

# AXEL-2018

## Introduction to Particle Accelerators

### Transverse instabilities:

- ✓ *How do they arise*
- ✓ *Single-bunch effects (“head-tail” instability)*
- ✓ *Multi-bunch modes (very brief)*
- ✓ *Possible cures*
- ✓ *Space charge effects*

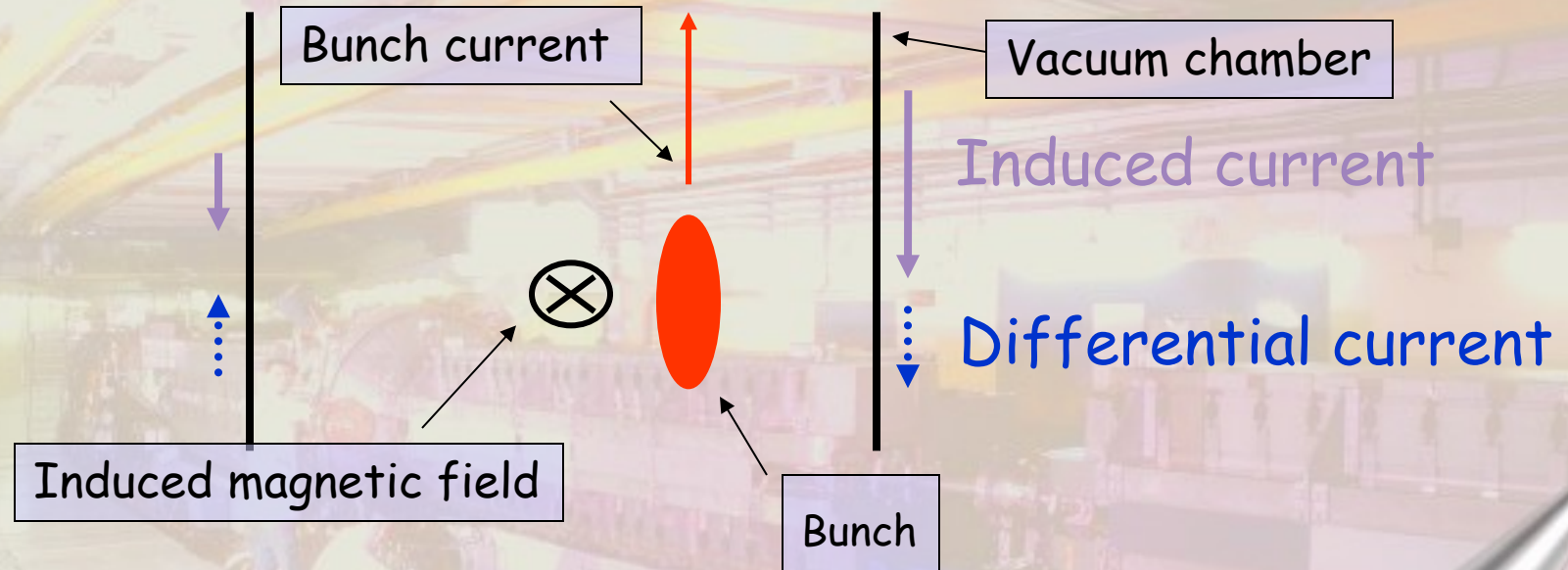
Rende Steerenberg (BE/OP)

9 March 2018

# Coherent Transverse Oscillation (1)

- # The complete bunch is displaced from side to side (or up and down)
- # A bunch of charged particles induces a charge in the vacuum chamber
- # This creates an image current in the vacuum chamber walls
- # How can these currents affect transverse motion?

# Coherent Transverse Oscillation (2)



- # If the bunch is displaced from the centre of the vacuum chamber it will drive a differential wall current
- # This leads to a magnetic field, which deflects the bunch



# Transverse coupling impedance (1)

- # We characterize the electromagnetic response to the bunch by a “transverse coupling impedance” (as for longitudinal case)

$$\int (Z_{\perp}(\omega) \times I(\omega)) d\omega = \int_0^S (E + v \times B) ds$$

Frequency spectrum  
of bunch current

Transverse E & B fields  
summed around the machine

- #  $Z_{\perp}$  (exactly as  $Z_{\parallel}$ ) is also a function of frequency
- #  $Z_{\perp}$  also has resistive, capacitive and inductive components
- # However, there is one big difference between  $Z_{\perp}$  &  $Z_{\parallel}$

# Transverse coupling impedance (2)

- # For a vacuum chamber with a short non-conducting section the direct image current sees a high impedance (large  $Z_{\parallel}$ )



- # For The differential current (current loops) is not greatly affected so  $Z_{\perp}$  is unchanged by the non-conducting section
- # Thus:
  - Any interruption to a smooth vacuum chamber increases  $Z_{\parallel}$
  - Any structure that will support current loops increases  $Z_{\perp}$

# Relationship with the longitudinal plane

- # **Longitudinal instabilities** are related to **synchrotron** oscillations
- # **Transverse instabilities** are related to **synchrotron and betatron** oscillations
- # Why....?....
- # Particles move around the machine and execute synchrotron and betatron oscillations
- # If the chromaticity  $\left( \xi = \frac{\Delta Q}{Q} / \frac{\Delta p}{p} \right)$  is non zero
- # Then the changing energy, due to synchrotron oscillations will also change the betatron oscillation frequency (Q)

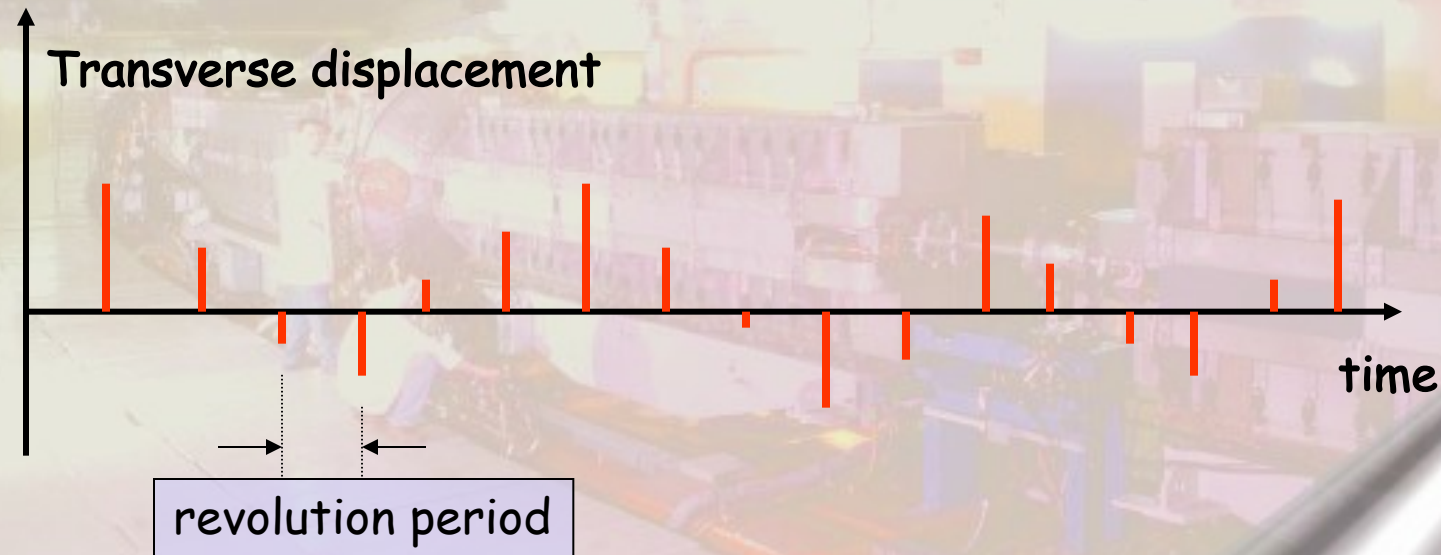


# Single bunch modes

- # As for longitudinal oscillation there are different modes for single bunch transverse oscillations
- # We can observe the transverse bunch motion from the difference signal on a position monitor

# Rigid bunch mode (1)

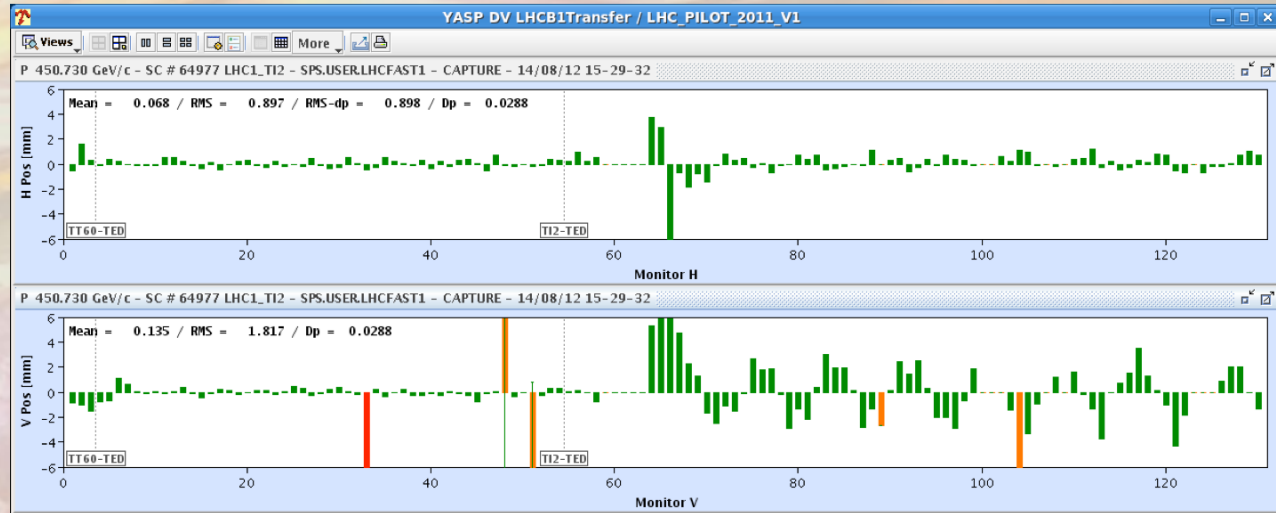
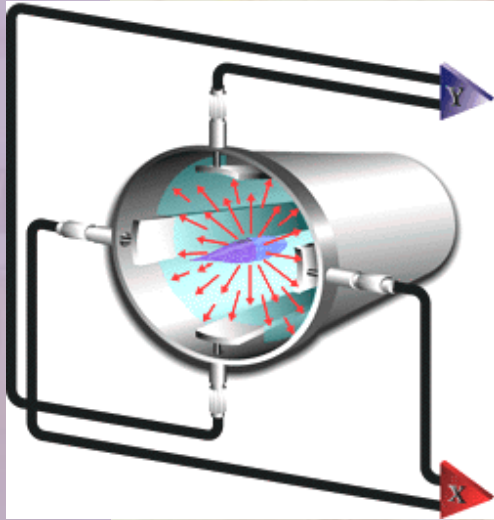
- # The bunch oscillates transversely as a rigid unit
- # On a single position sensitive pick-up we can observe the following:



Change in position/turn  $\Rightarrow$  betatron phase advance/turn



# Beam Position Measurement



$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta}{\Sigma}$$

# Rigid bunch mode (2)

Transverse displacement



Lets superimpose successive turns

# Rigid bunch mode (3)

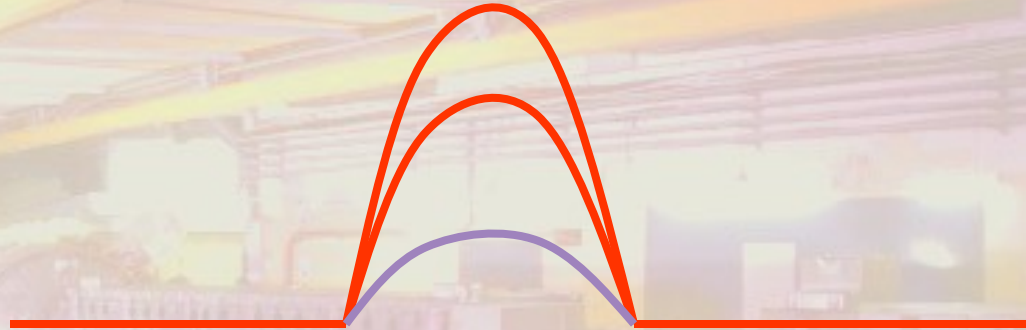
Transverse displacement





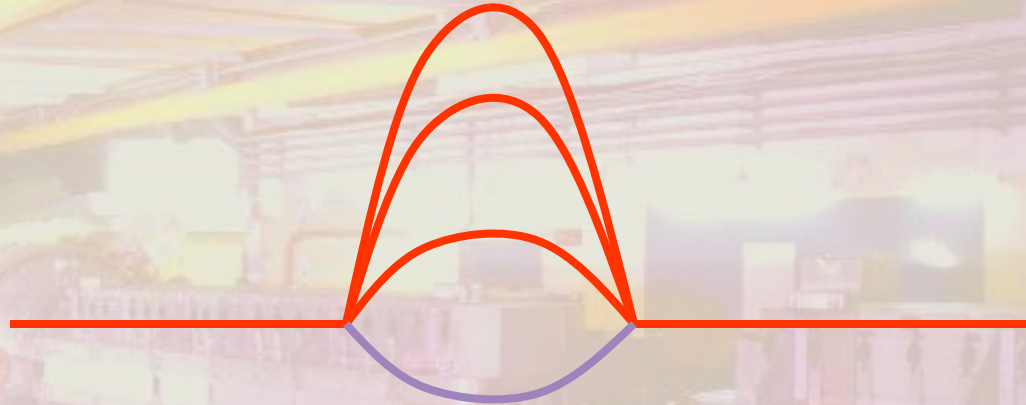
# Rigid bunch mode (4)

Transverse displacement



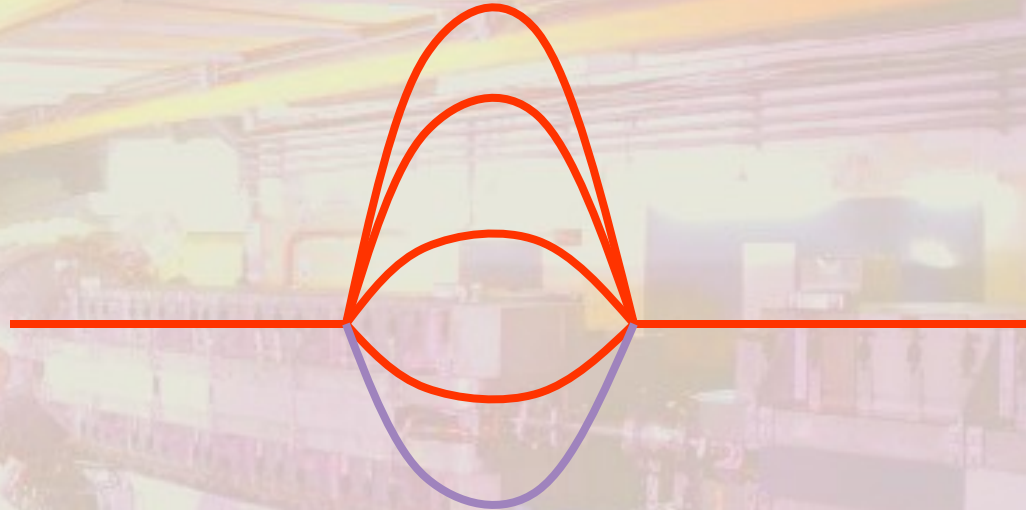
# Rigid bunch mode (5)

Transverse displacement



# Rigid bunch mode (6)

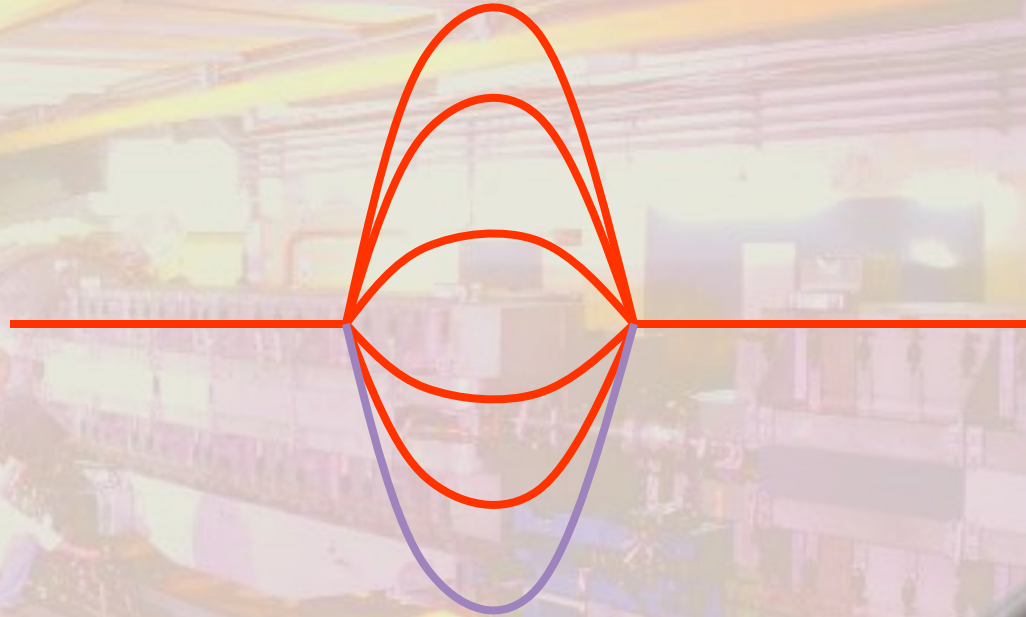
Transverse displacement





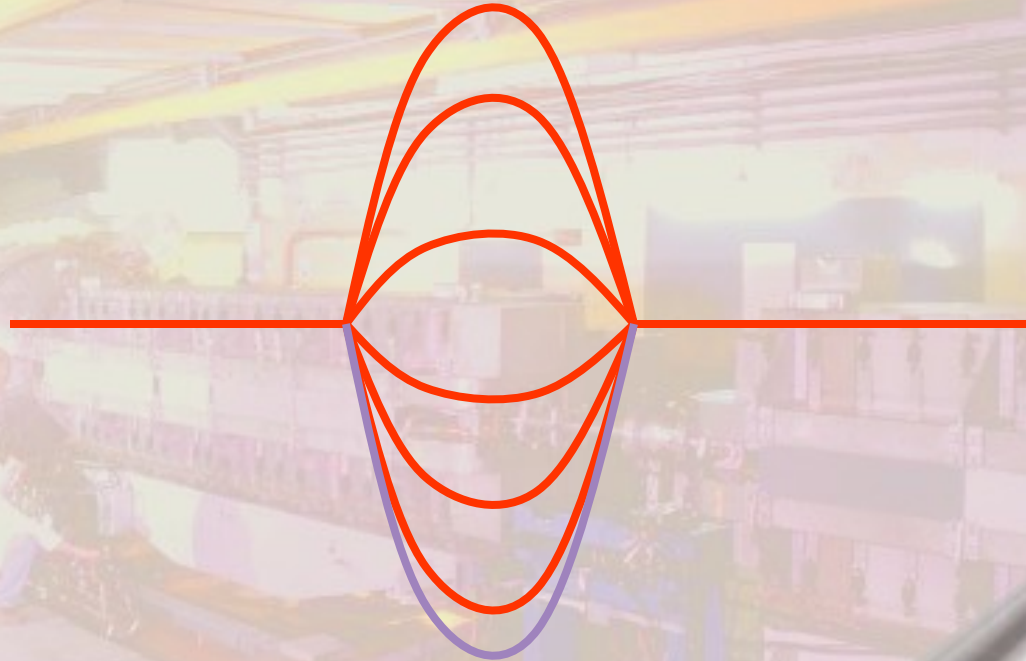
# Rigid bunch mode (7)

Transverse displacement



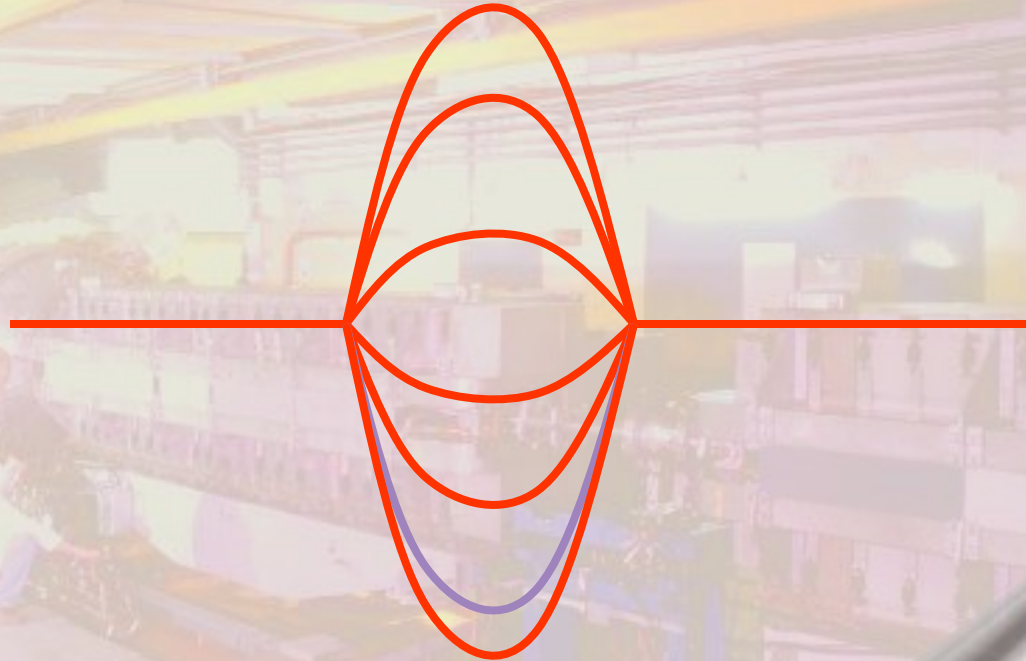
# Rigid bunch mode (8)

Transverse displacement



# Rigid bunch mode (9)

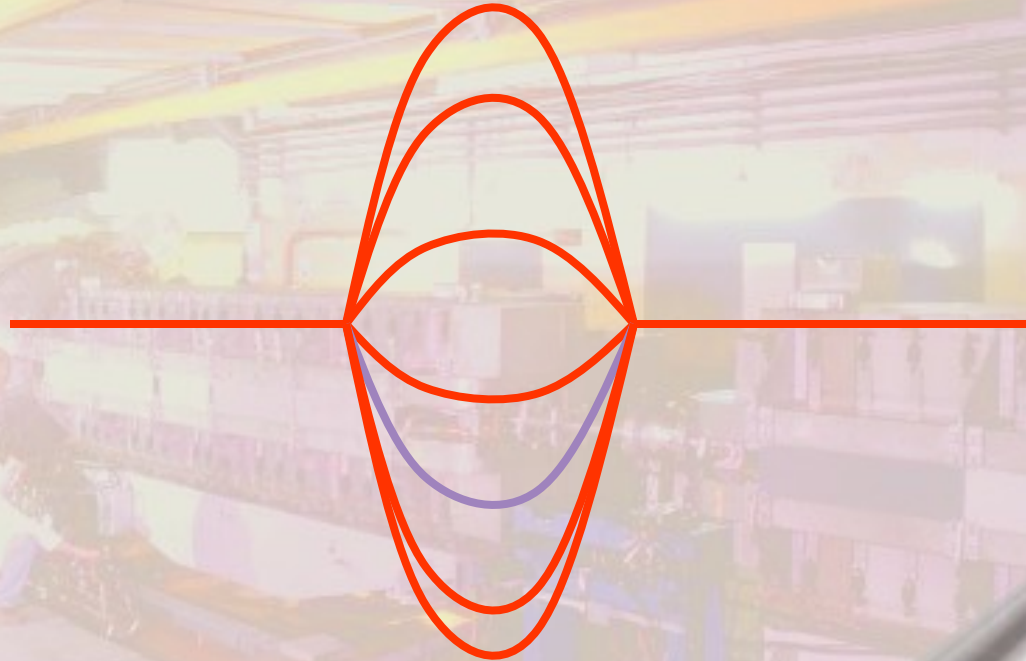
Transverse displacement





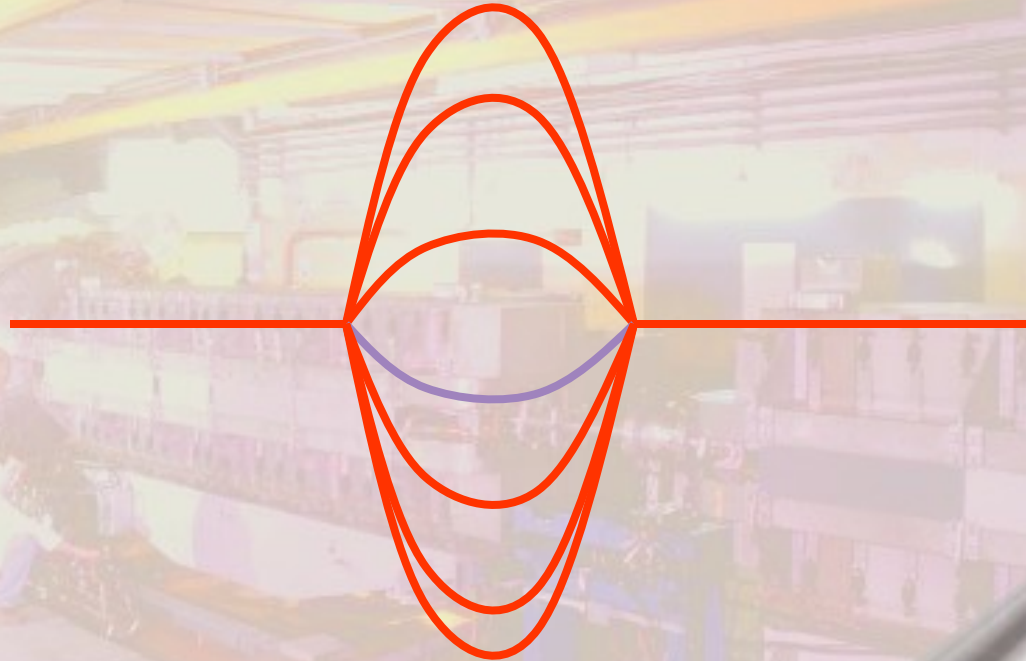
# Rigid bunch mode (10)

Transverse displacement



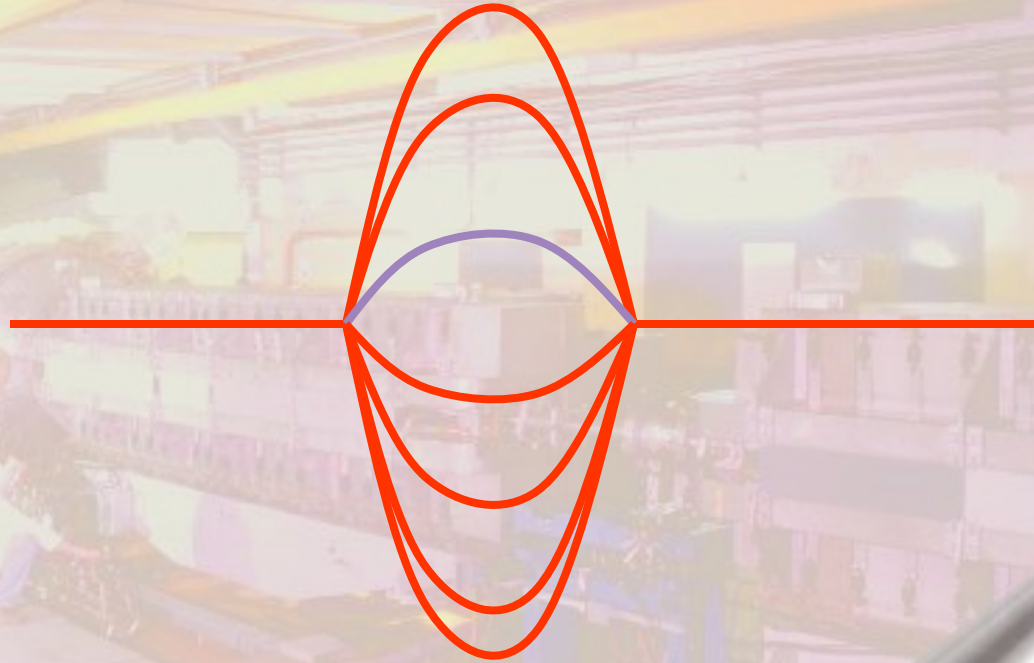
# Rigid bunch mode (11)

Transverse displacement



# Rigid bunch mode (12)

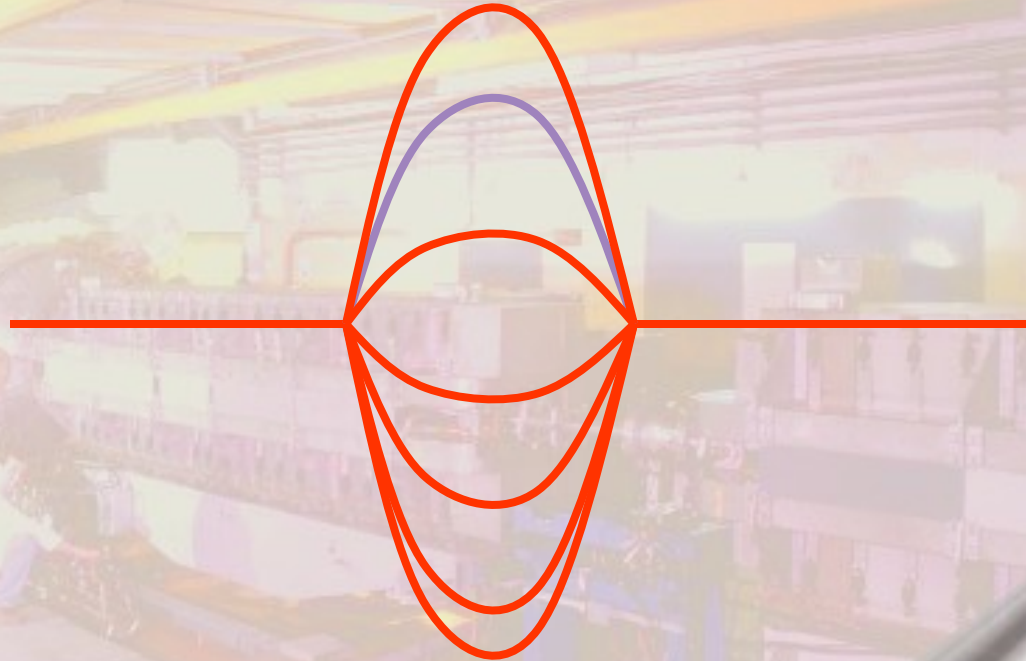
Transverse displacement





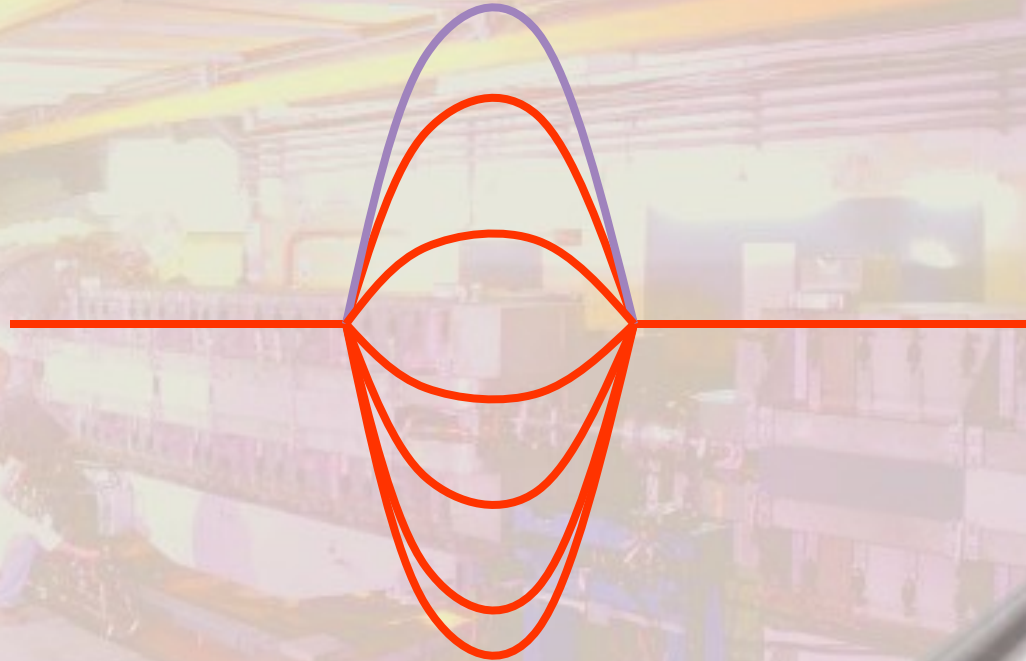
# Rigid bunch mode (13)

Transverse displacement



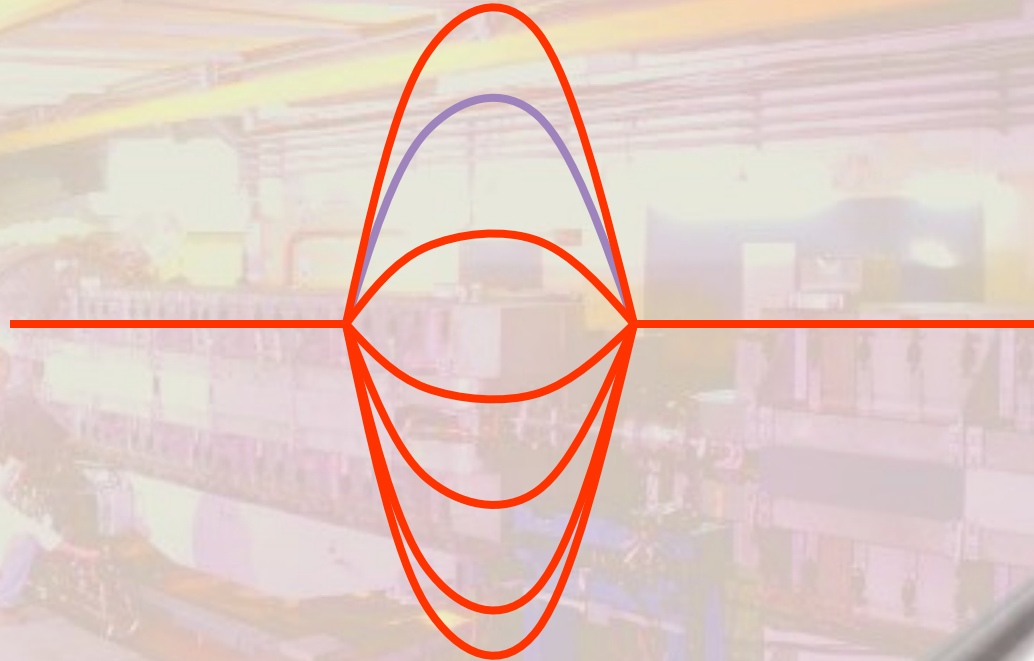
# Rigid bunch mode (14)

Transverse displacement



# Rigid bunch mode (15)

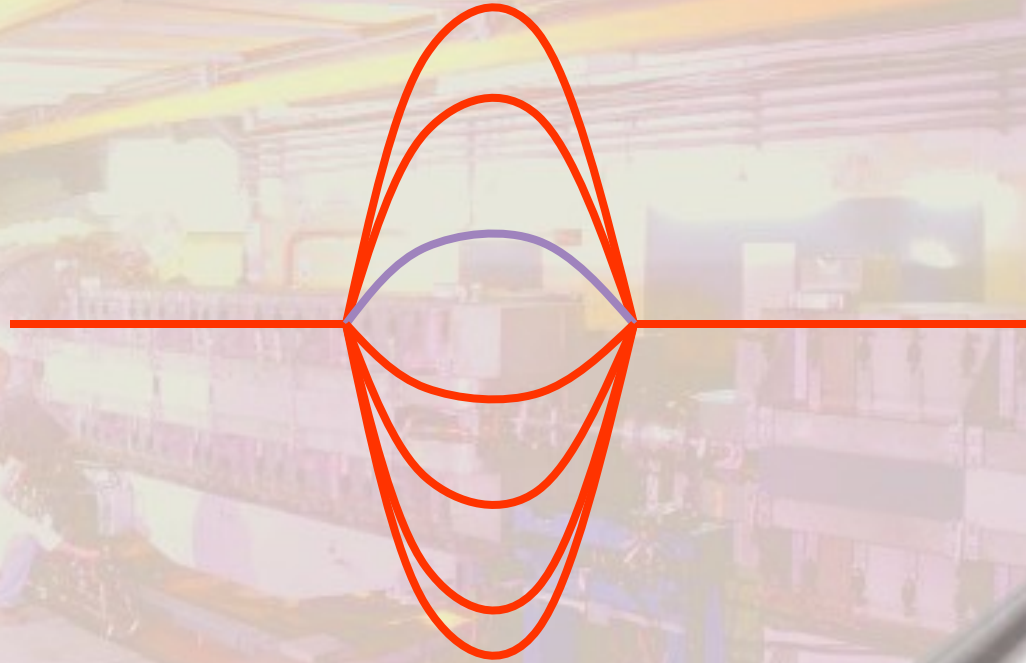
Transverse displacement





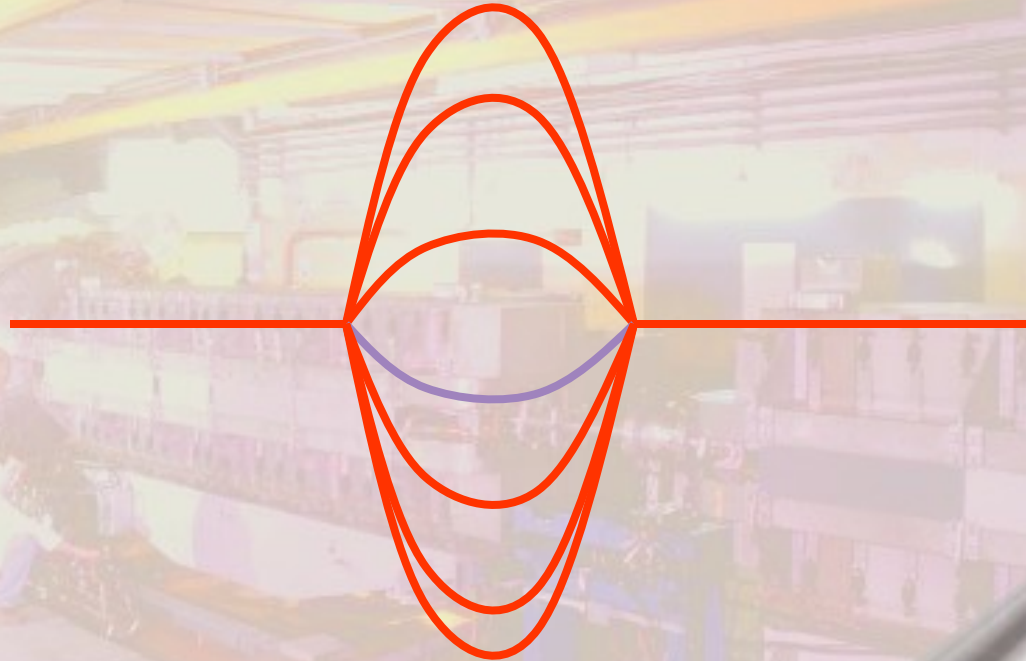
# Rigid bunch mode (16)

Transverse displacement



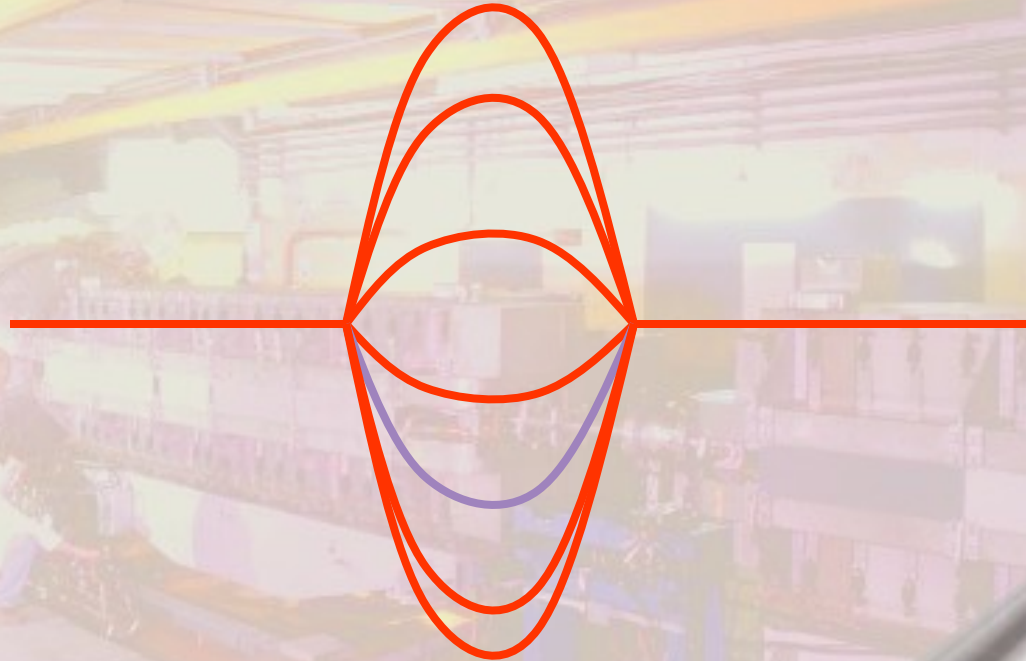
# Rigid bunch mode (17)

Transverse displacement



# Rigid bunch mode (18)

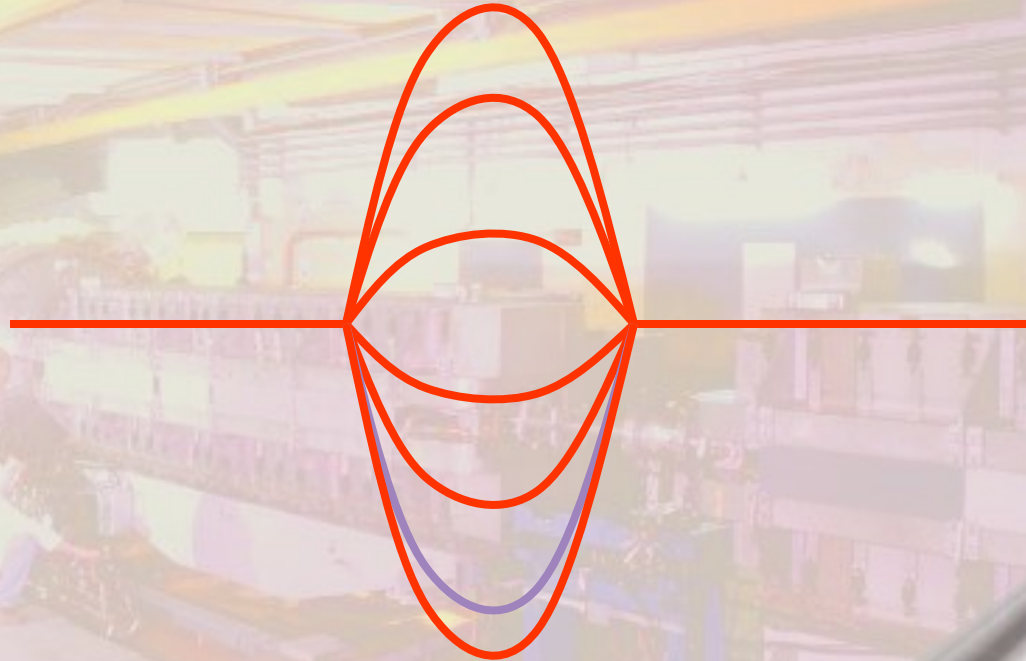
Transverse displacement





# Rigid bunch mode (19)

Transverse displacement



Standing wave without node  $\Rightarrow$  Mode M=0

# Cure for rigid bunch mode instability

- # To help avoid this instability we need a **non-zero chromaticity**

$$\left( \xi = \frac{\Delta Q}{Q} / \frac{\Delta p}{p} \right)$$

- # The bunch has an **energy/momentum spread**

- # The Particles will have a **spread in betatron frequencies**

- # A **spread in betatron frequencies** will mean that any coherent transverse oscillation (all particles moving together) will very quickly become **incoherent** again.

# Higher order bunch modes

- # Higher order modes are called “Head-tail” modes as the electro-magnetic fields induced by the head of the bunch excite oscillation of the tail
- # However, these modes may be harder to observe as the centre of gravity on the bunch may not move.....
- # Nevertheless, they are very important and cannot be neglected

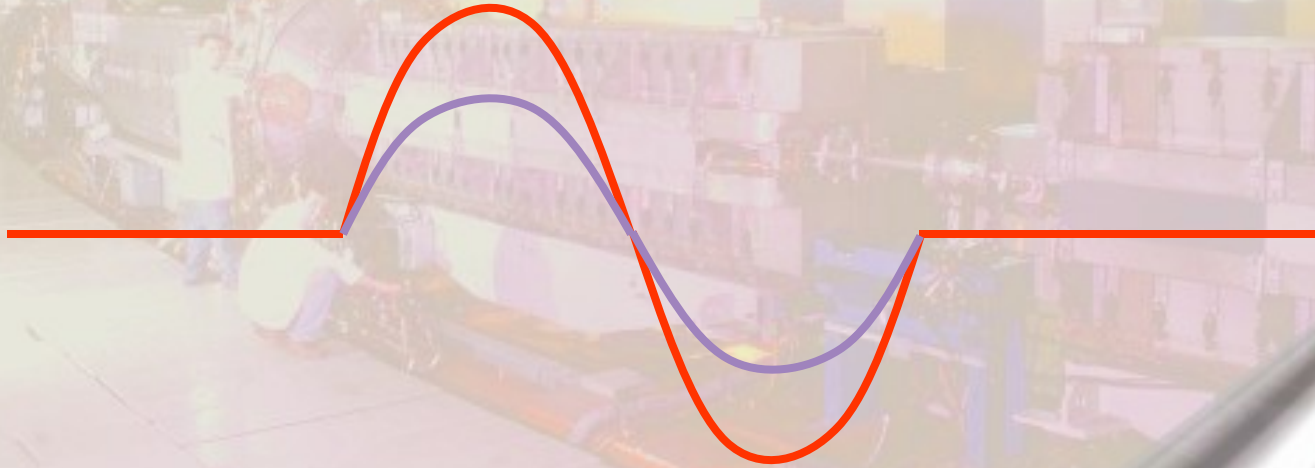


# Head-tail modes (1)

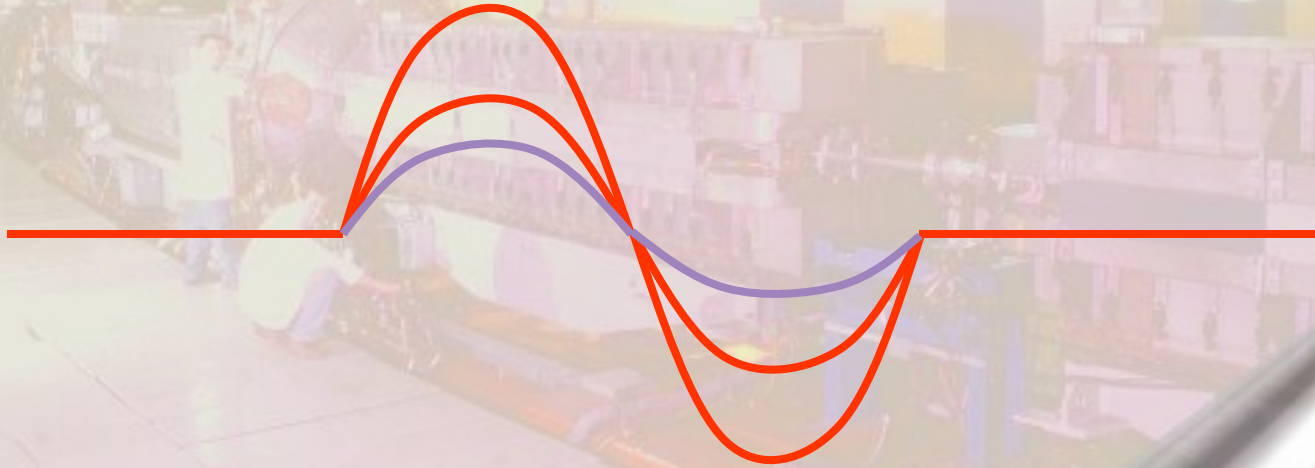
- # Head & Tail of bunch move  $\pi$  out of phase with each other
- # Again, lets superimpose successive turns



# Head-tail modes (2)

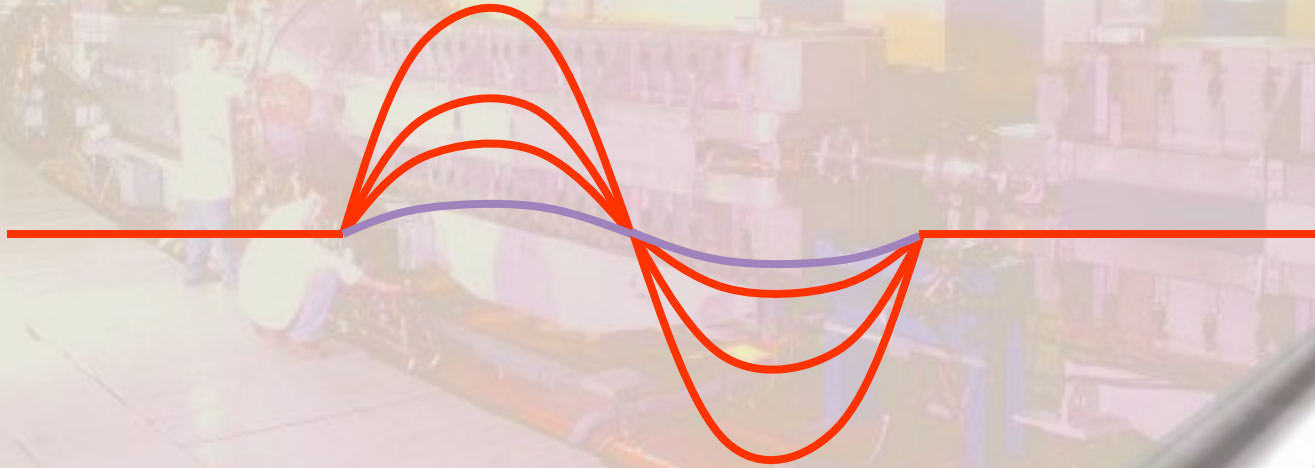


# Head-tail modes (3)

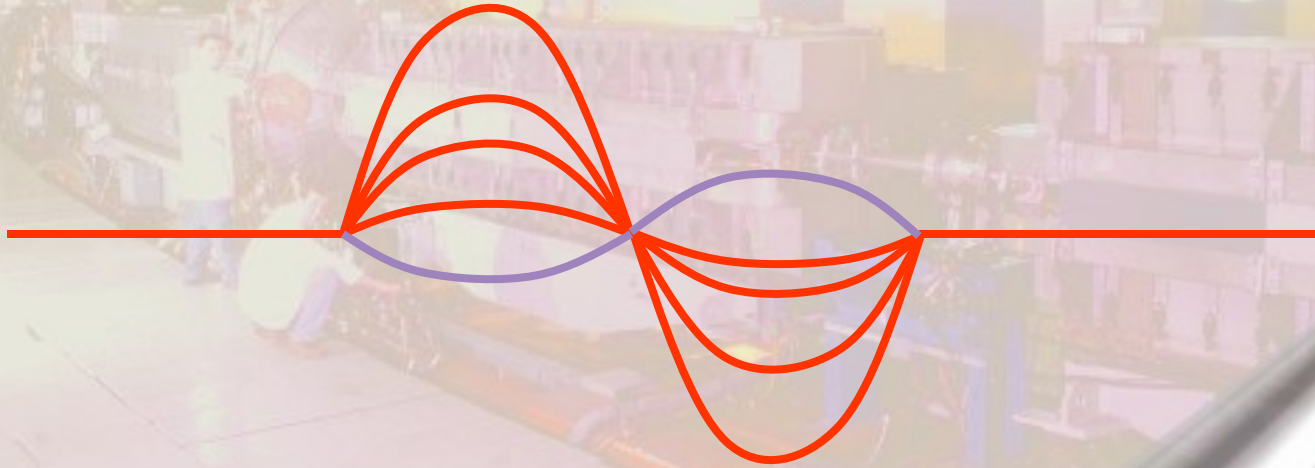




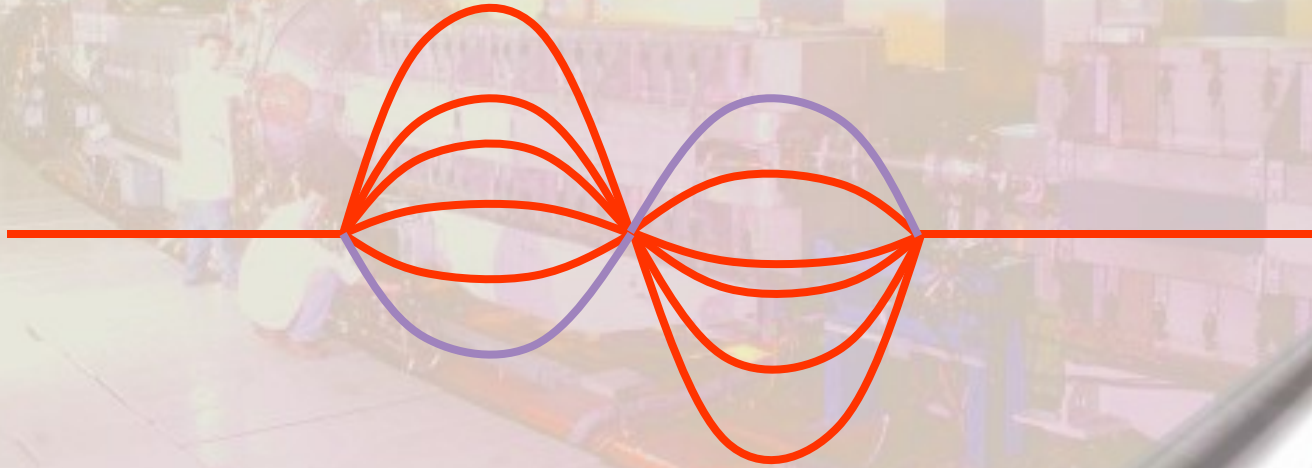
# Head-tail modes (4)



# Head-tail modes (5)



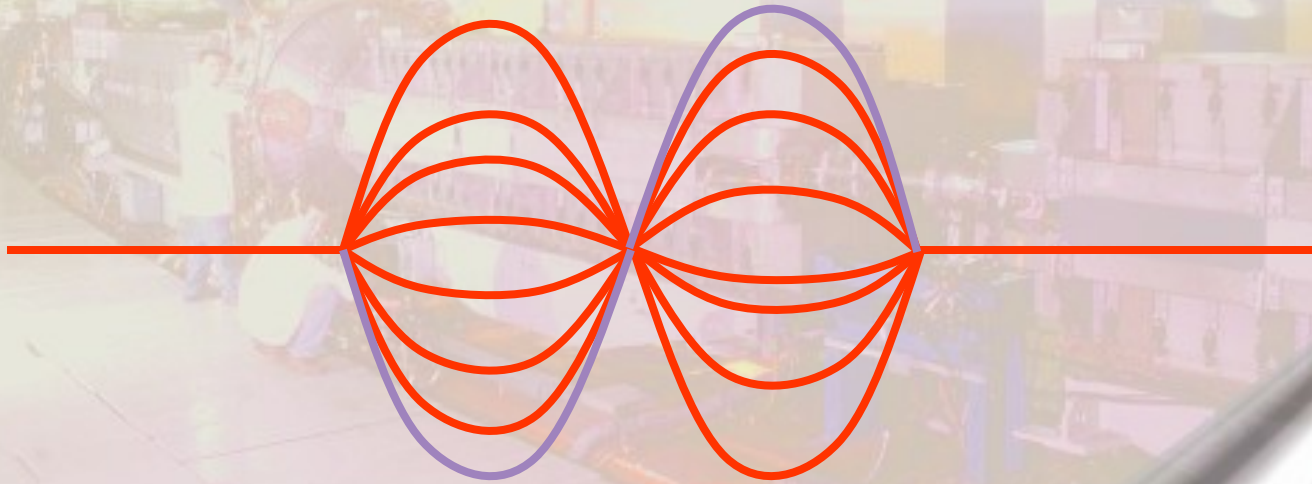
# Head-tail modes (6)





# Head-tail modes (7)

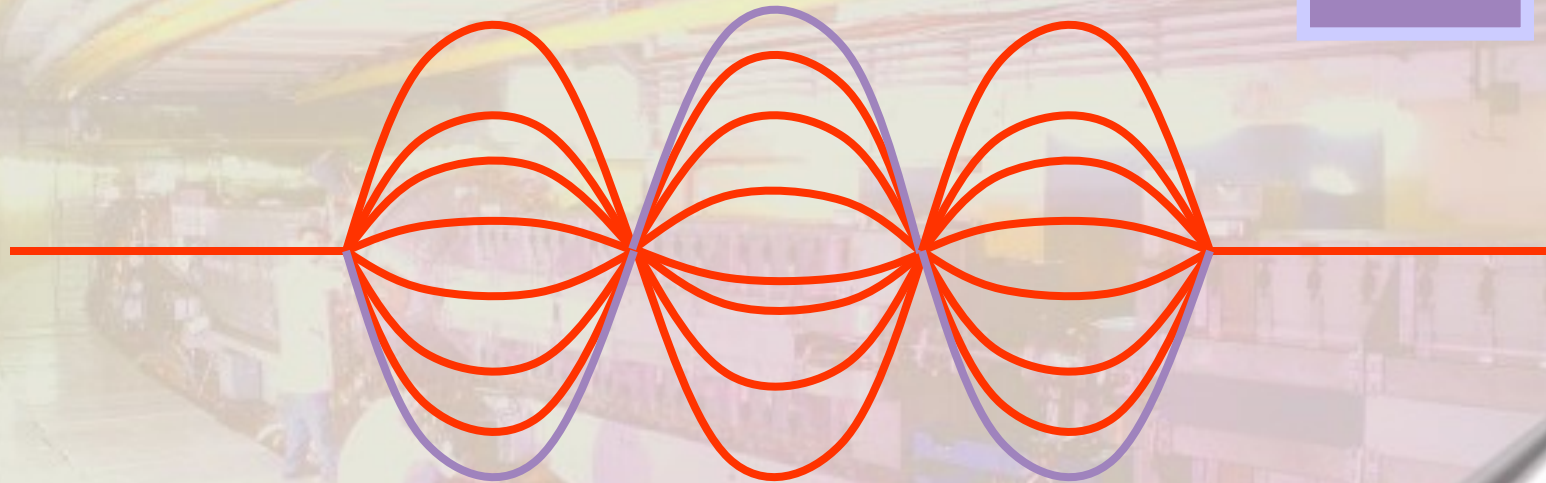
- # This is a standing wave with one node
- # Thus: Mode M=1



# Head-tail modes (8)

# This is (obviously!) Mode:

**M=2**

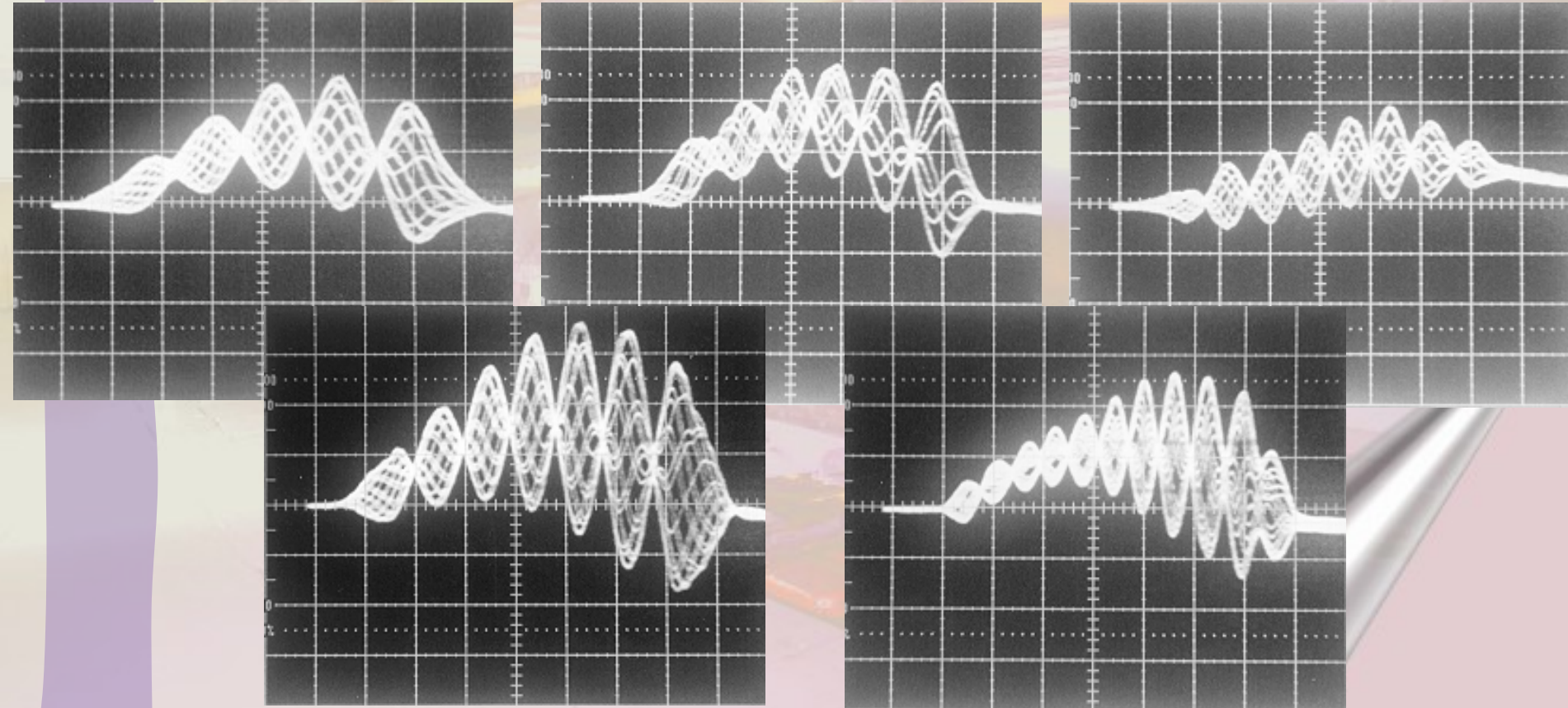


# Let's look more in detail at the M=1 "head-tail" mode

# But first some real life examples.....

# Head-tail modes (8)

# Some real life examples:





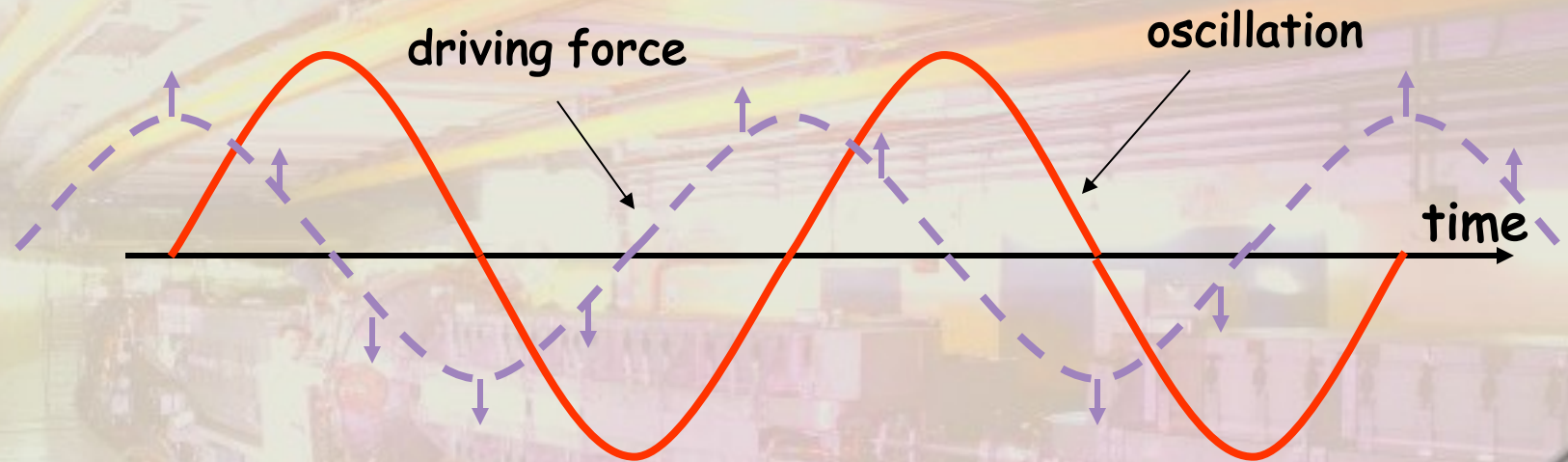
# Oscillation and the driving force (1)

- # Before continuing, first a memory refresher....



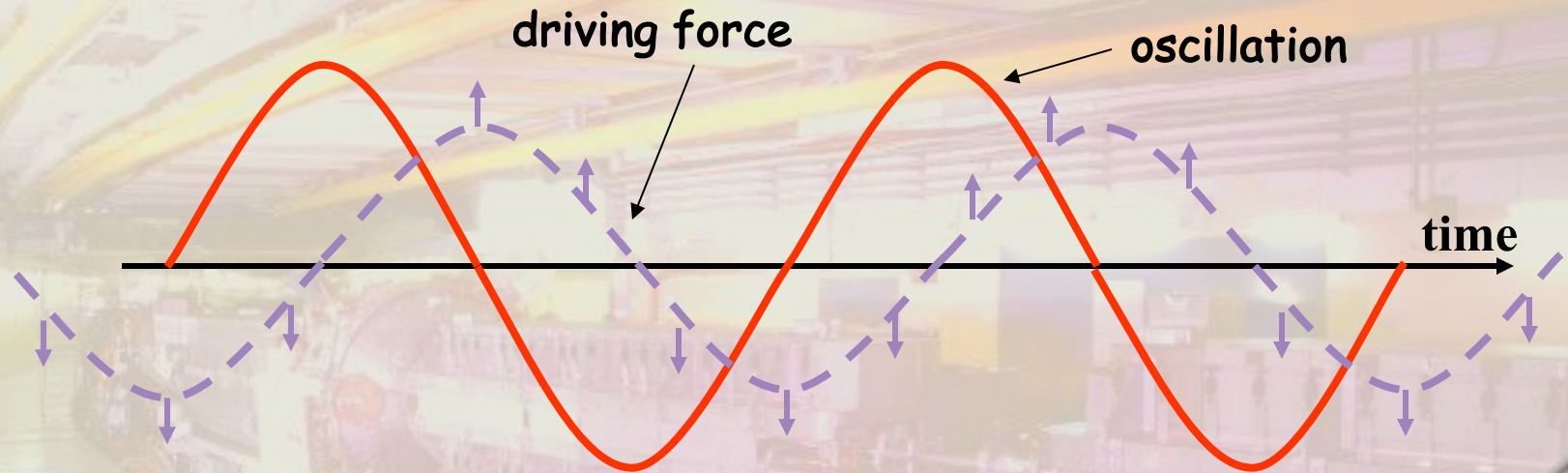
- # In order to increase the amplitude of a driven oscillator the driving force must be ahead (in phase) of the motion
- # Anyone who has pushed a child on a swing will know this.....

# Oscillation and the driving force (2)



Driving force ahead of oscillation  $\Rightarrow$  increasing amplitude  
Makes children happy but the beam unstable  
**INSTABILITY**

# Oscillation and the driving force (3)



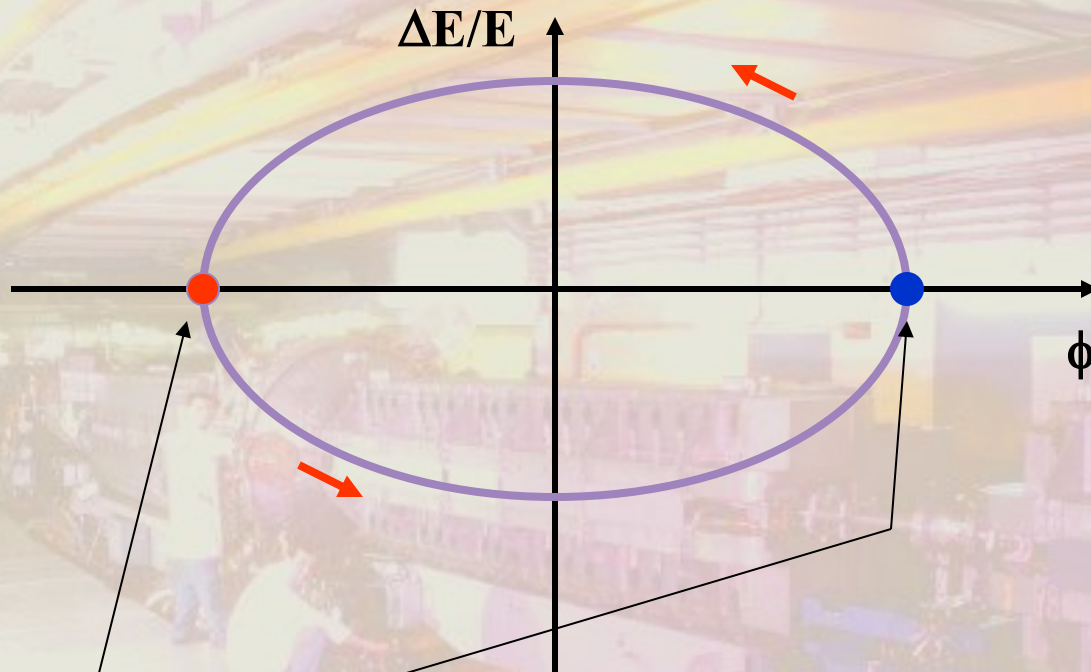
Driving force behind the oscillation  $\Rightarrow$  decreasing amplitude  
Makes children unhappy but the beam stable  
**DAMPING**



# M=1 Head-tail mode (1)

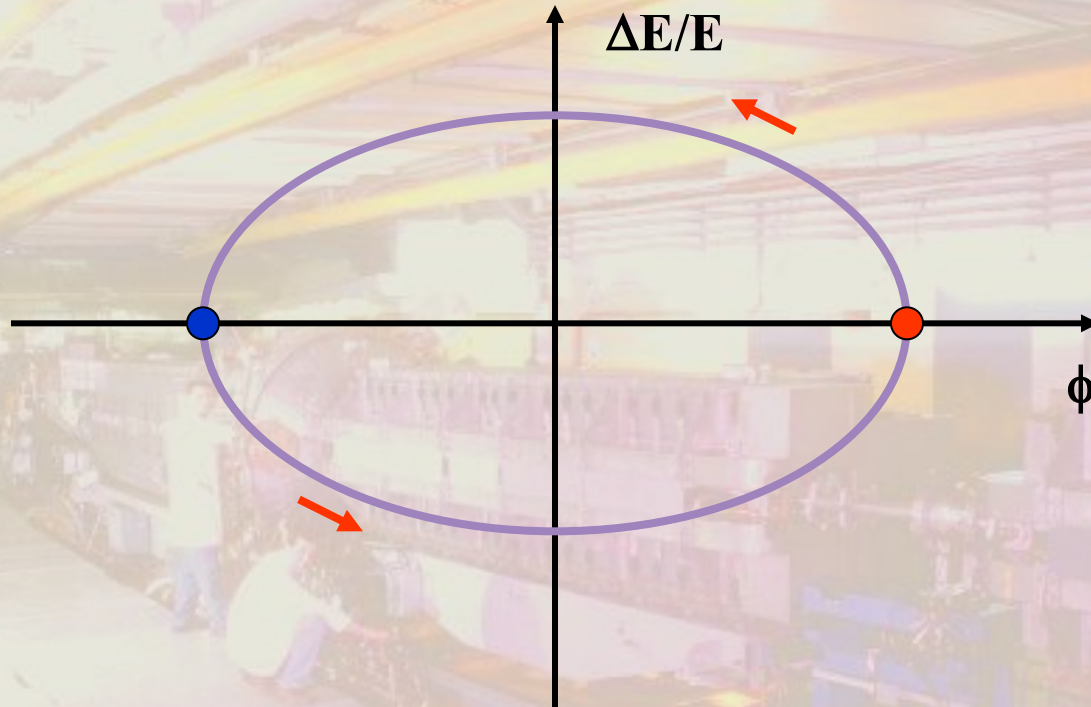
- # The M=1 head tail mode includes both **betatron** and **synchrotron** oscillations
- # There are many betatron oscillations during one synchrotron oscillation
- # Thus:  $Q_s \ll Q_h$  and also  $Q_s \ll Q_v$
- # Lets set up an M=1 mode transverse bunch oscillation
- # This means that the particles in the tail of the bunch are deflected by the electro-magnetic field left behind by the head of the bunch

# M=1 Head-tail mode (2)



Two particles in longitudinal phase space:  
Transverse oscillation of the blue particle is exactly out of phase with red one  $\Rightarrow$  red particle is exactly out of phase with the field left by the blue particle  
**NO EXCITATION**

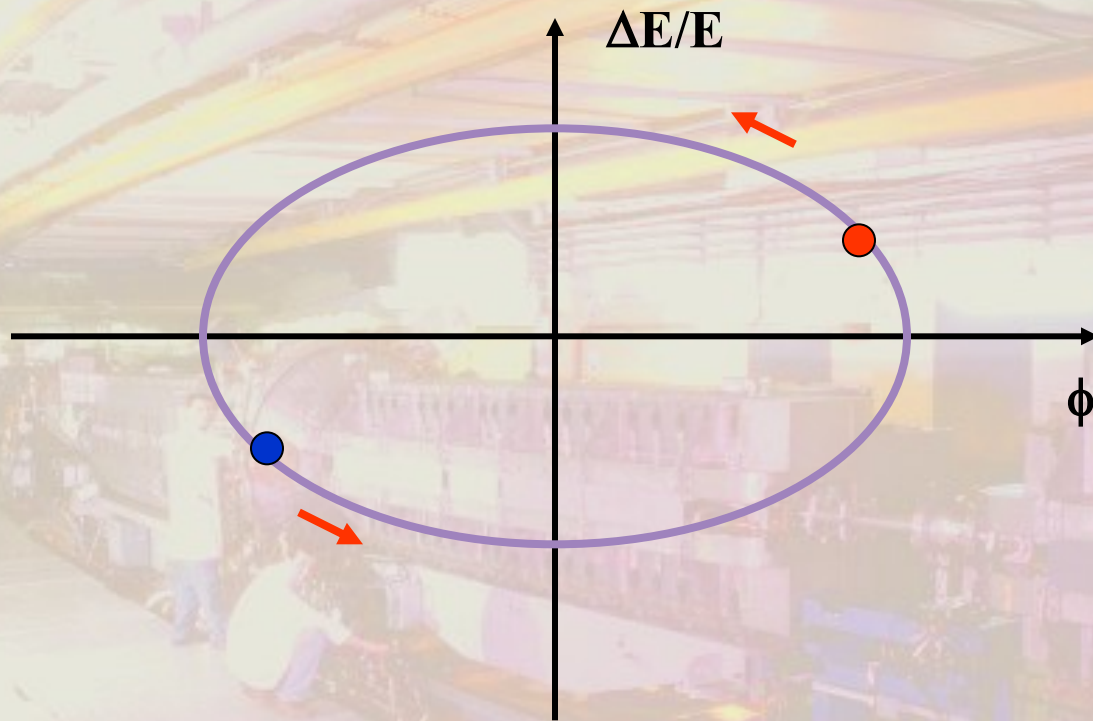
# M=1 Head-tail mode (3)



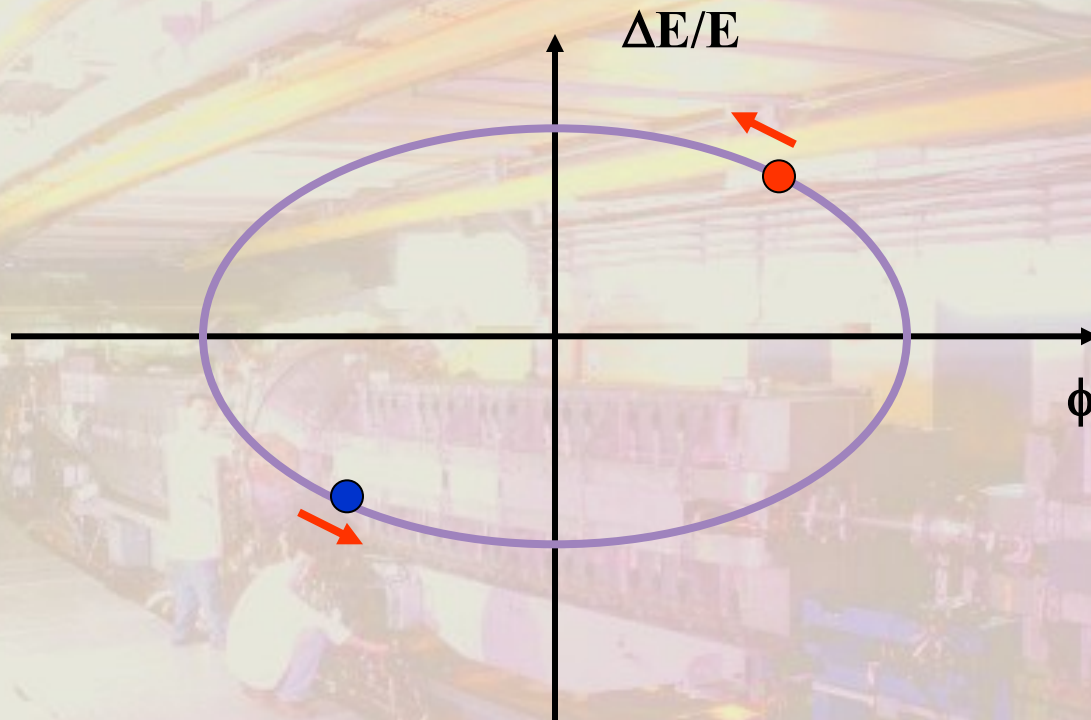
However in 1/2 of a synchrotron period the particles will change places



# M=1 Head-tail mode (4)

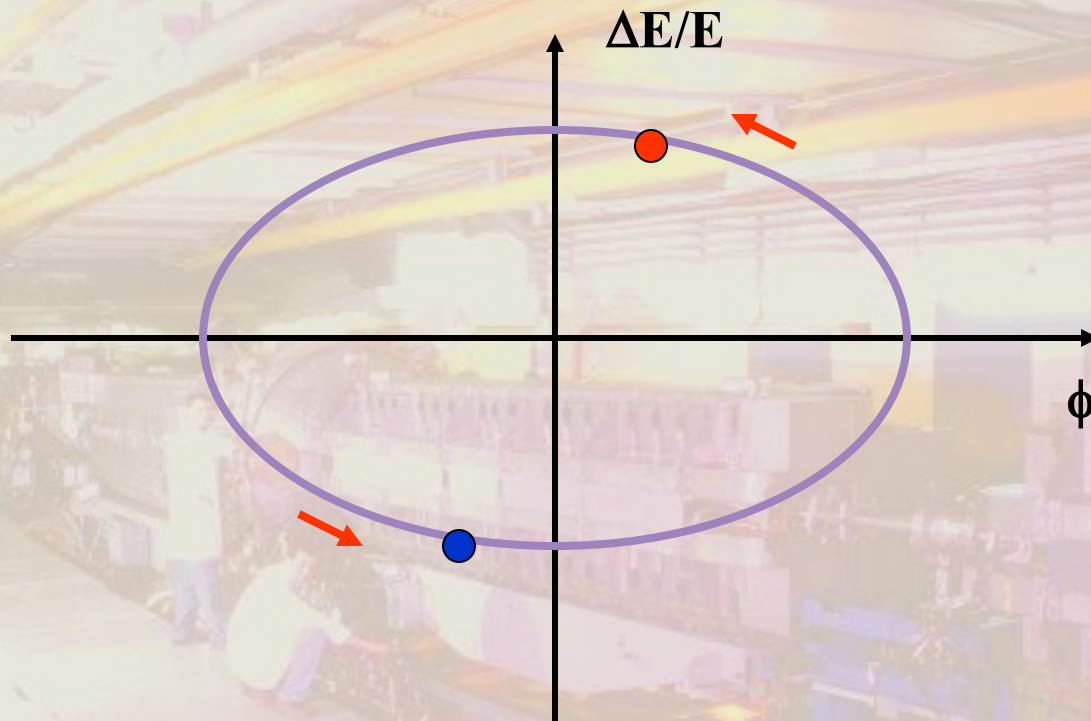


# M=1 Head-tail mode (5)



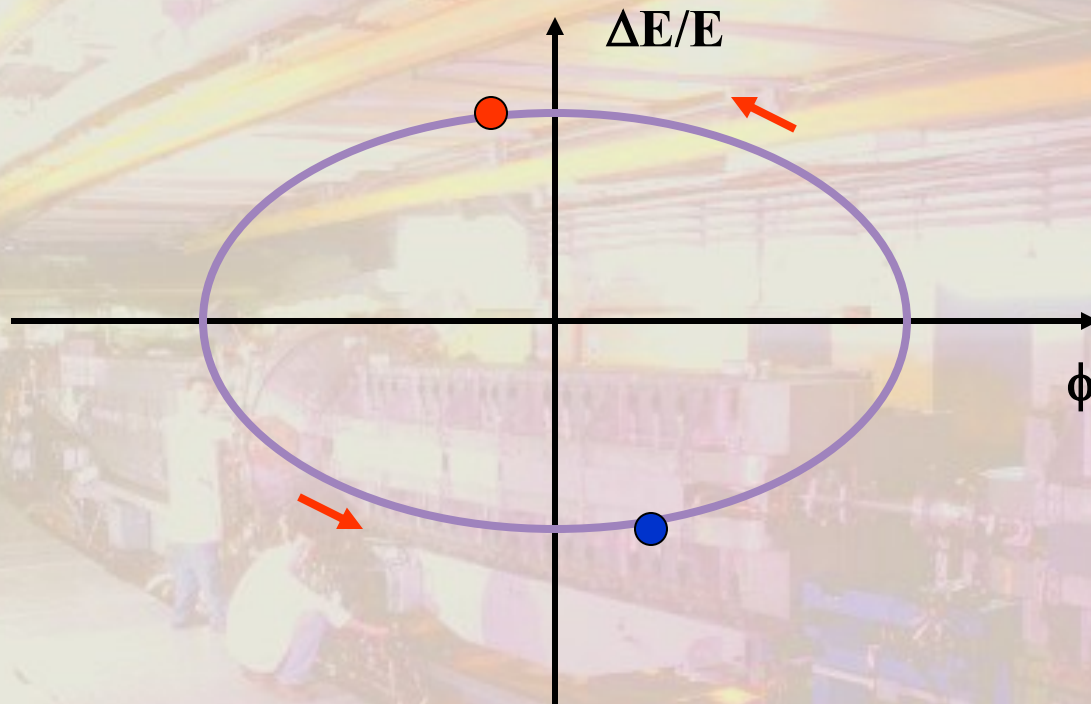
The energy of **red** particle is increasing  
The energy of **blue** particle is decreasing

# M=1 Head-tail mode (6)

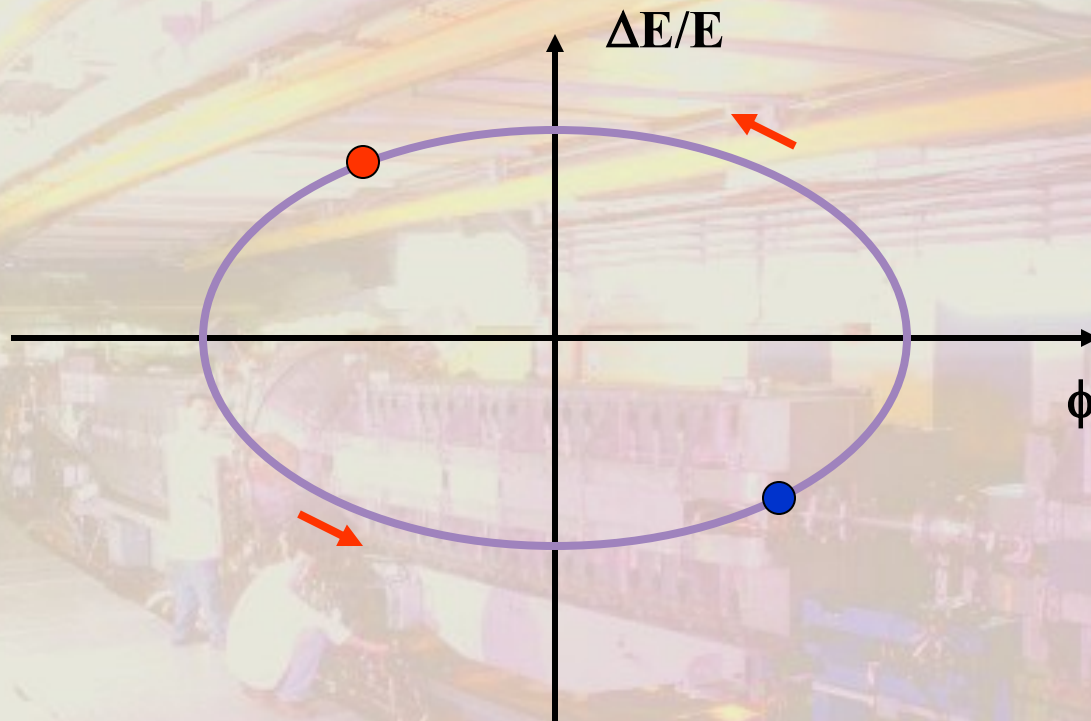




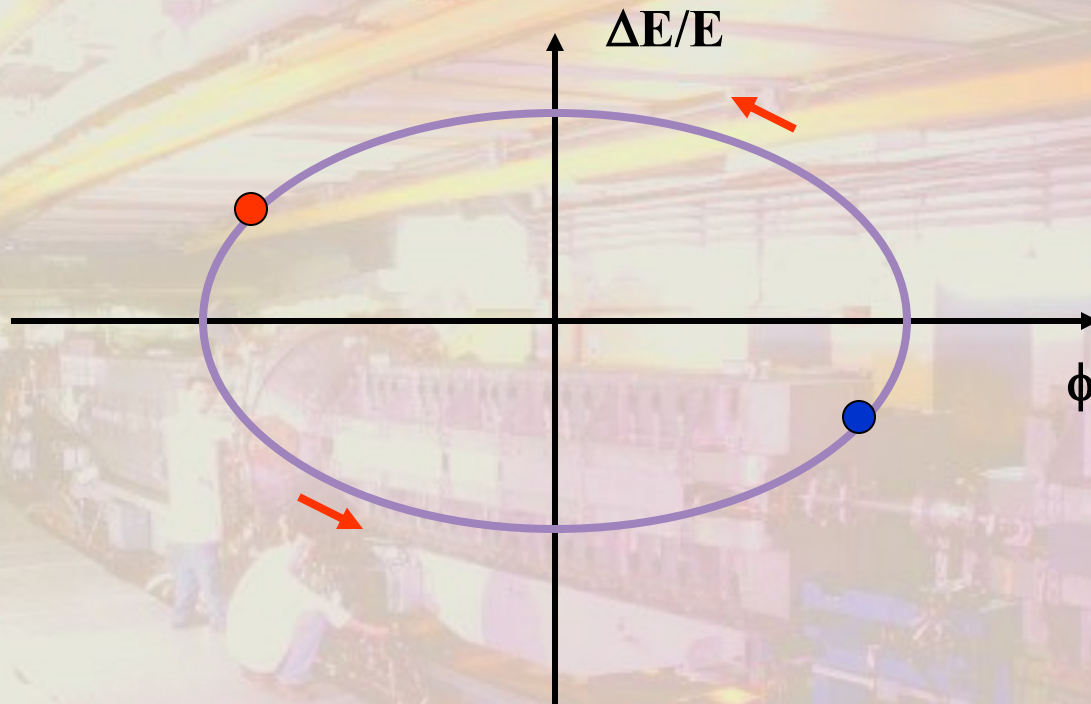
# M=1 Head-tail mode (7)



# M=1 Head-tail mode (8)

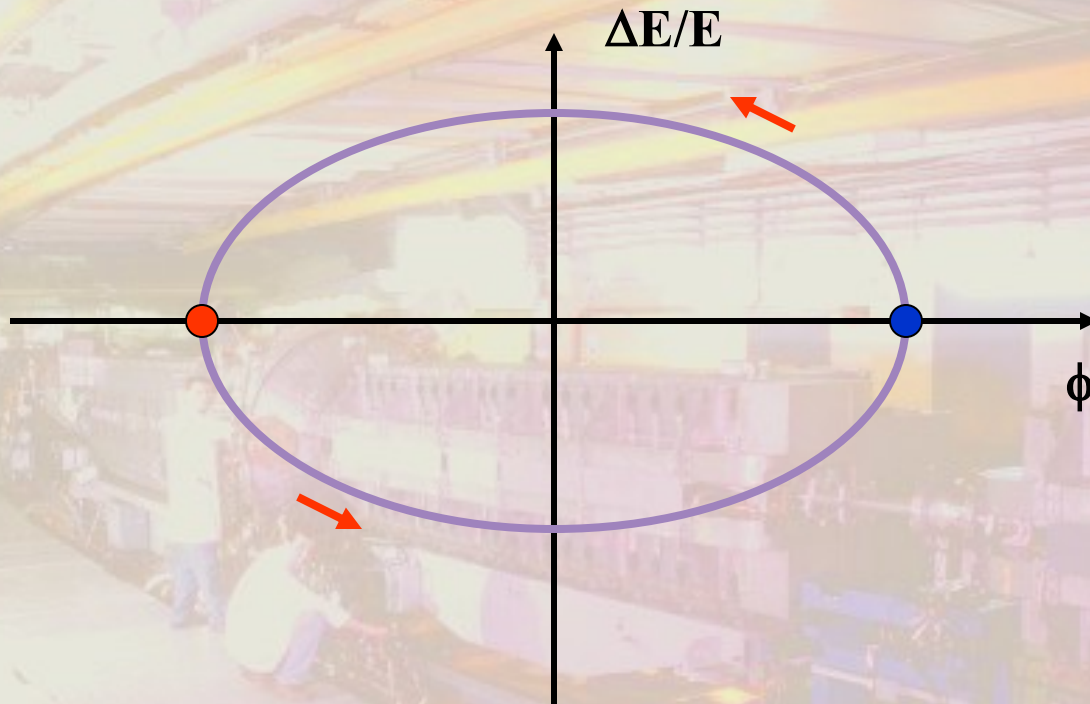


# M=1 Head-tail mode (9)



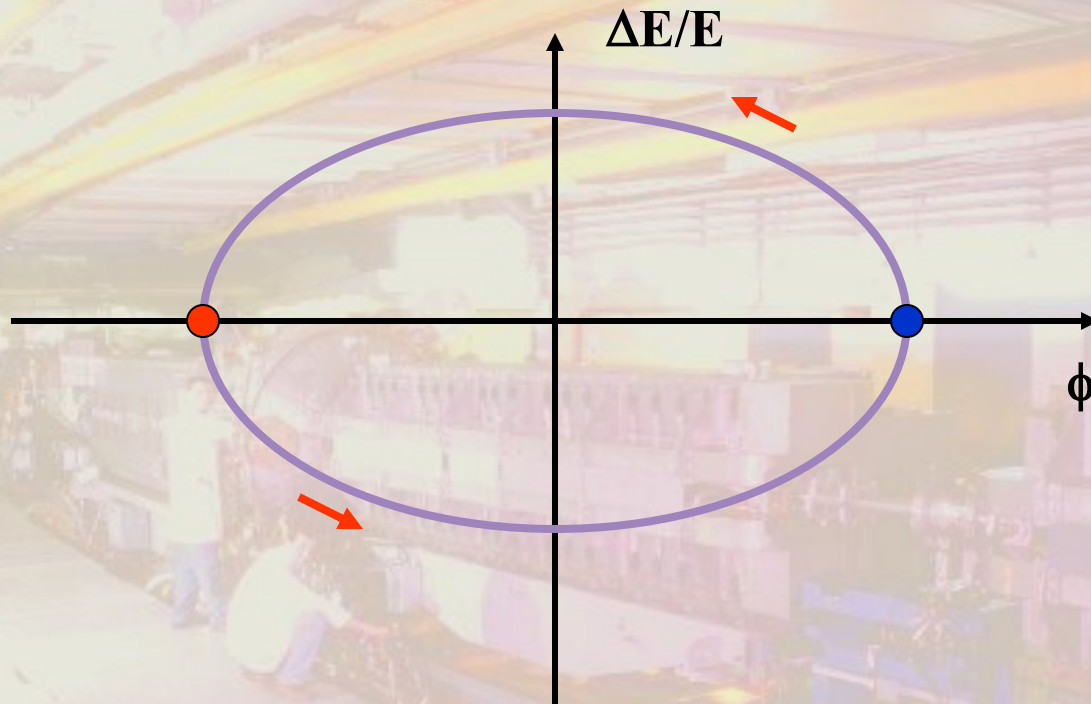


# M=1 Head-tail mode (10)



Now they have changed places and have returned to their original energies

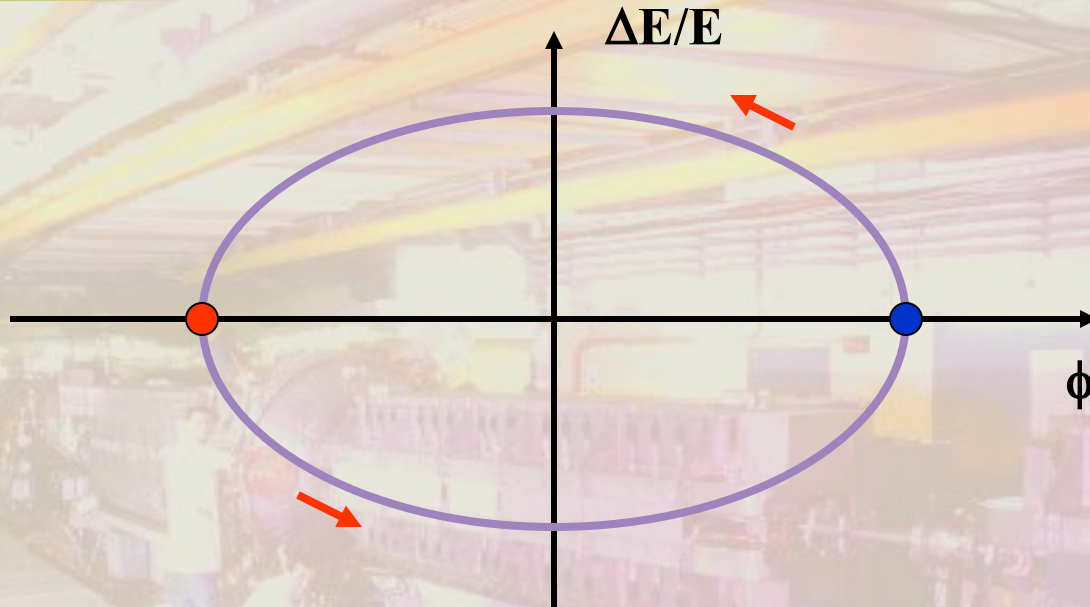
# M=1 Head-tail mode (11)



If the chromaticity is zero **red** will still be exactly out of phase with the wake field left behind by **blue**

**STABLE CONDITION**

# M=1 Head-tail mode (12)

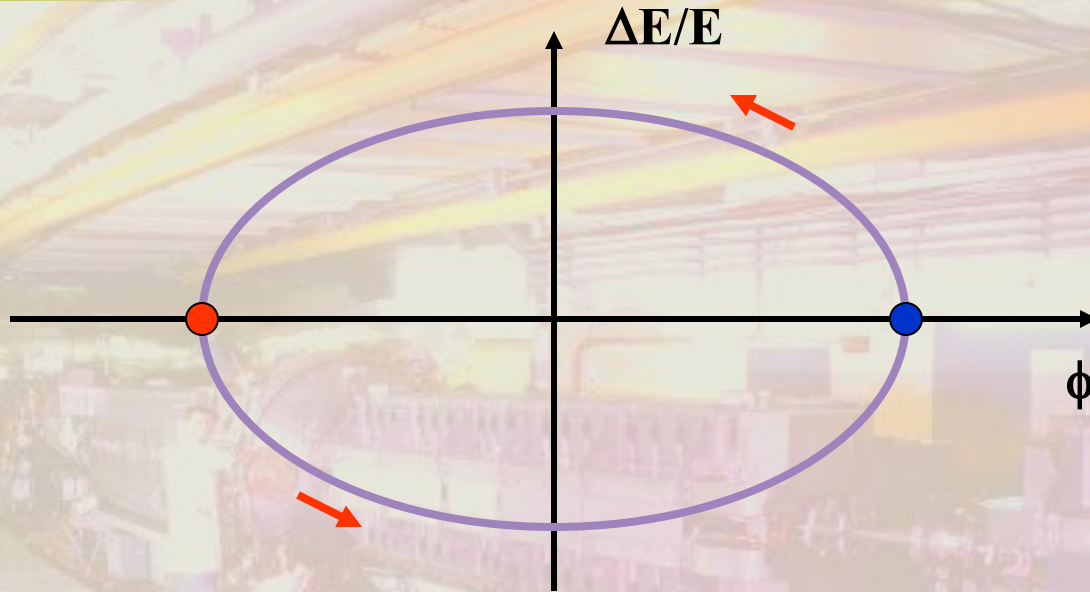


If Chromaticity is negative **red** would have made slightly less betatron oscillations than **blue**  
Then **red's** transverse oscillation would **lag slightly behind the wake field** left by **blue**

INSTABLE



# M=1 Head-tail mode (13)



If Chromaticity is positive **red** would have made slightly more betatron oscillations than **blue**  
Then **red's** transverse oscillation would be **slightly ahead of the wake field** left by **blue**

STABLE

# M=1 Head-tail mode (14)

## # Conclusion:

- # Above transition we must have a **positive chromaticity** to avoid the M=1 mode Head-Tail instability.
  - # Below transition we must have a **negative chromaticity**.
- # The natural chromaticity of the machine without sextupoles is normally negative ( $E\uparrow \rightarrow Q\uparrow$ )
- # We therefore we need sextupoles to be able to correct the chromaticity.

# Transverse multi-bunch modes

- # Longitudinal multi-bunch instabilities limit the bunch intensity before the transverse modes become a problem
- # However, once a longitudinal feed back system has been built, one may need to consider a transverse feed back system too.....

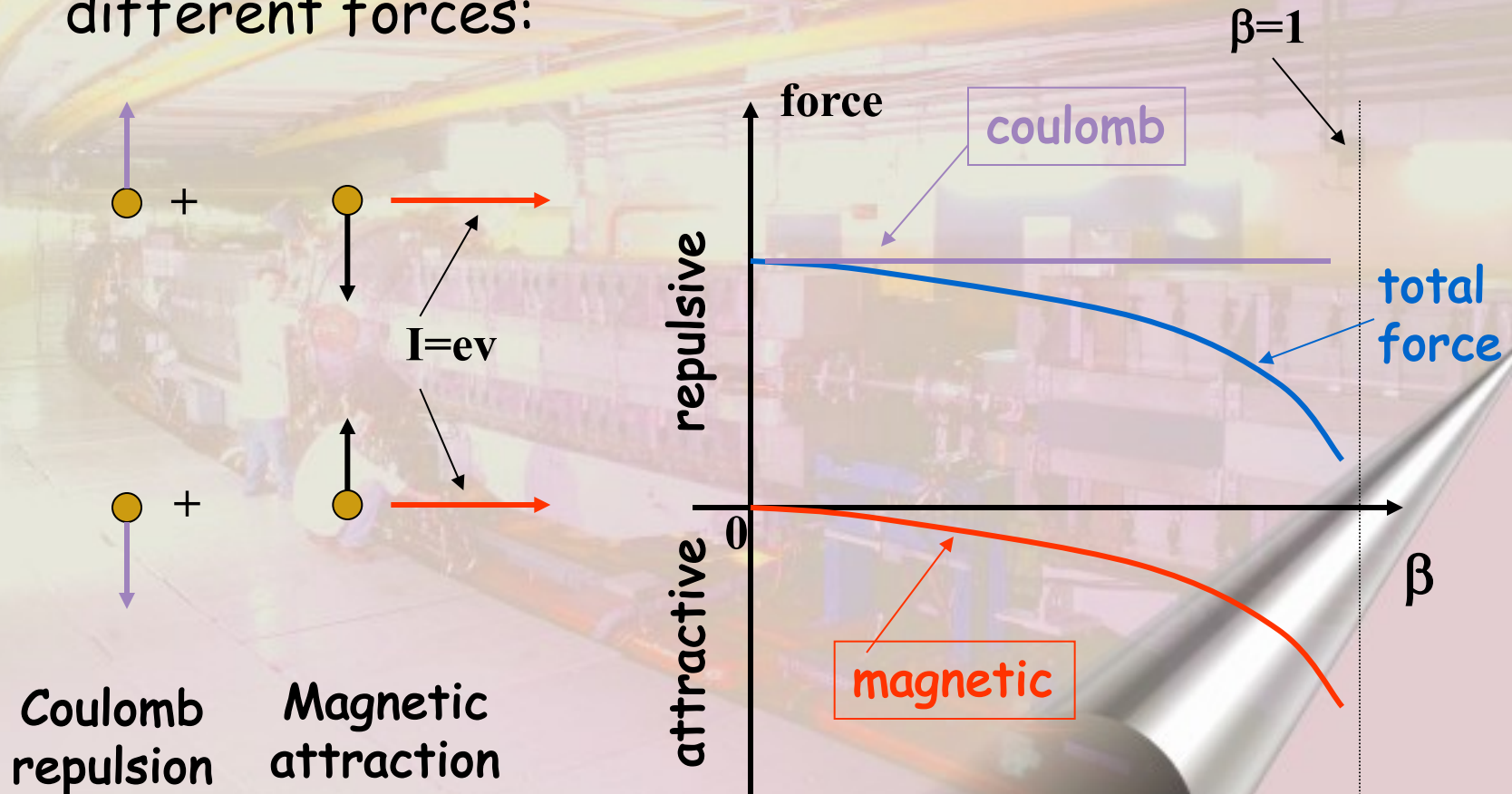


# Cures

- # Correct the natural chromaticity of the machine (set chromaticity negative below transition and positive above transition, but not zero)
- # Install a feed-back system.
  - Detect a coherent oscillation and damp it using a transverse kicker
- # Damp transverse modes in cavities, where they will remain longest, using a damping antenna

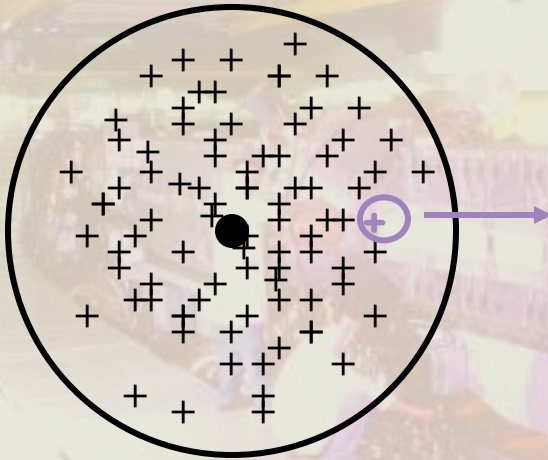
# Space Charge effects (1)

# Between two charged particles in a beam we have different forces:

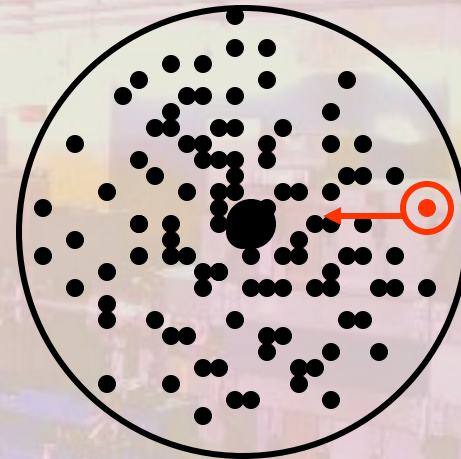


# Space Charge effects (2)

- # For many particles in a beam we can represent it as following:



**Charges  $\Rightarrow$  repulsion**

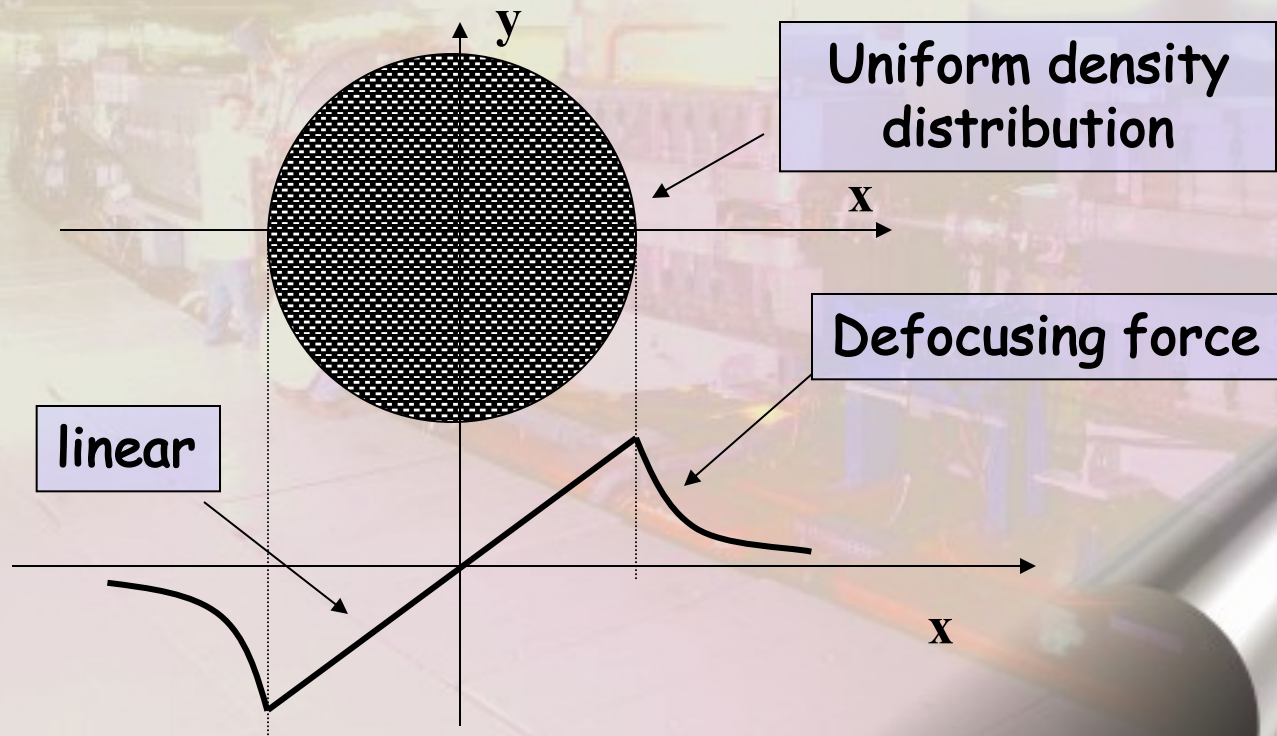


**Parallel currents  $\Rightarrow$  attraction**



# Space Charge effects (2)

- # At low energies, which means  $\beta \ll 1$ , the force is mainly repulsive  $\Rightarrow$  defocusing
- # It is zero at the centre of the beam and maximum at the edge of the beam



# Space Charge effects (3)

- # For the uniform beam distribution, this linear defocusing leads to a tune shift given by:

$$\Delta Q_{h,v} = - \frac{r_0 N}{2\pi \epsilon_{h,v} \beta^2 \gamma^3}$$

Machine radius →  $r_0$

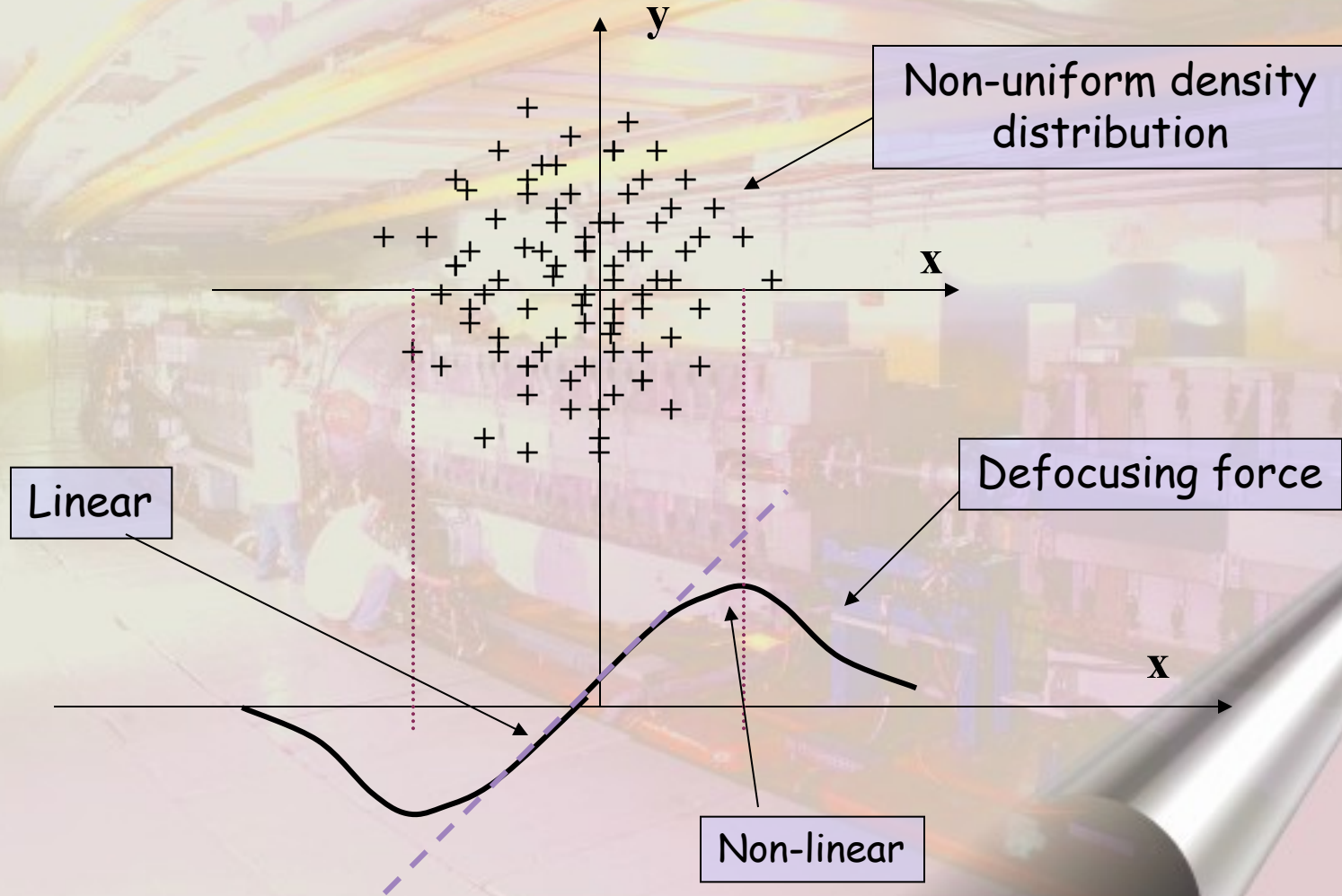
Number of particles in the beam →  $N$

Transverse emittance →  $\epsilon_{h,v}$

Relativistic parameters →  $\beta^2 \gamma^3$

- # This tune shift is the **same for all particles** and vanishes at high momenta ( $\beta=1, \gamma \gg 1$ )
- # However in reality the beam distribution is not uniform....

# Space charge effects (4)





# Laslett tune shift (1)

- # For the non-uniform beam distribution, this non-linear defocusing means the  $\Delta Q$  is a function of  $x$  (transverse position)
- # This leads to a spread of tune shift across the beam
- # This tune shift is called the 'LASLETT tune shift'

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi \epsilon_{h,v} \beta^2 \gamma^3}$$

half of the  
uniform tune shift

- # This tune spread cannot be corrected and does get very large at high intensity and low momentum

# Laslett tune shift (2)

Tune Shift

$$\Delta Q_{h,v} \approx -\frac{r_0 N}{4\pi\epsilon_{h,v}\beta^2\gamma^3}$$

Large neck tie  
in tune diagram

- # At injection into the PS Booster
  - $E = 0.988 \text{ GeV}, \gamma = 1.053, \beta = 0.313 \Rightarrow \Delta Q \approx 0.3$
- # For the same beam at injection into the PS
  - $E = 2.3826 \text{ GeV}, \gamma = 2.475, \beta = 0.915 \Rightarrow \Delta Q \approx 0.005$
- # For the same beam at injection into the SPS
  - $E = 14 \text{ GeV}, \gamma = 14.93, \beta = 0.998 \Rightarrow \Delta Q \approx 0.00001$
- # We accelerate the beam in the PSB as quickly as possible to avoid problems of blow-up due to betatron resonances

# Questions....,Remarks...?

*Single bunch  
modes*

*Head-tail modes*

*Space charge*

*Tune shift*





# Beam Break-up around transition....

