as seen today}$
- Amplitudes seen today depend on:
  - primordial density fluctuations
  - Baryon/photon ratio
  - Amount of dark matter
  - ....

Fit of $C_\ell$ as function of $\ell$ yields cosmological parameters
Position and height of peaks

• Position and height of acoustic peaks depend on curvature, matter and energy content and other cosmological parameters.
Physics of Young universe
Large wavelengths

Physics of primordial fluctuations
Short wavelengths

Angular Scale

TT Cross Power Spectrum

1° = horizon at decoupling

Sachs-Wolfe effect

Silk damping
overview

• problems in Standard Model of Cosmology: horizon and flatness problems – presence of structures

• Need for an exponential expansion in the early universe
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• Primordial fluctuations in inflaton field
• Growth of density fluctuations during radiation and matter dominated eras

• Observations related to primordial fluctuations: CMB and LSS
Examen

• Book Perkins 2\textsuperscript{nd} edition chapters 5, 6, 7, 8, + slides
• Few examples worked out: ex5.1, ex5.3, ex6.1, ex7.2
• ‘Open book’ examination