AXEL-2018

Introduction to Particle Accelerators

Longitudinal instabilities:

- ✓ *Single bunch longitudinal instabilities*
- ✓ *Multi bunch longitudinal instabilities*
- ✓ *Different modes*
- ✓ *Bunch lengthening*

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Instabilities (1)

- Until now we have only considered independent particle motion.
- We call this incoherent motion.
  - single particle synchrotron/betatron oscillations
  - each particle moves independently of all the others
- Now we have to consider what happens if all particles move in phase, coherently, in response to some excitations

Synchrotron & betatron oscillations

Instabilities (2)

- We cannot ignore interactions between the charged particles
- They interact with each other in two ways:

Space charge effects, intra beam scattering

  - Direct Coulomb interaction between particles

Longitudinal and transverse beam instabilities

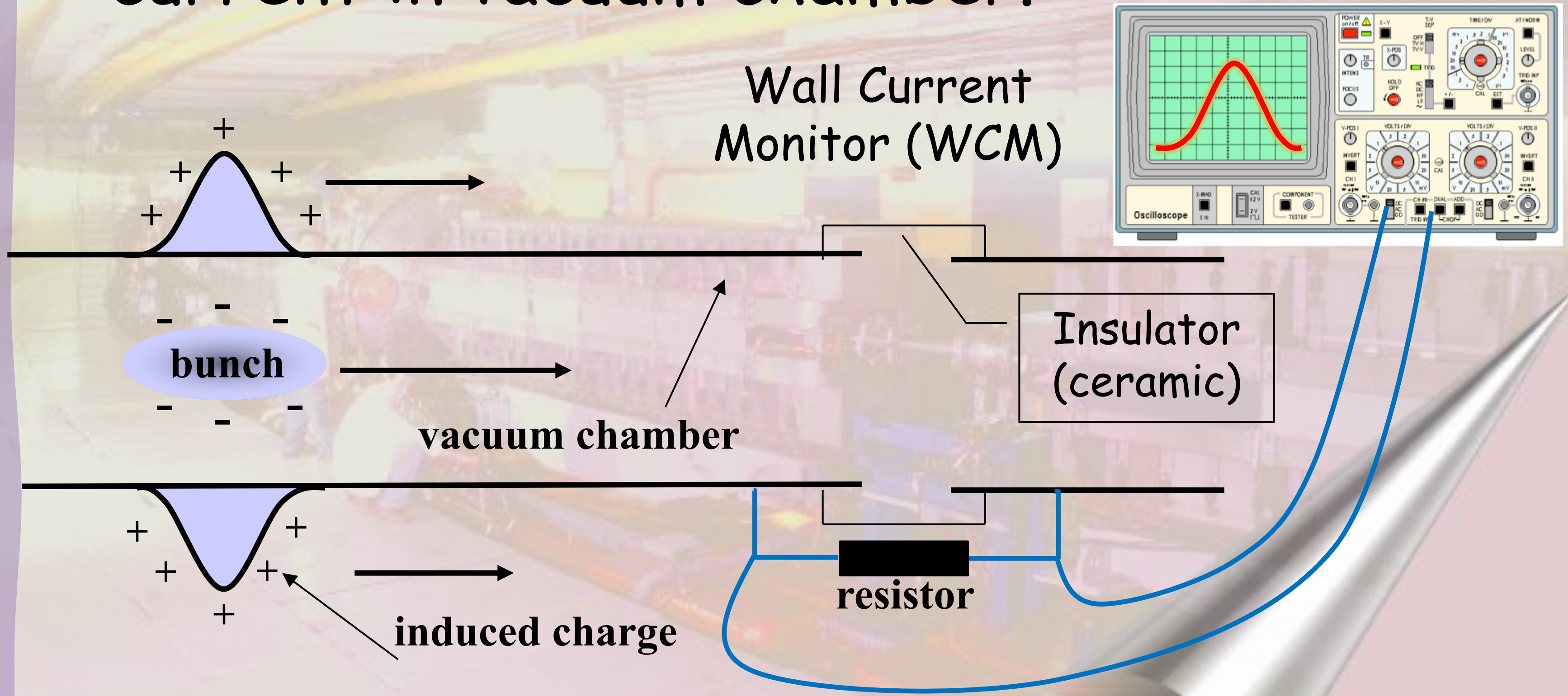
  - Via the vacuum chamber

Why do Instabilities arise?

- A circulating bunch induces electro magnetic fields in the vacuum chamber
- These fields act back on the particles in the bunch
- Small perturbations to the bunch motion, change the induced EM fields
- If this change amplifies the perturbation then we have an instability

Measuring Longitudinal Instabilities

- A circulating bunch creates an image current in vacuum chamber.



- The induced image current is the same size but has the opposite sign to the bunch current.

Impedance and Wall current (1)

- The vacuum chamber presents an impedance to this induced wall current (changes of shape, material etc.)
- The image current combined with this impedance induces a voltage, which in turn affects the charged particles in the bunch

$$\int (Z(\omega) \times I(\omega))\, d\omega = \int_0^s E\, ds$$

Impedance & current ⇒ voltage ⇒ electric field

Resistive, inductive, capacitive

$$Z = Z_r + iZ_i$$

Strong frequency dependence

Real & Imaginary components

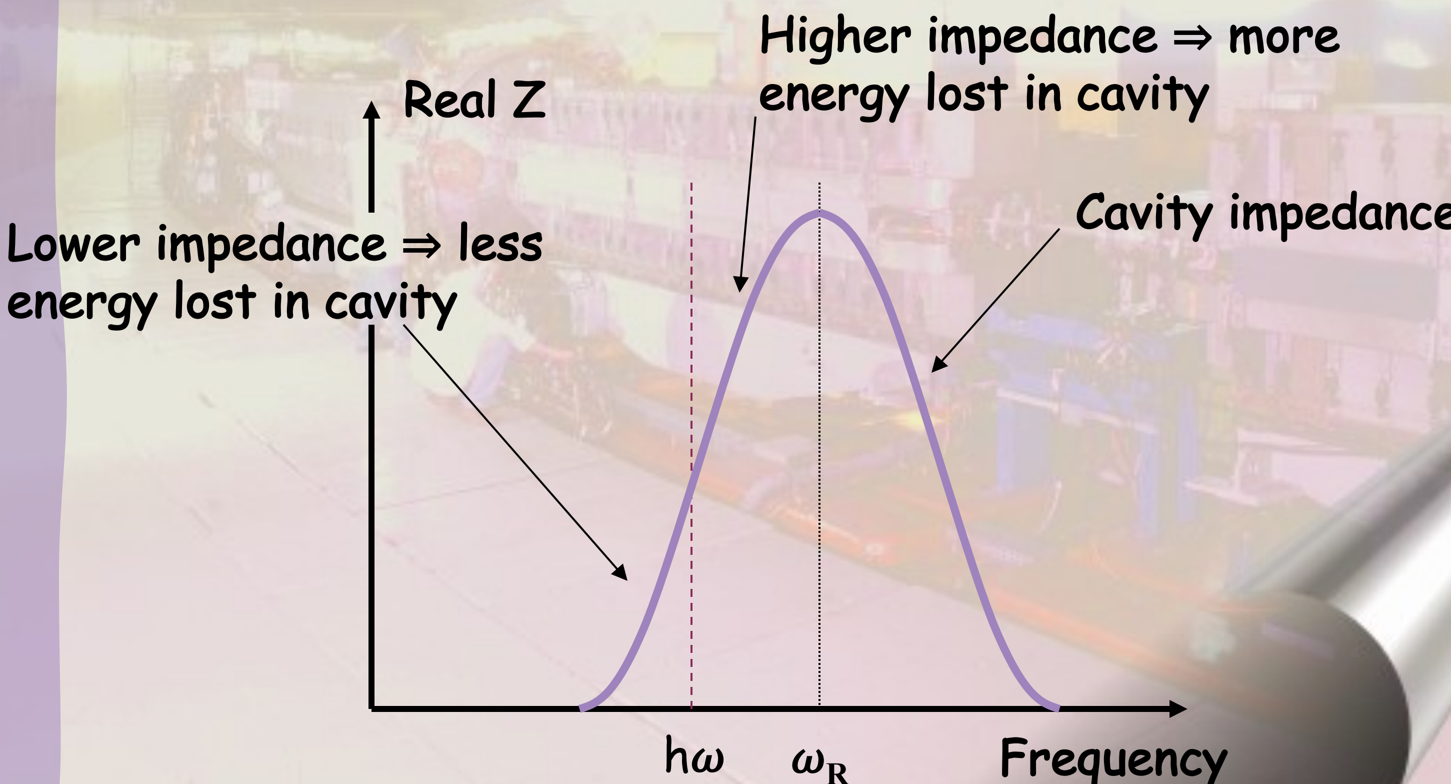
Impedance and Wall current (2)

- Any change of cross section or material leads to a finite impedance
- We can describe the vacuum chamber as a series of cavities
  - Narrow band - High Q resonators - RF Cavities tuned to some harmonic of the revolution frequency
  - Broad band - Low Q resonators - rest of the machine
- For any cavity two frequencies are important:
  - ω = Excitation frequency (bunch frequency)
  - ω_R= Resonant frequency of the cavity
- If $h\omega \approx \omega_R$ then the induced voltage will be large and will build up with repeated passages of the bunch

h is an integer

Single bunch Longitudinal Instabilities (1)

- Lets consider:
  - A single bunch with a revolution frequency = ω
  - That this bunch is not centered in the long. Phase Space
  - A single high-Q cavity which resonates at ω_R ($\omega_R \approx h\omega$)

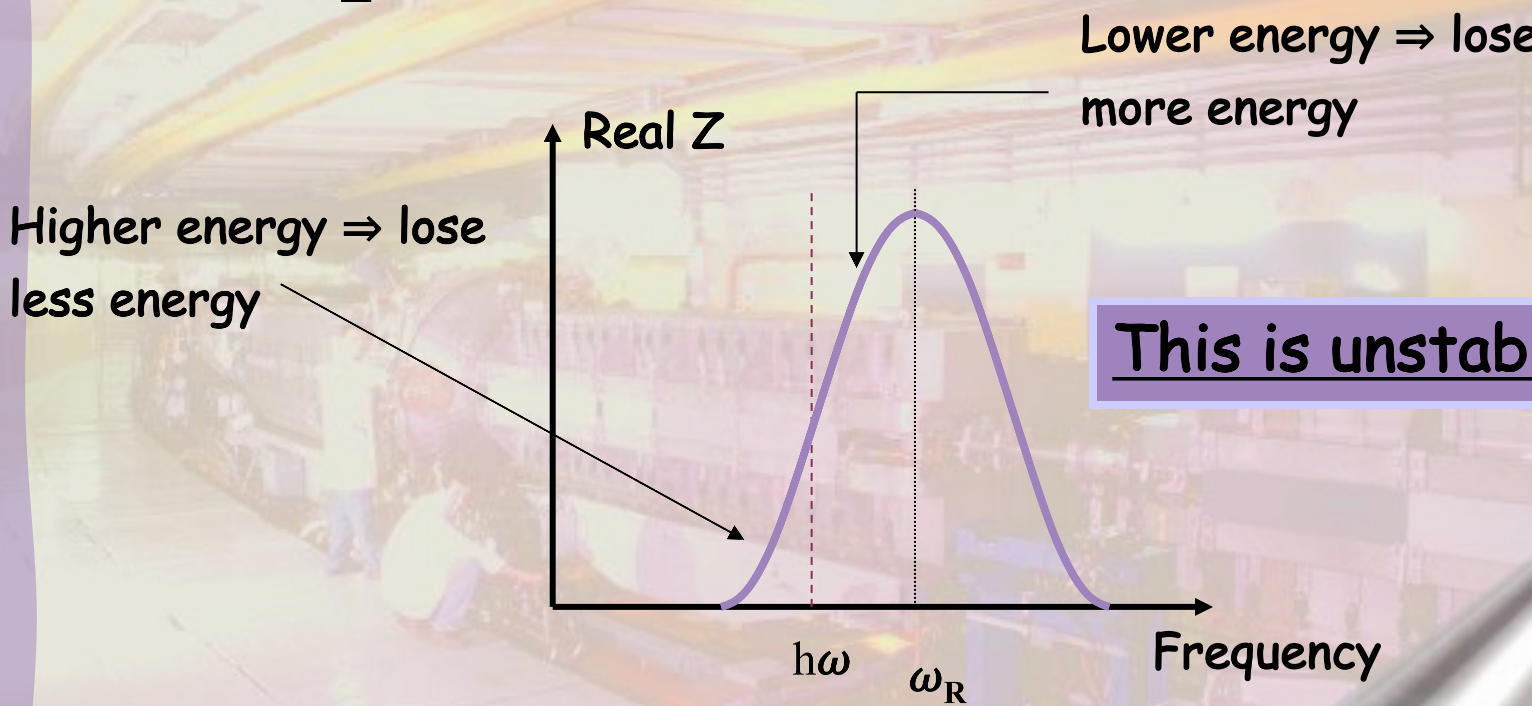


Single bunch Longitudinal Instabilities (2)

- Lets start a coherent synchrotron oscillation (above transition)
- The bunch will gain and loose energy/momentum
- There will be a decrease and increase in revolution frequency
- Therefore the bunch will see changing cavity impedance
- Lets consider two cases:
  - First case, consider $\omega_R > h\omega$
  - Second case, consider $\omega_R < h\omega$

Single bunch Longitudinal Instabilities (3)

- Case: $\omega_R > h\omega$

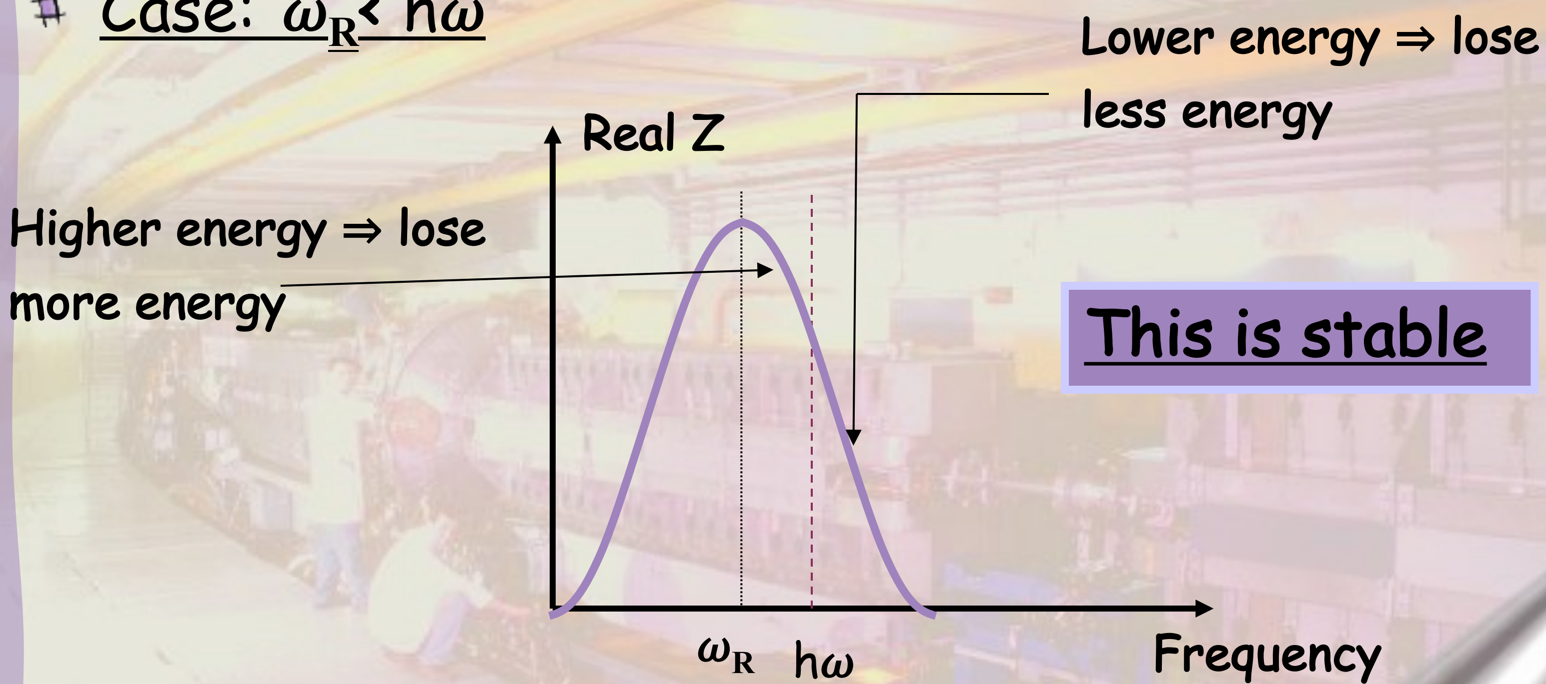


This is unstable

- The cavity tends to increase the energy oscillations
- Now retune cavity so that $\omega_R < h\omega$

Single bunch Longitudinal Instabilities (3)

- Case: $\omega_R < h\omega$

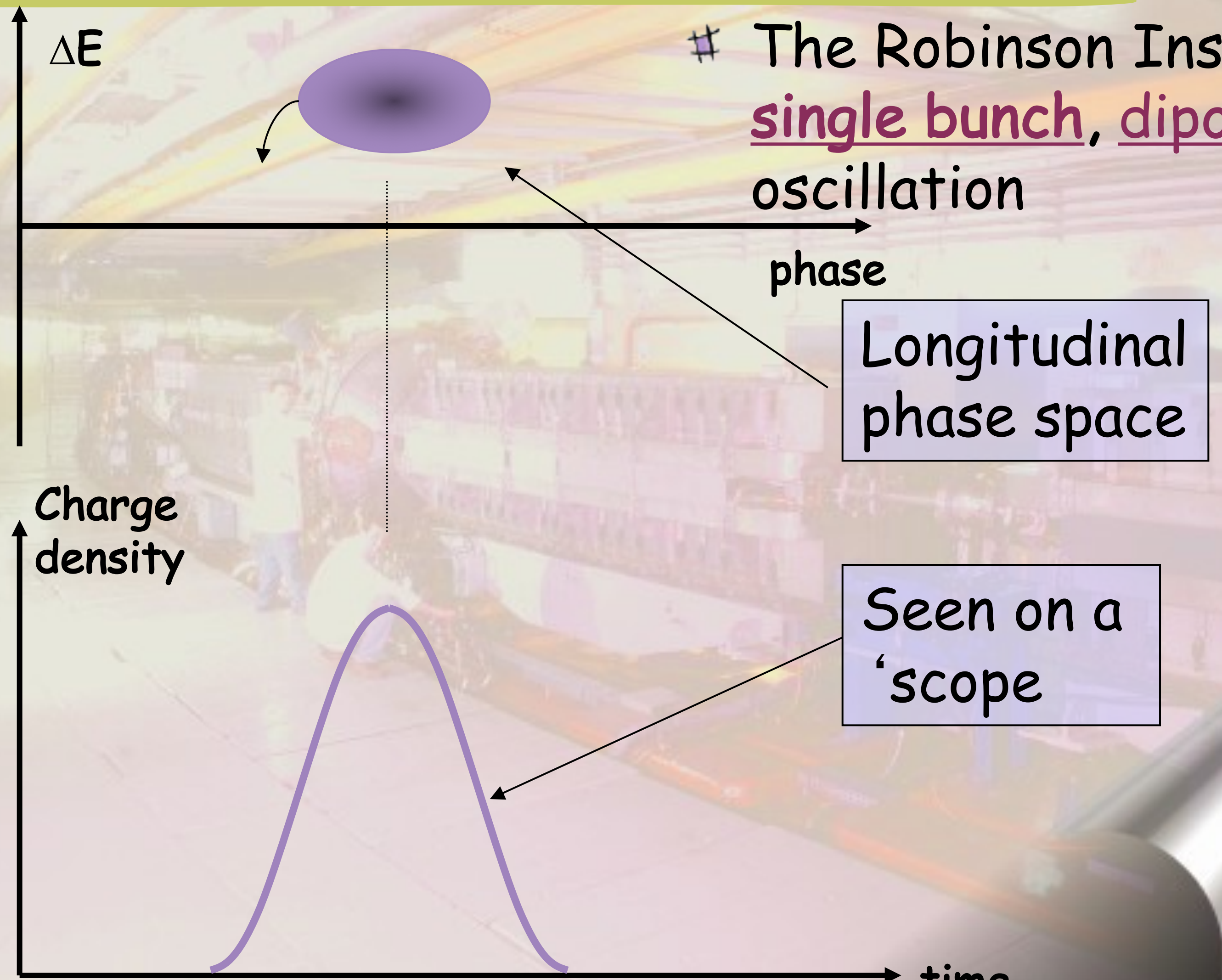


This is stable

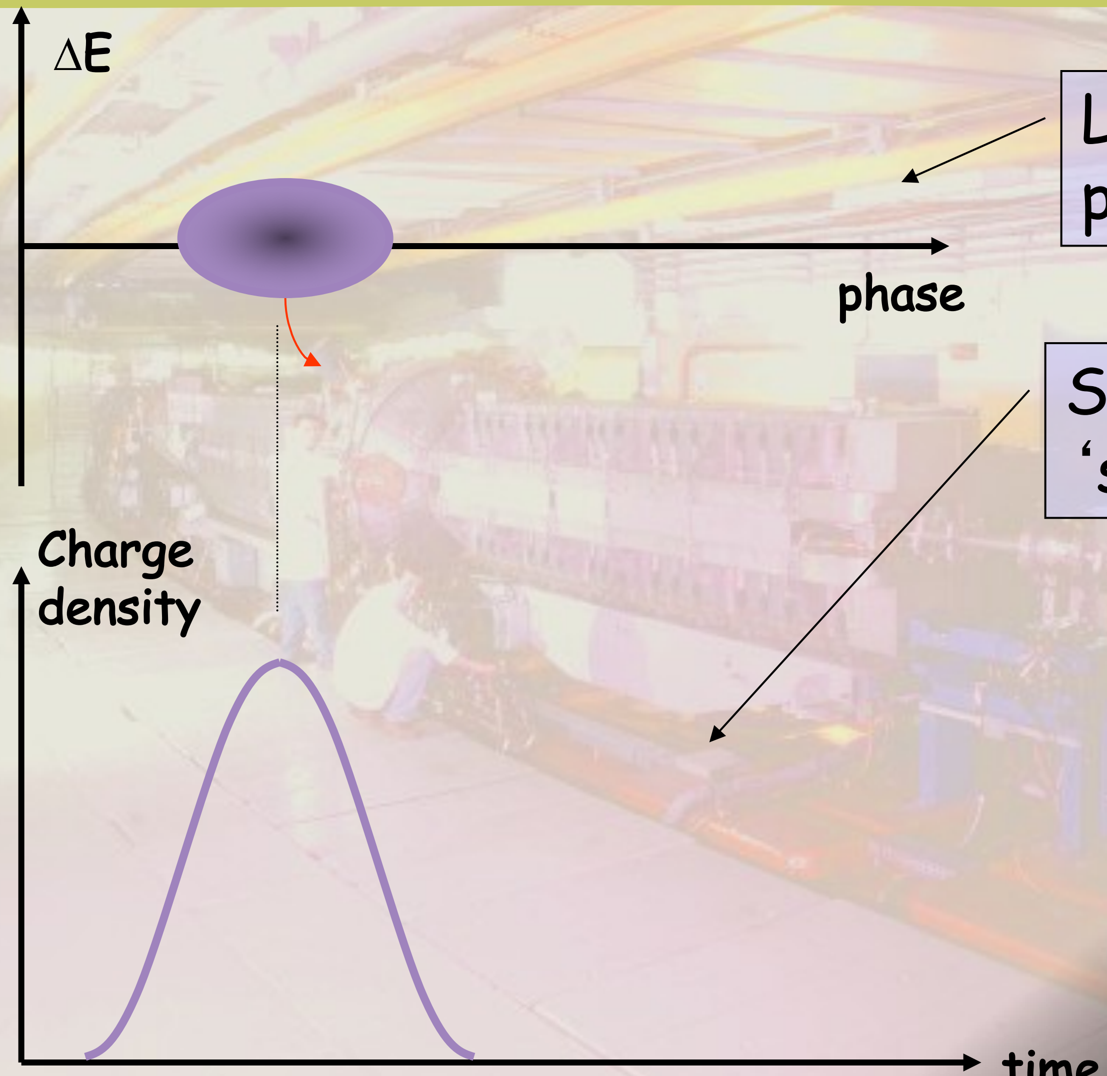
- This is is known as the 'Robinson Instability'
- **To damp this instability one should retune the cavity so that $\omega_R < h\omega$**

Robinson Instability (1)

- The Robinson Instability is a single bunch, dipole mode oscillation



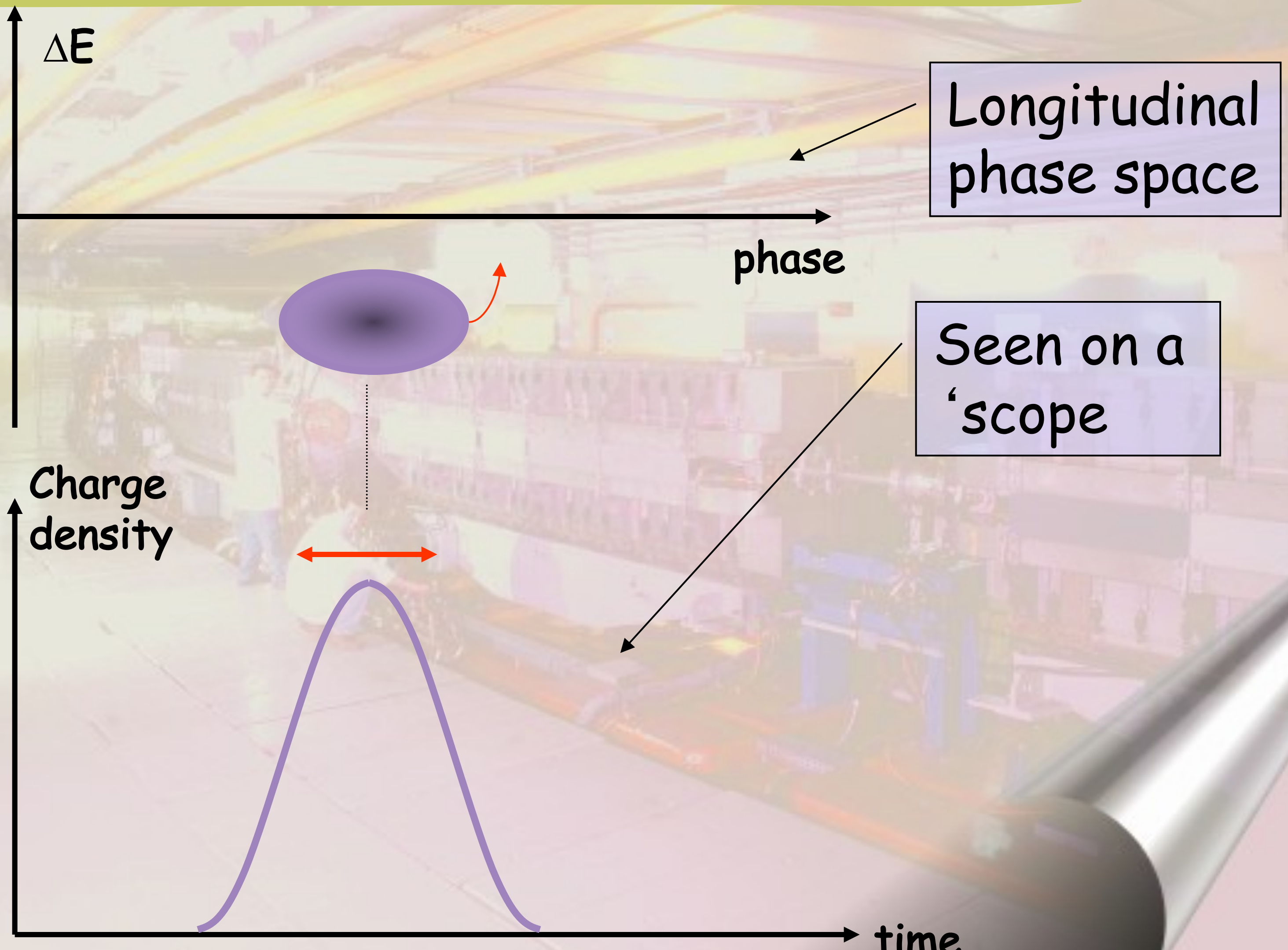
Robinson Instability (2)



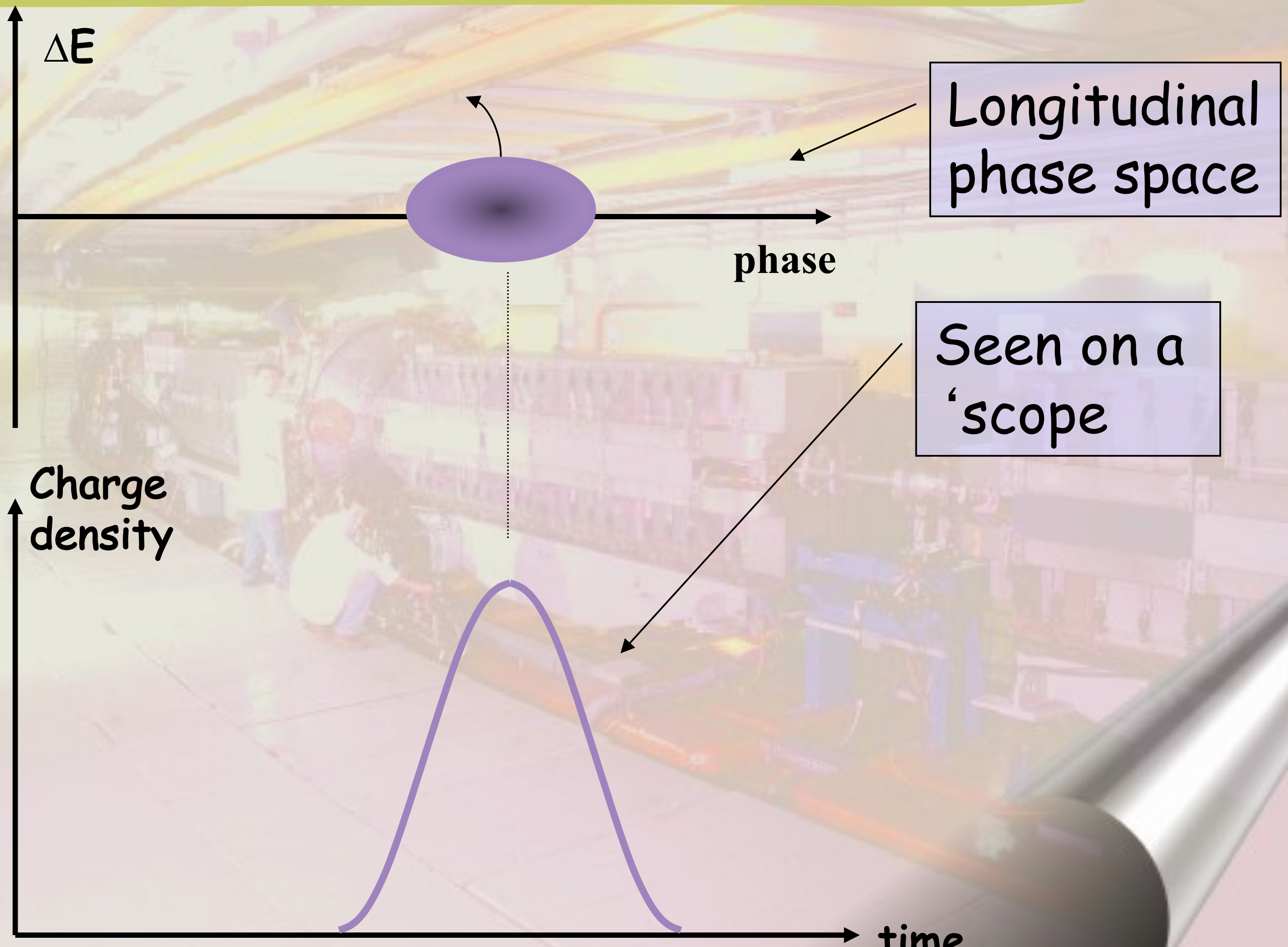
Longitudinal phase space

Seen on a 'scope

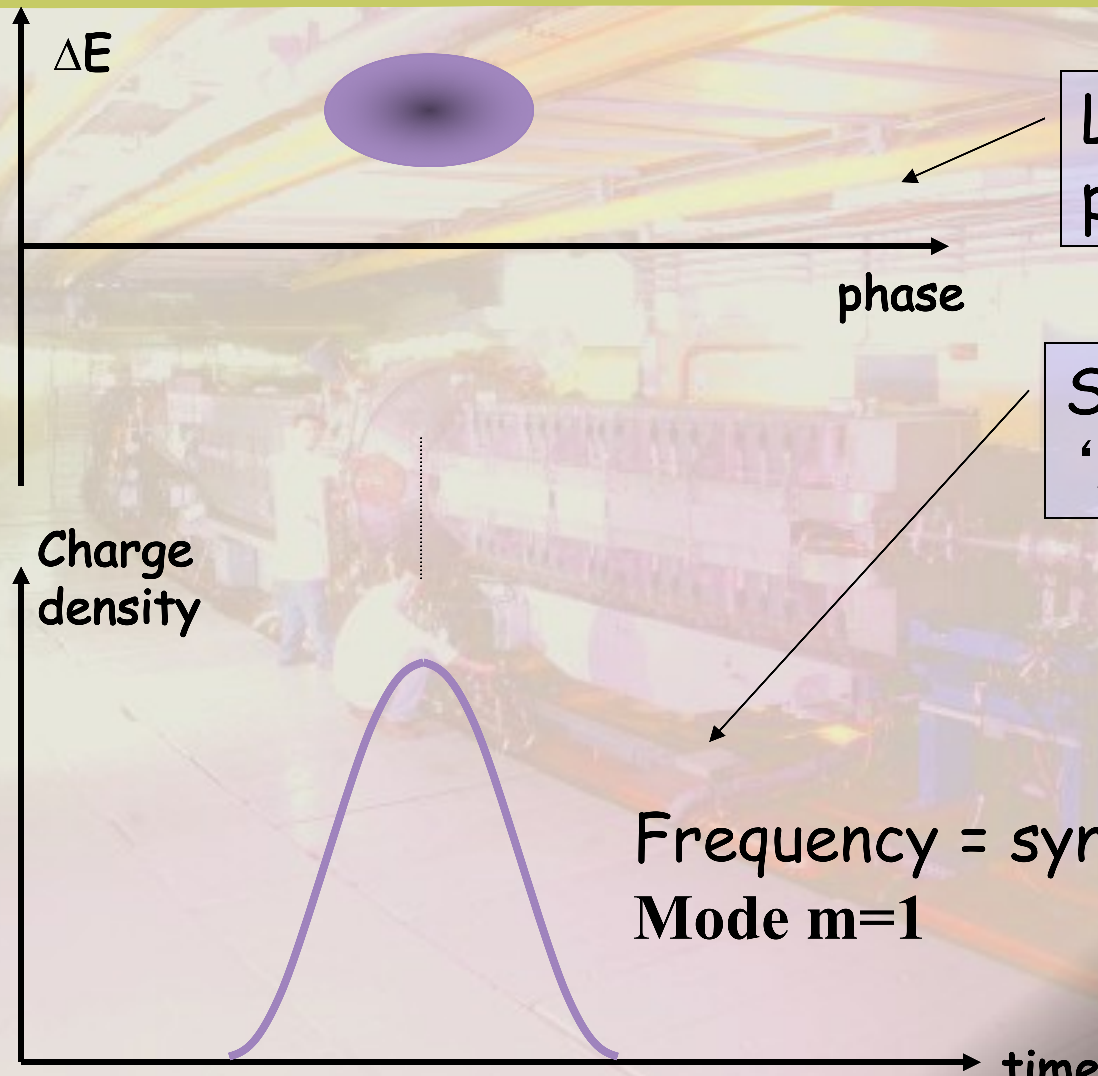
Robinson Instability (3)



Robinson Instability (4)



Robinson Instability (5)



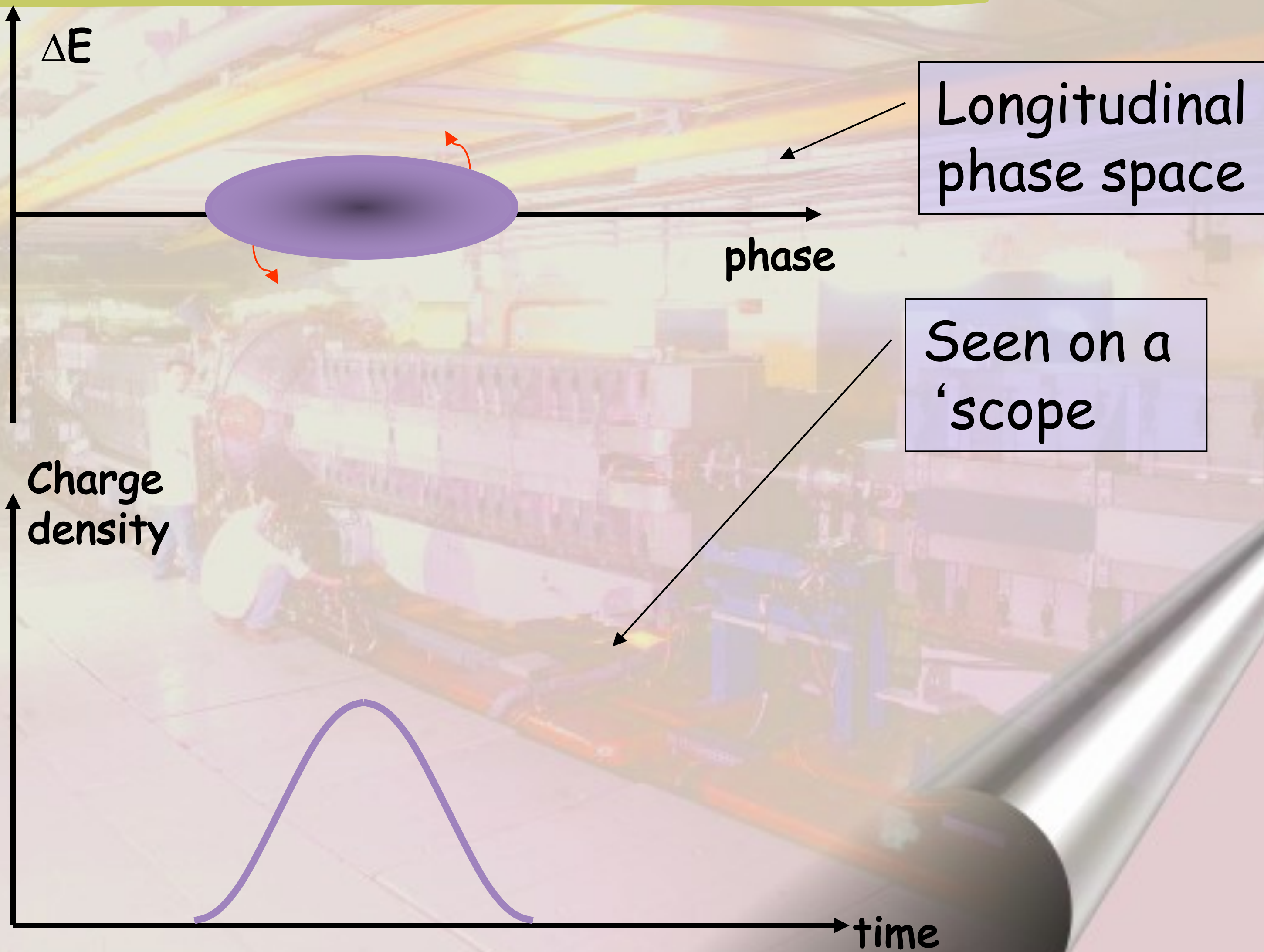
Longitudinal phase space

Seen on a 'scope

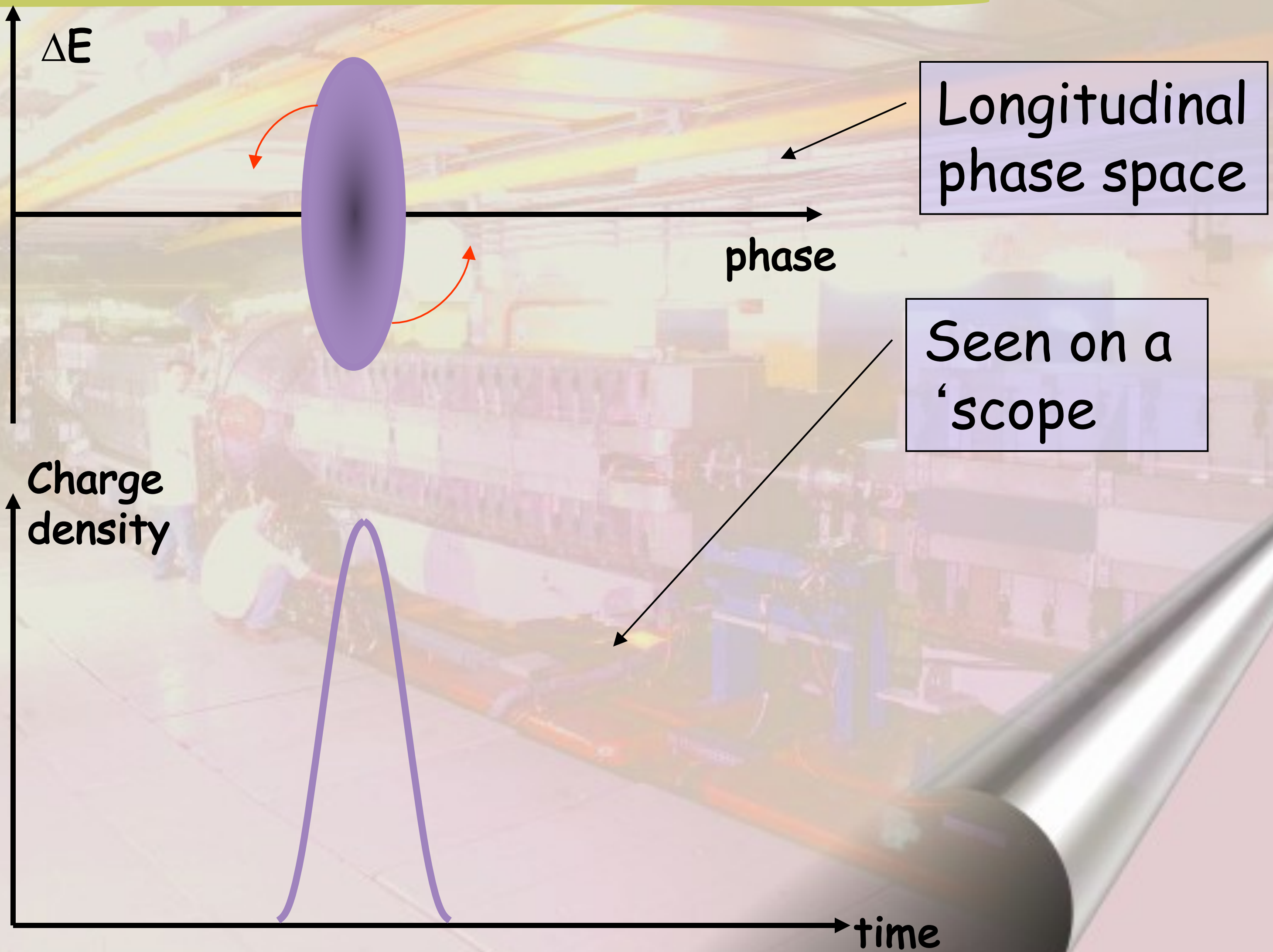
Frequency = synchrotron frequency

Mode m=1

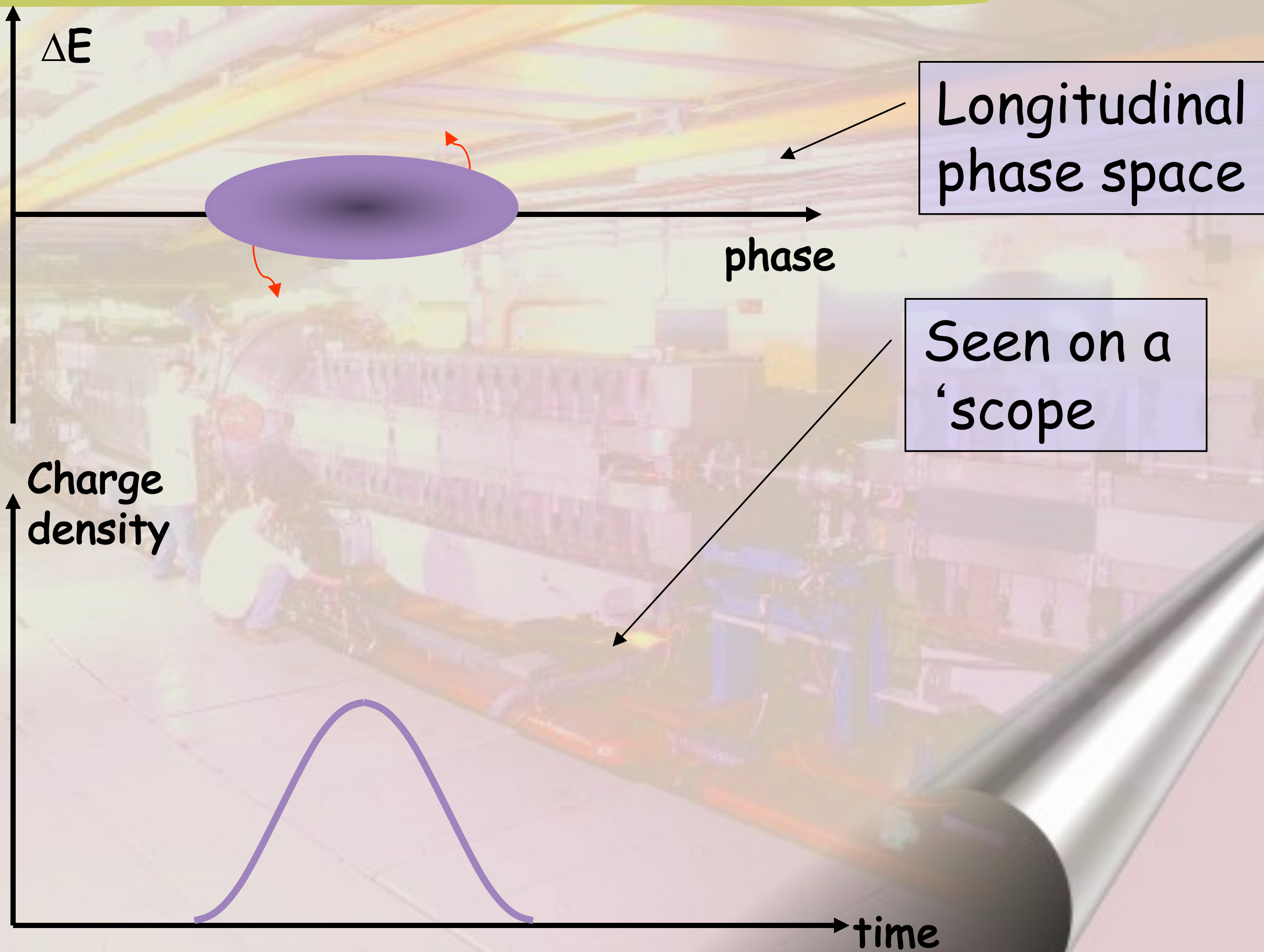
Higher order modes m=2 ….. (1)



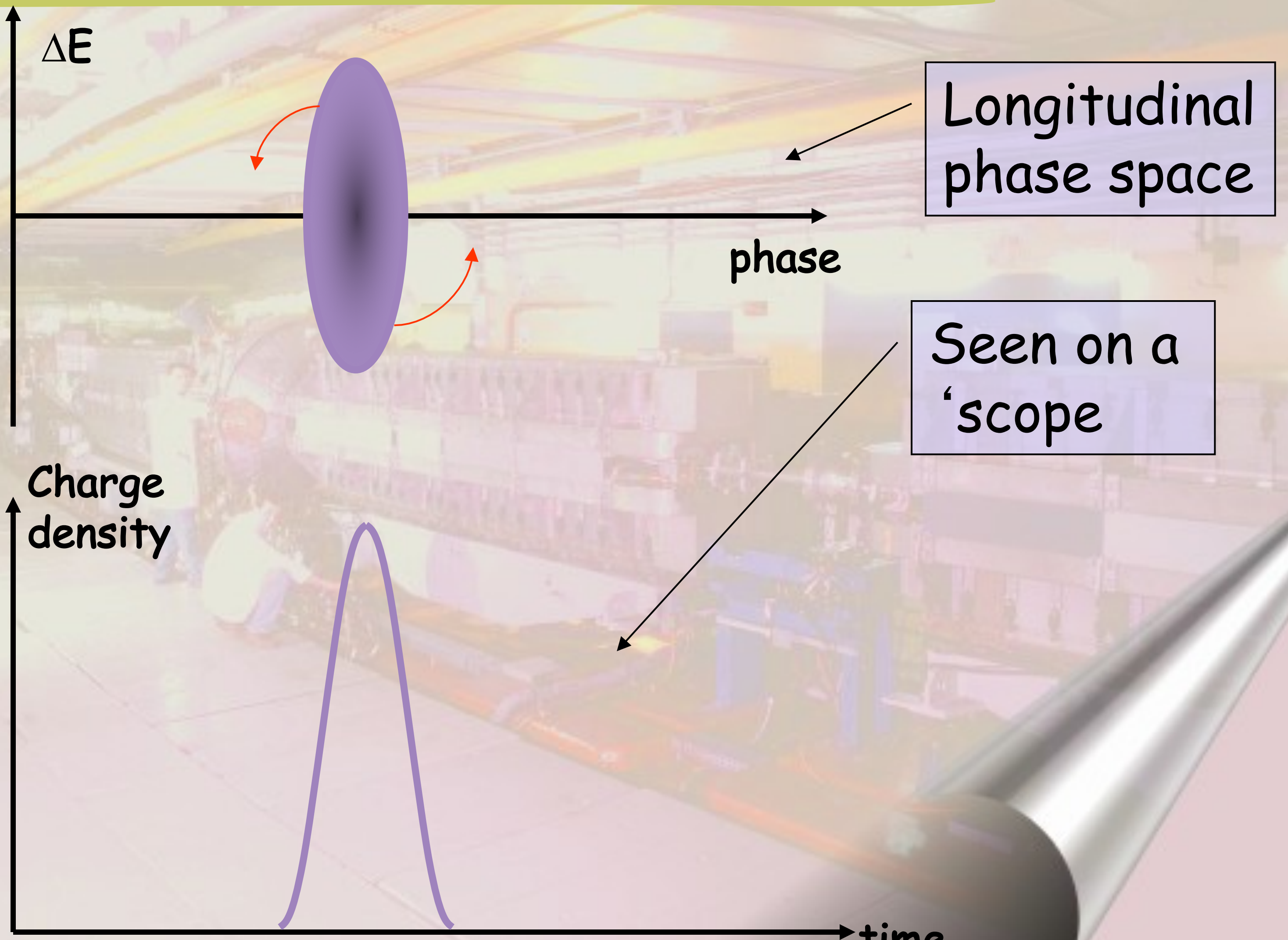
Higher order modes m=2 ..... (2)



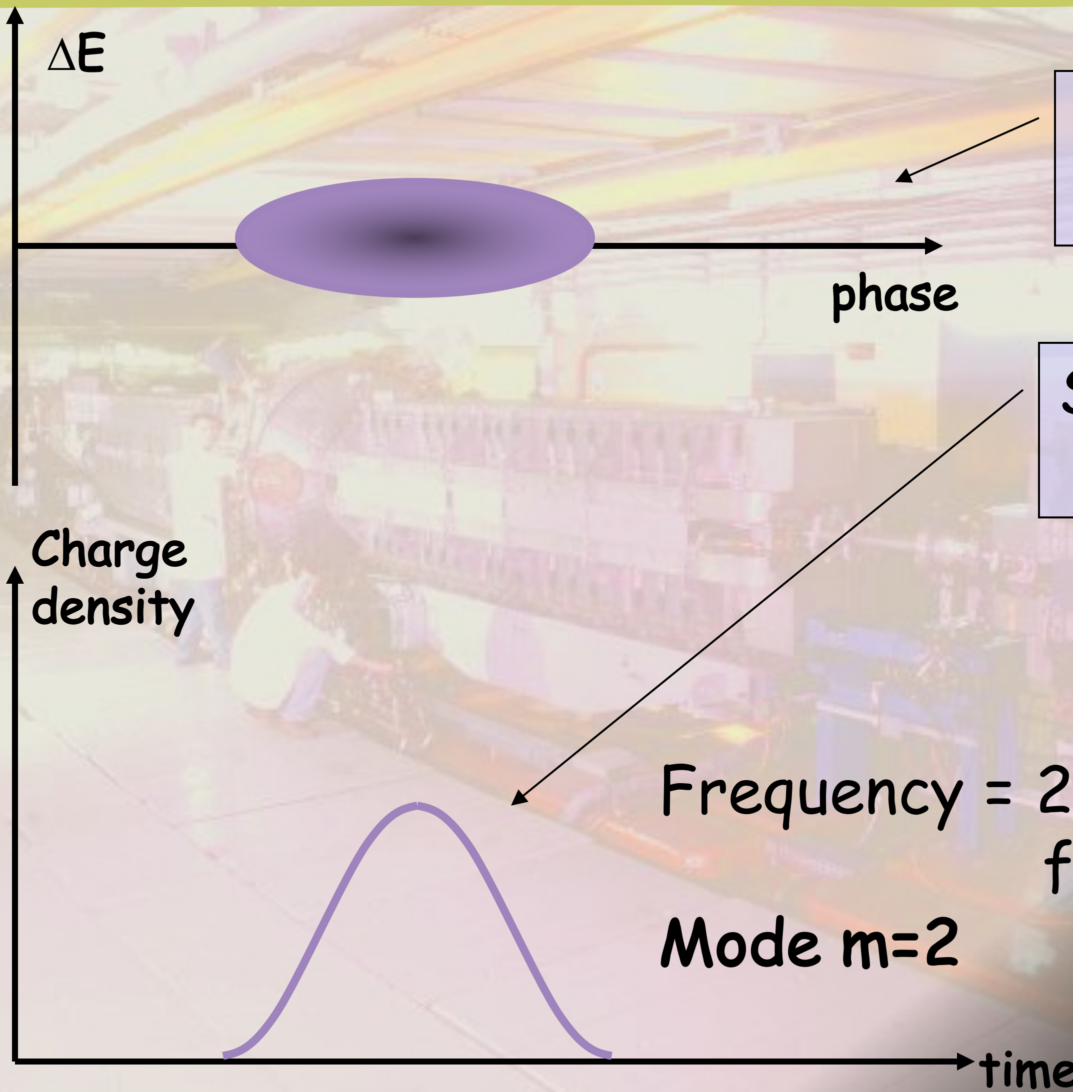
Higher order modes m=2 ..... (3)



Higher order modes m=2 ….. (4)



Higher order modes m=2 ..... (5)



Longitudinal phase space

Seen on a 'scope

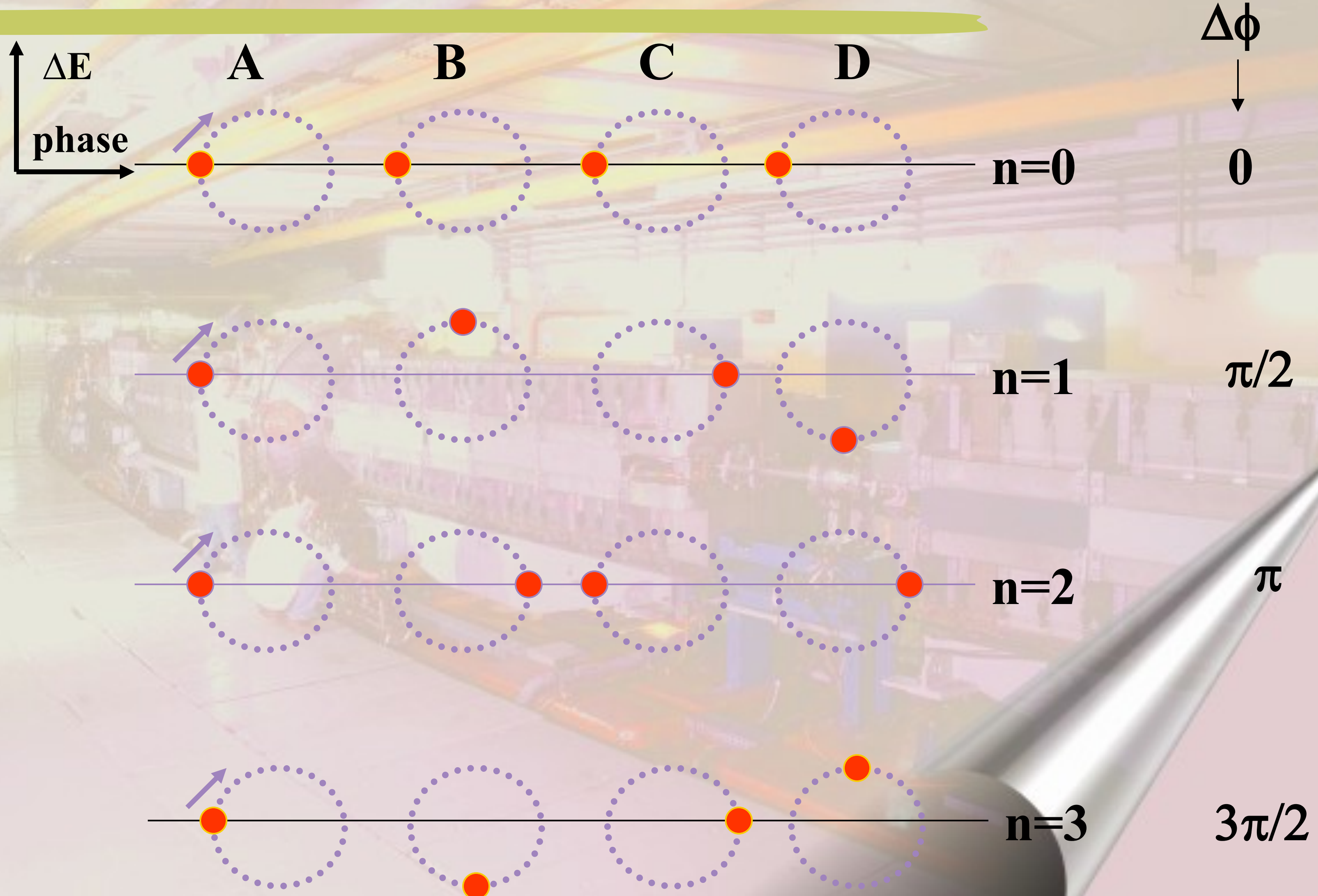
Frequency = 2 × synchrotron frequency

Mode m=2

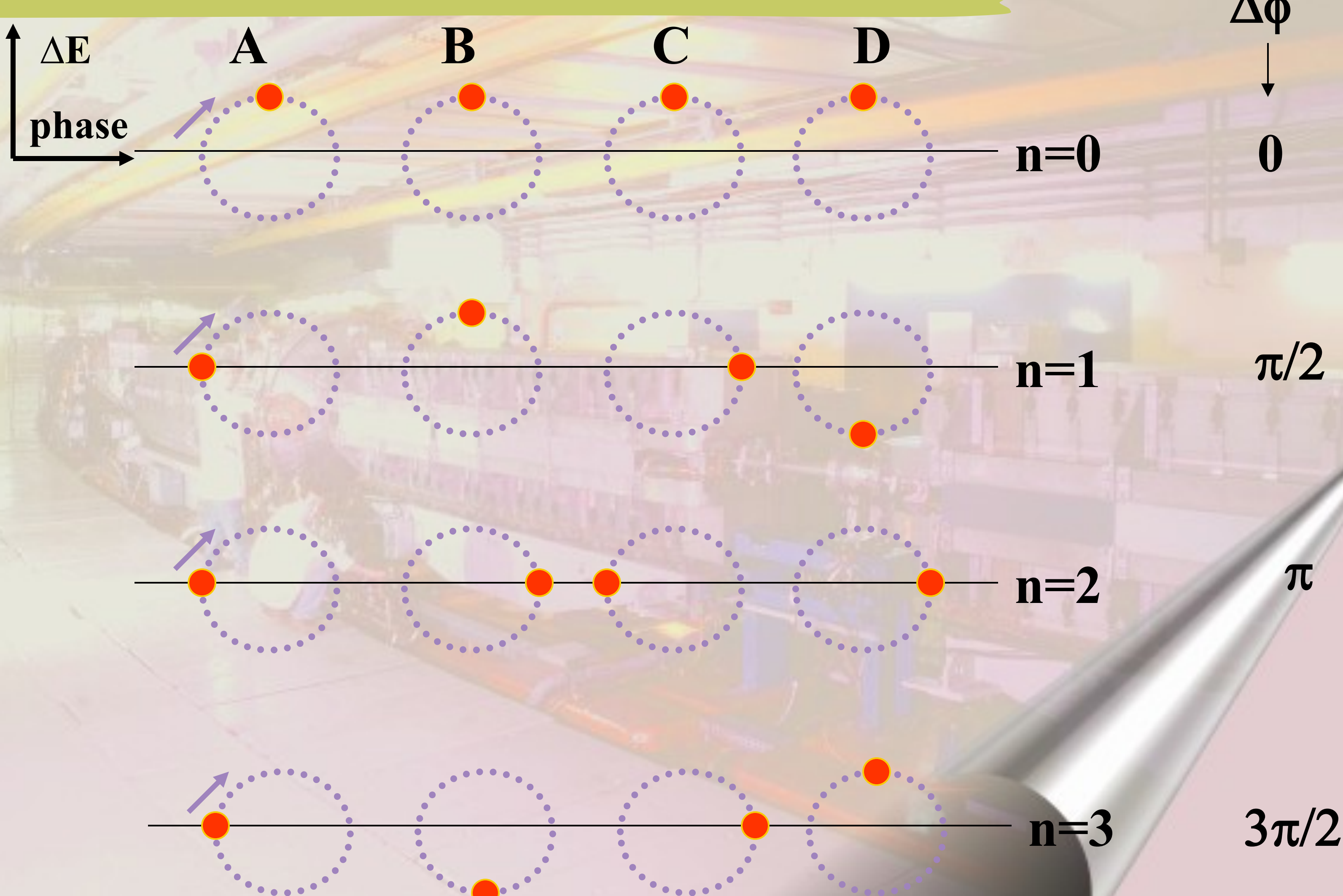
Multi-bunch instabilities (1)

- What if we have more than one bunch in our ring.....?
- Lets take 4 equidistant bunches **A**, **B**, **C** & **D**
- The field left in the cavity by bunch **A** alters the coherent synchrotron oscillation motion of **B**, which changes field left by bunch **B**, which alters bunch **C**......to bunch **D**, etc...etc..
- Until we get back to bunch **A**.....
- For 4 bunches there are 4 possible modes of coupled bunch longitudinal oscillation

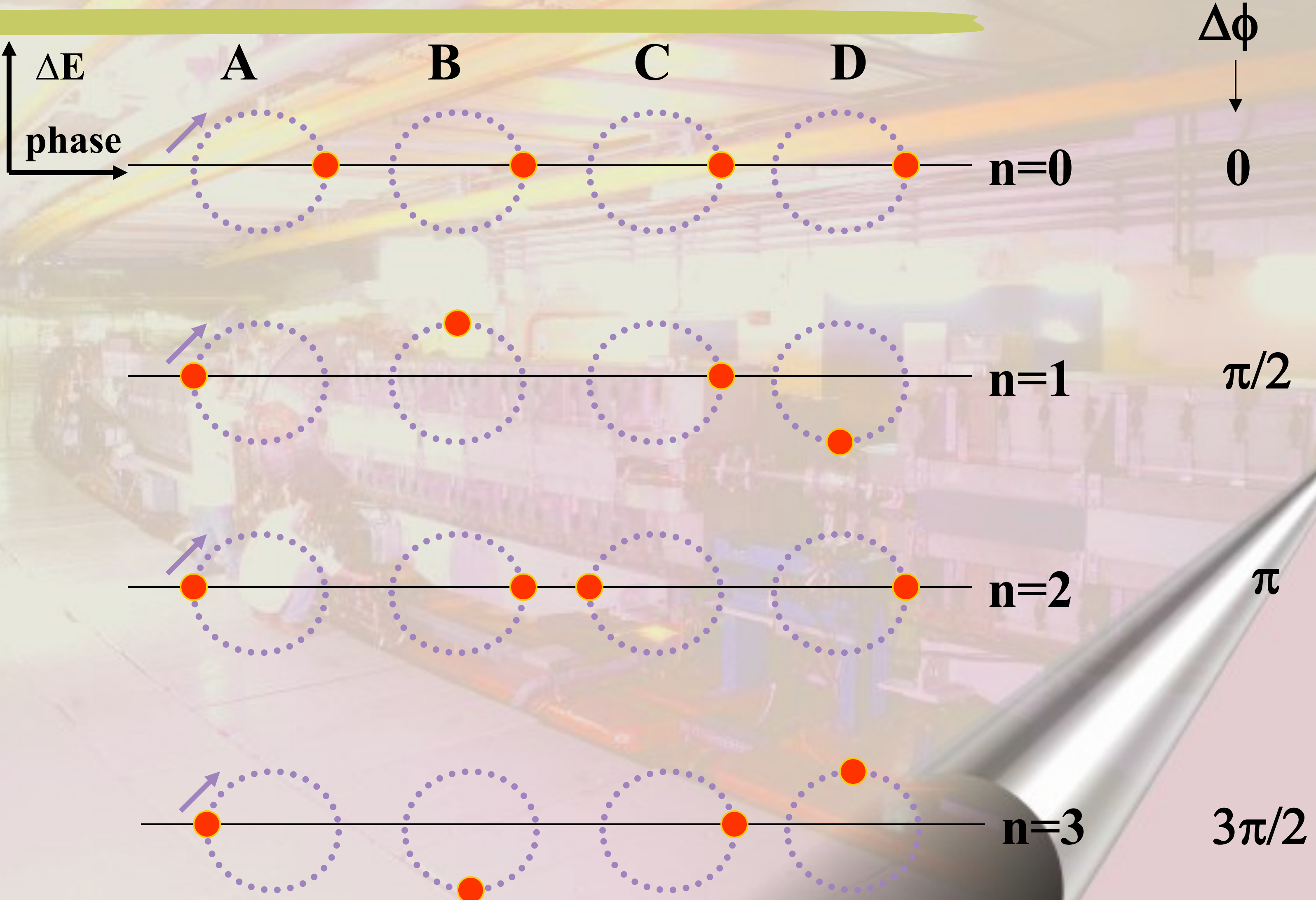
Multi-bunch instabilities (2)



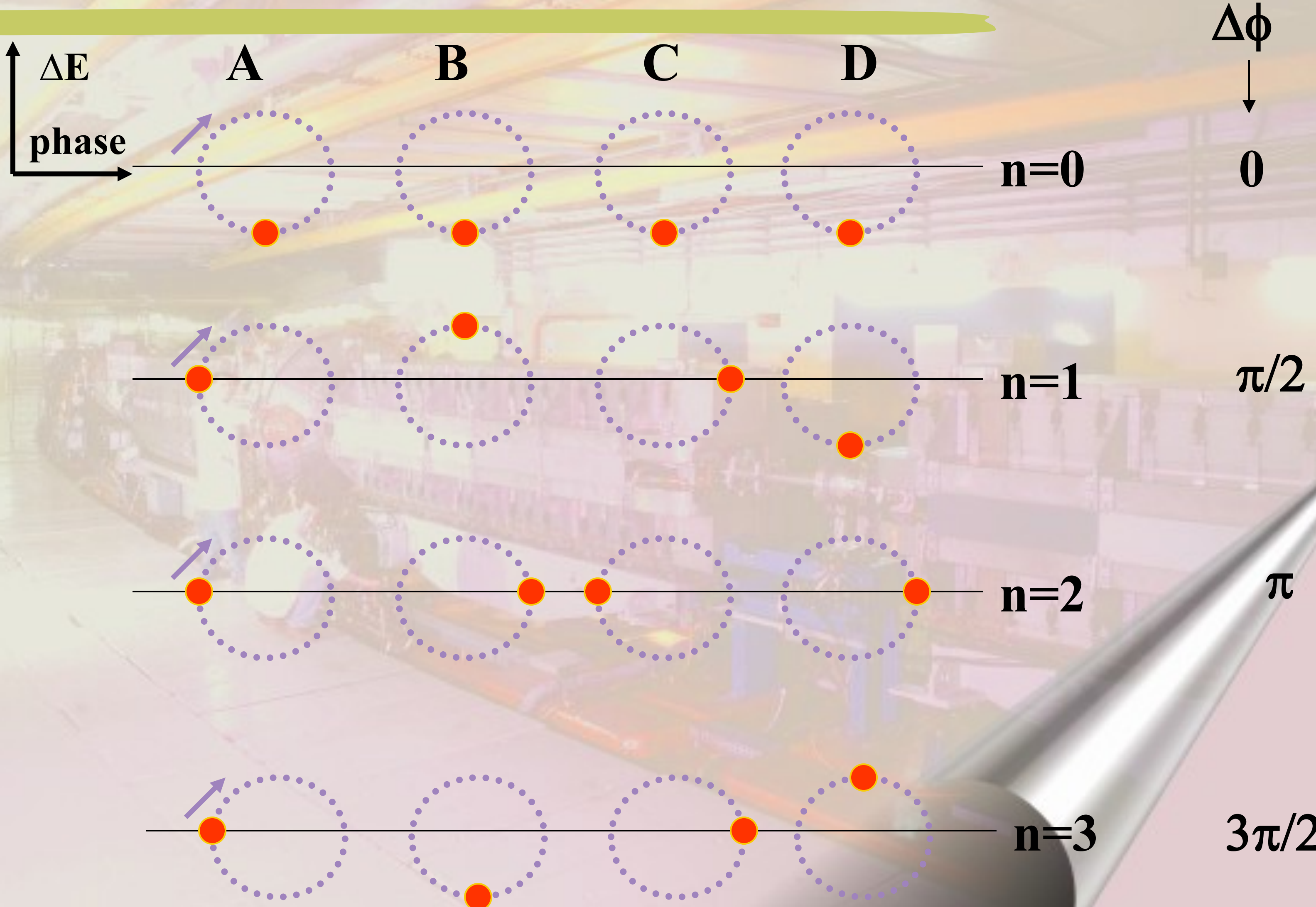
Multi-bunch instabilities (3)



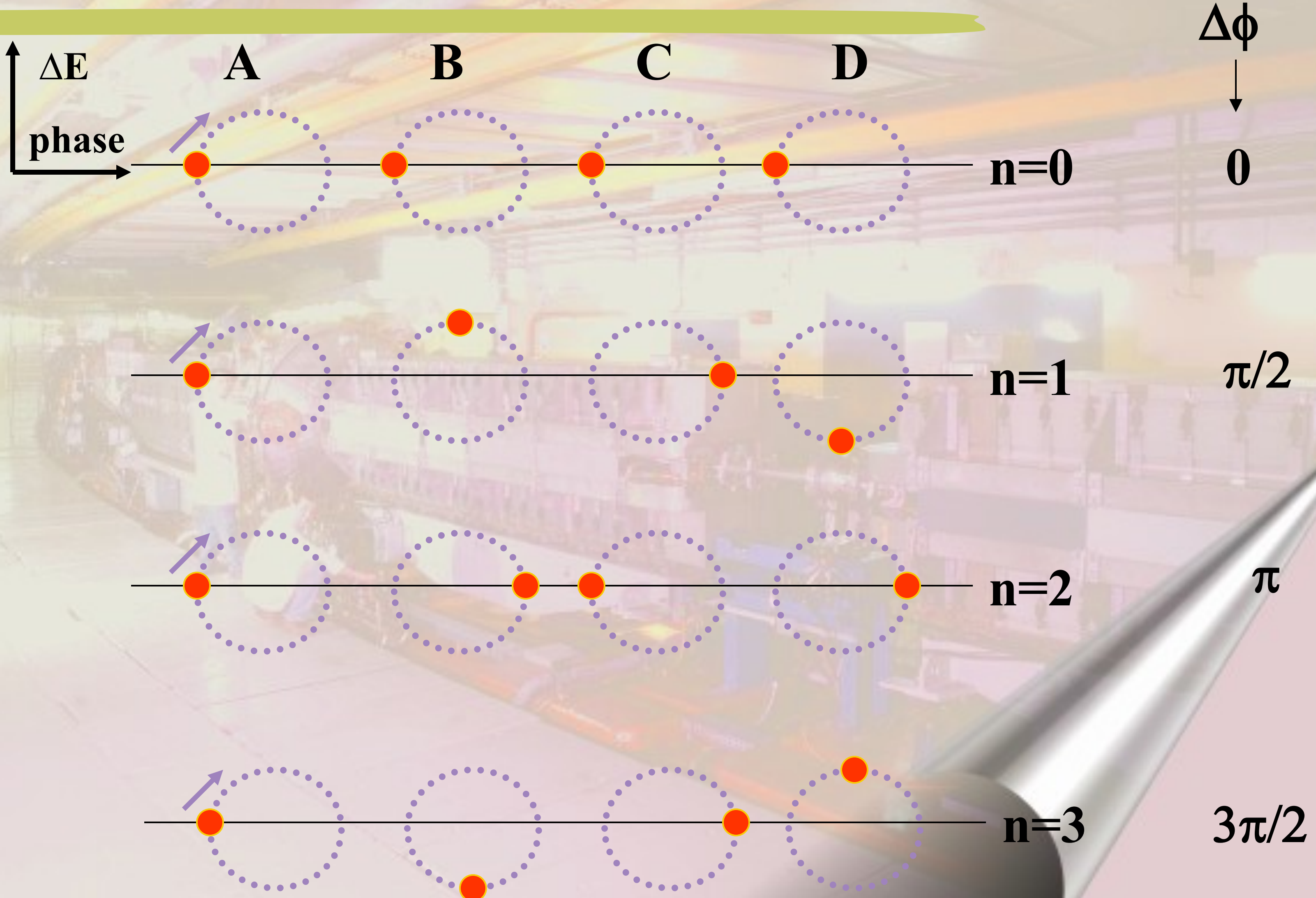
Multi-bunch instabilities (4)



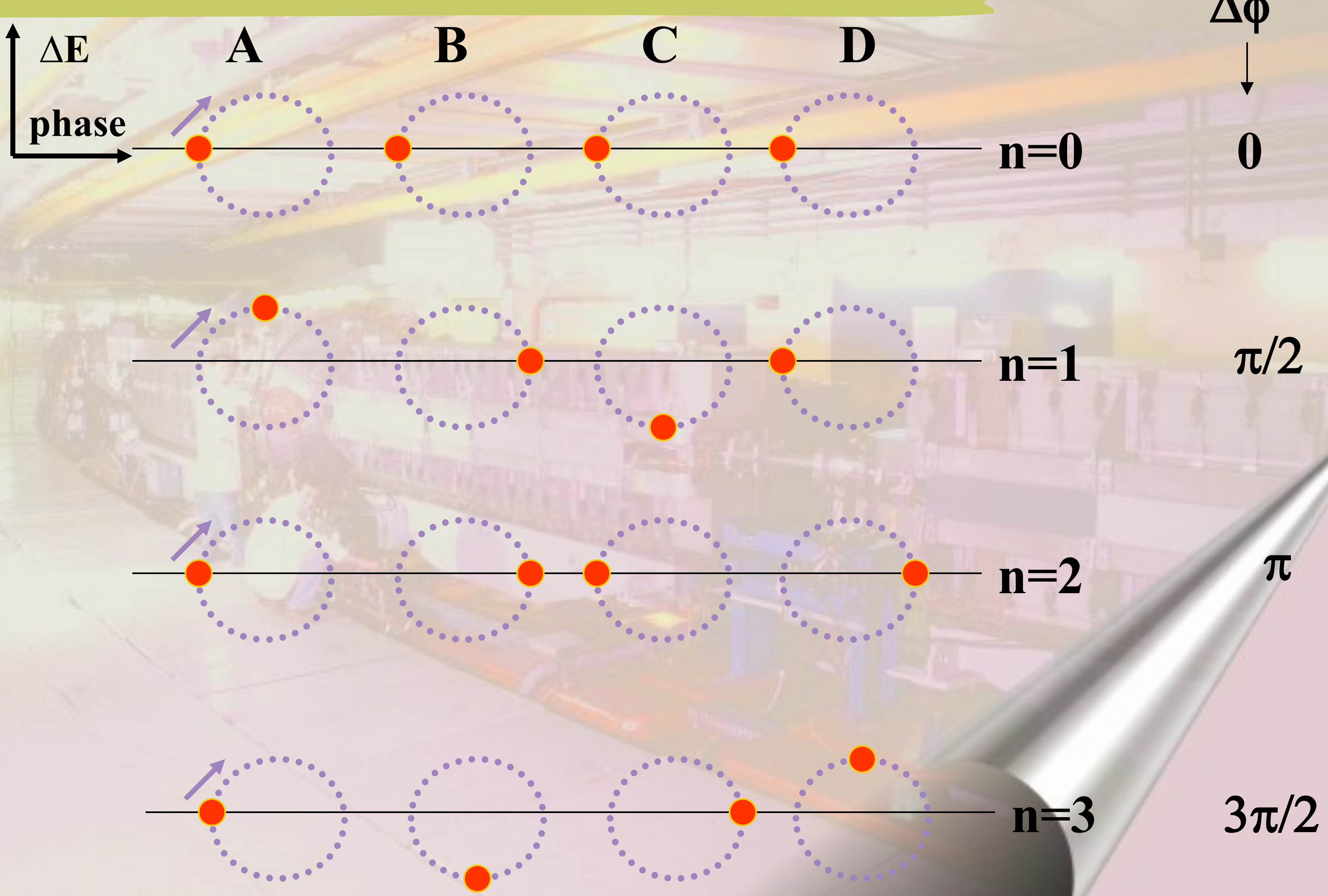
Multi-bunch instabilities (5)



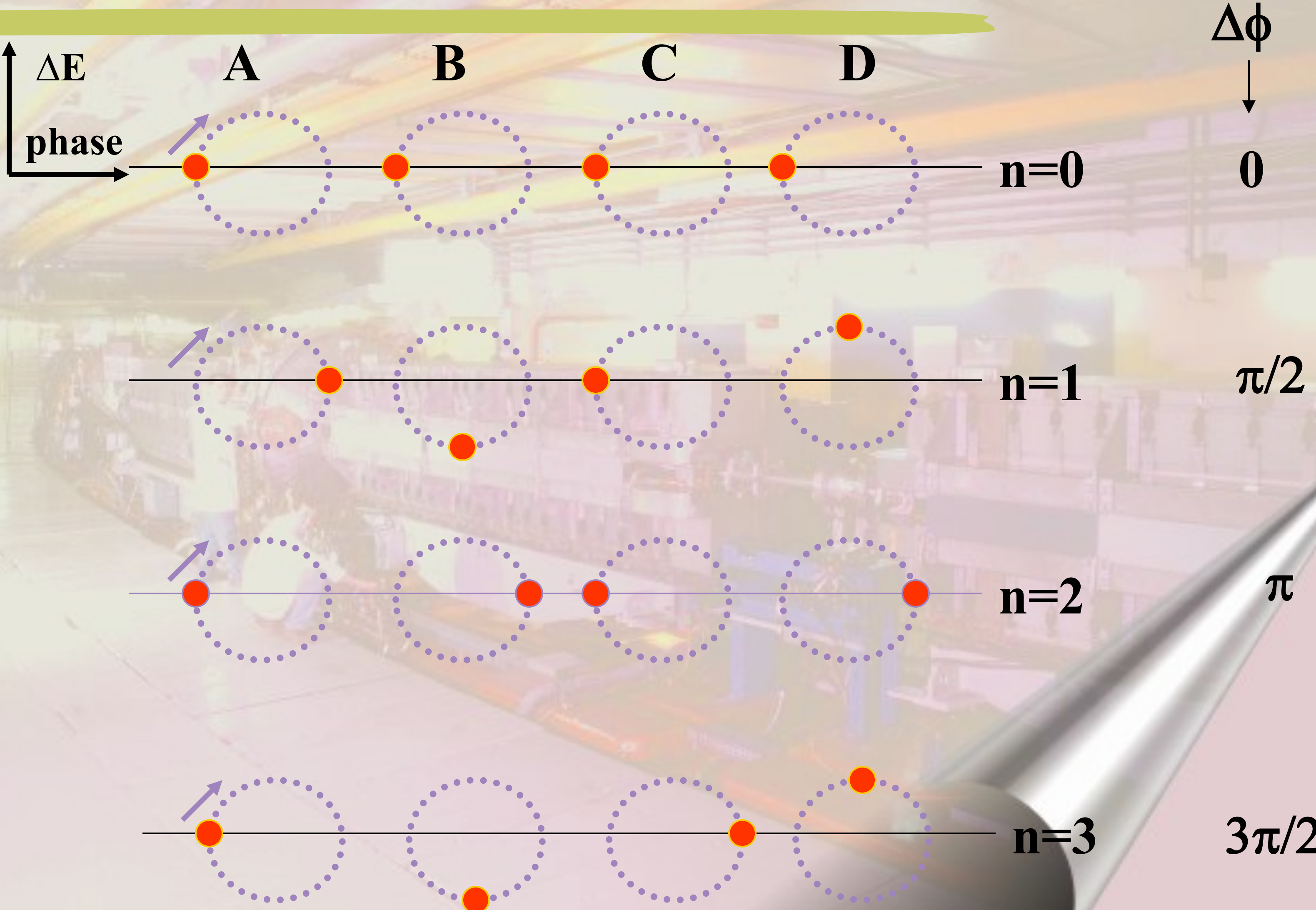
Multi-bunch instabilities (6)



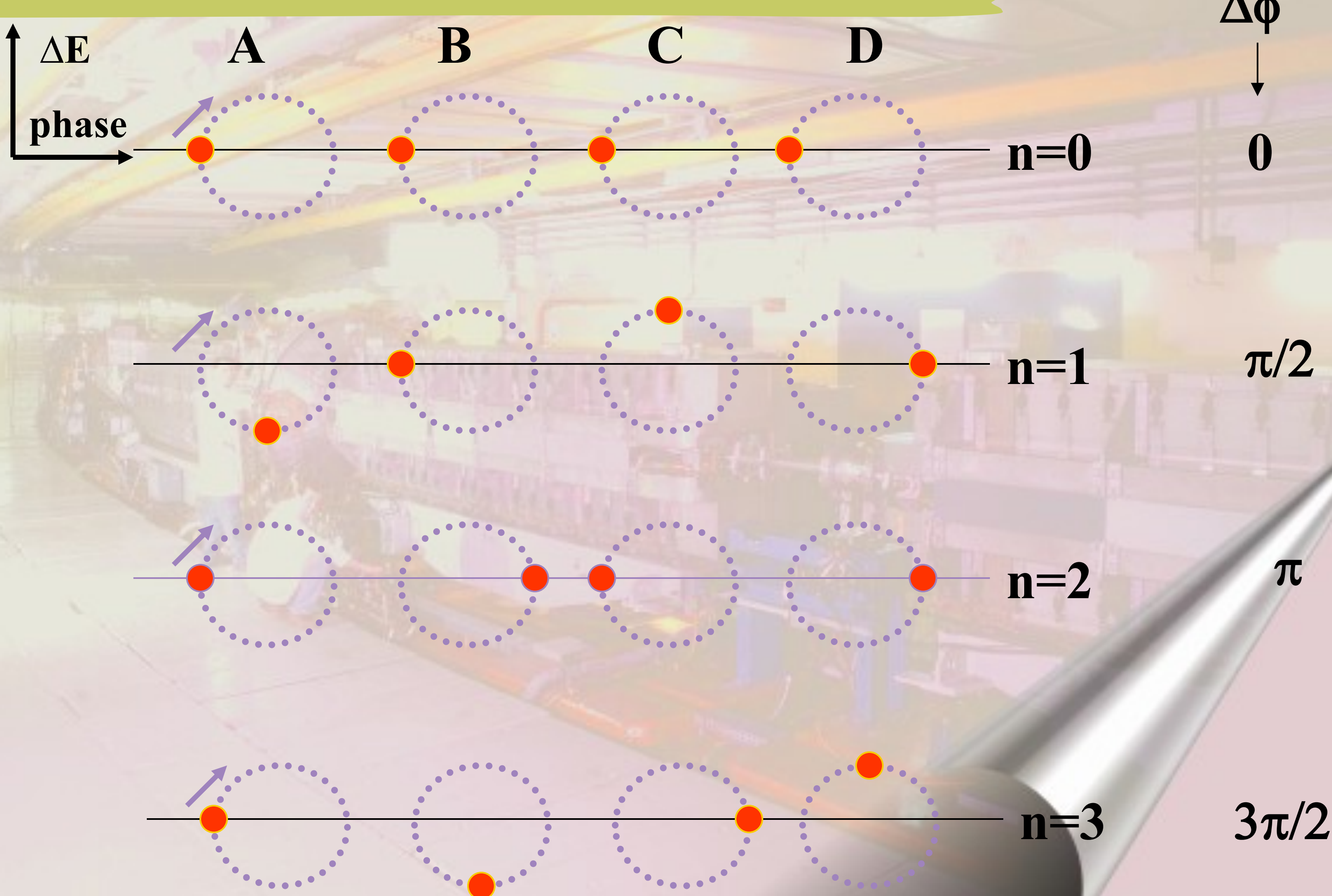
Multi-bunch instabilities (7)



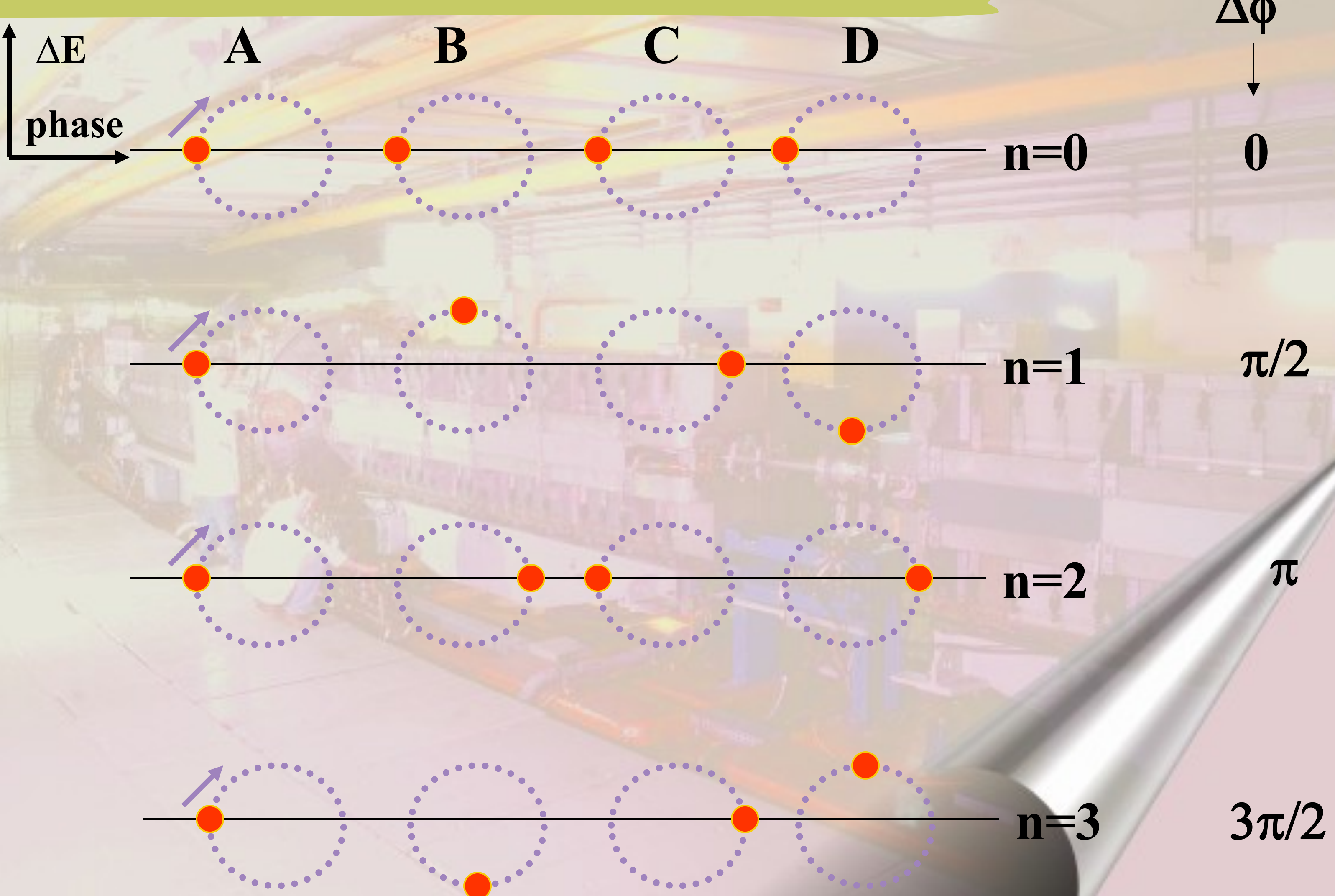
Multi-bunch instabilities (8)



Multi-bunch instabilities (9)

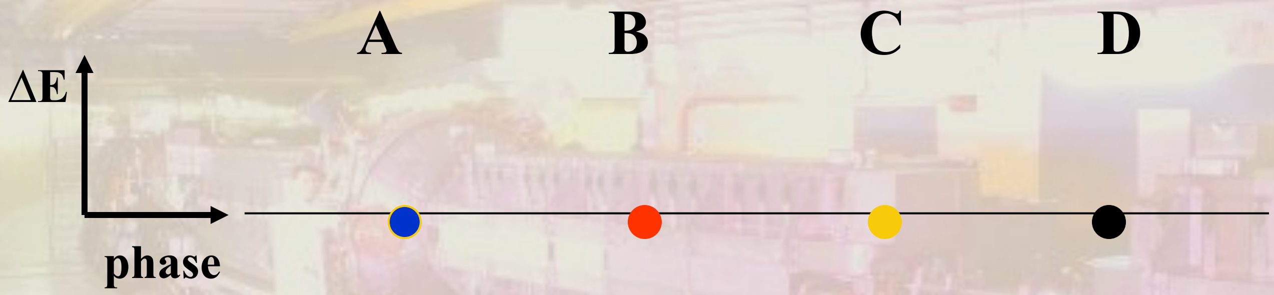


Multi-bunch instabilities (10)



Multi-bunch instabilities (11)

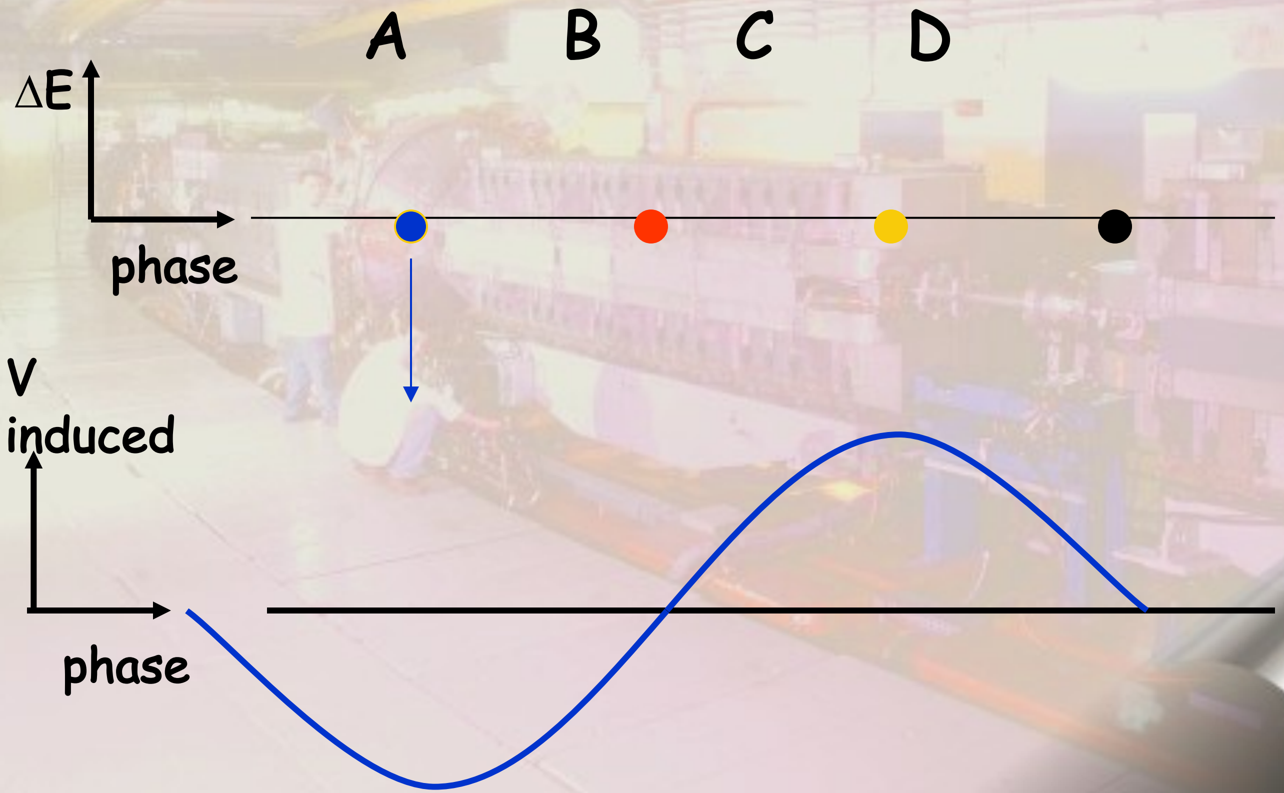
- For simplicity assume we have a single cavity which resonates at the revolution frequency
- With no coherent synchrotron oscillation we have:



- Lets have a look at the voltage induced in a cavity by each bunch

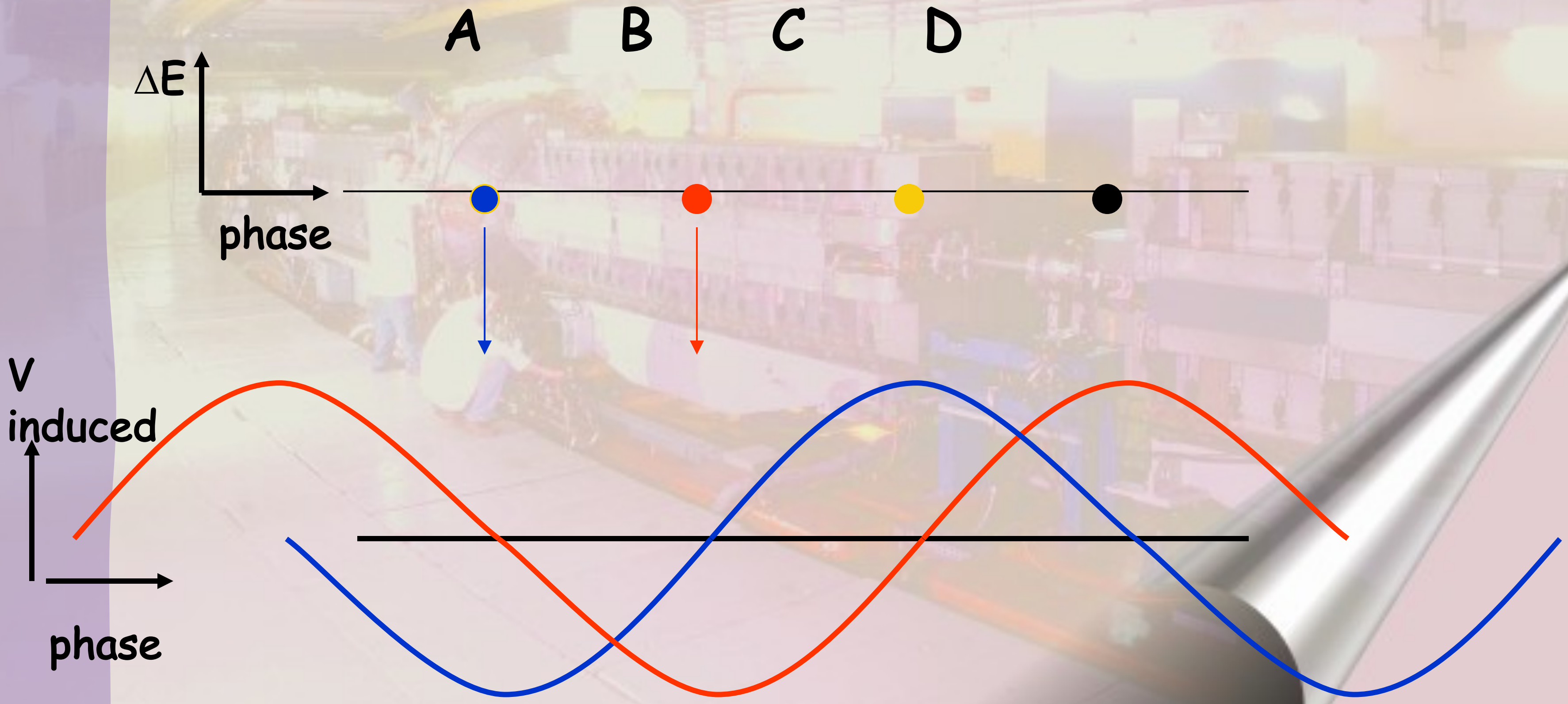
Multi-bunch instabilities (12)

Bunch A



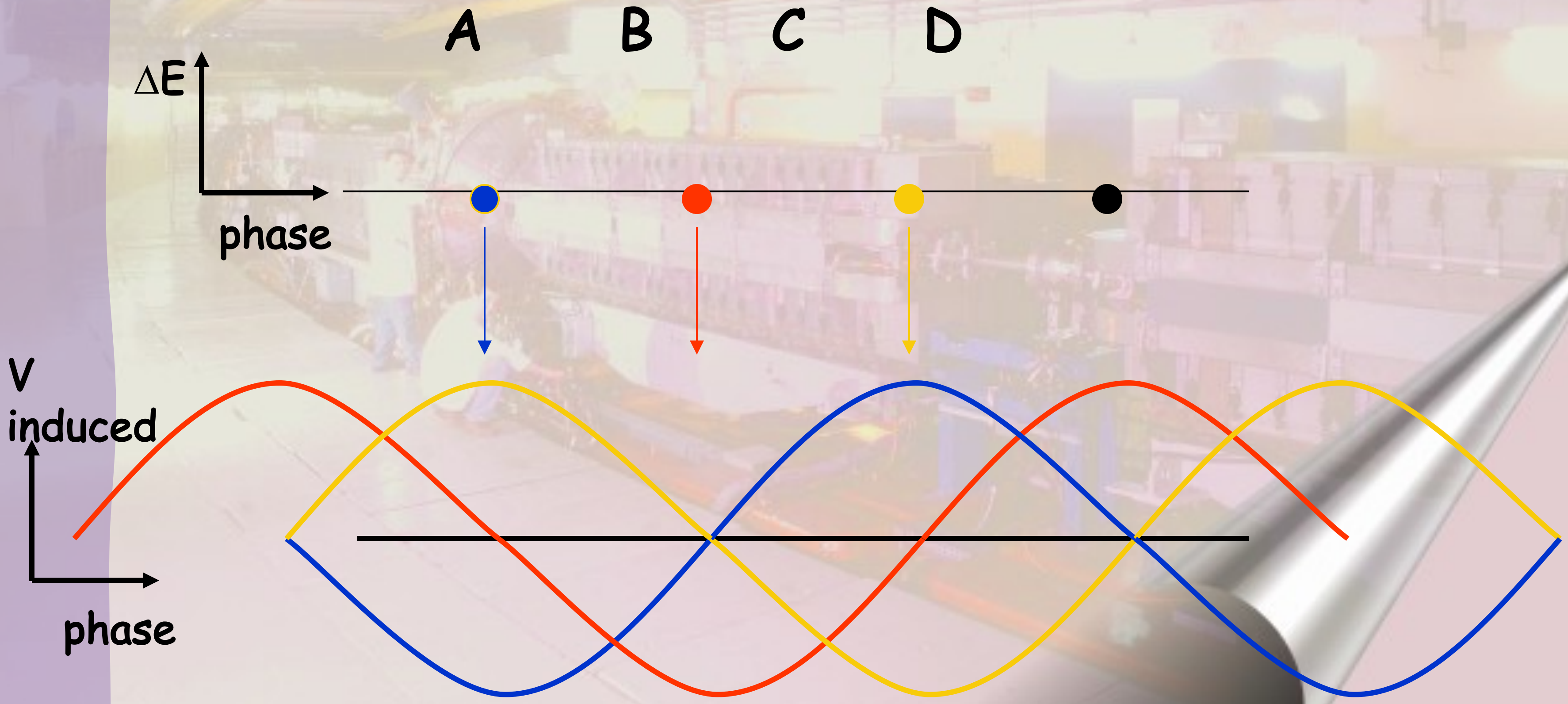
Multi-bunch instabilities (13)

Bunch B



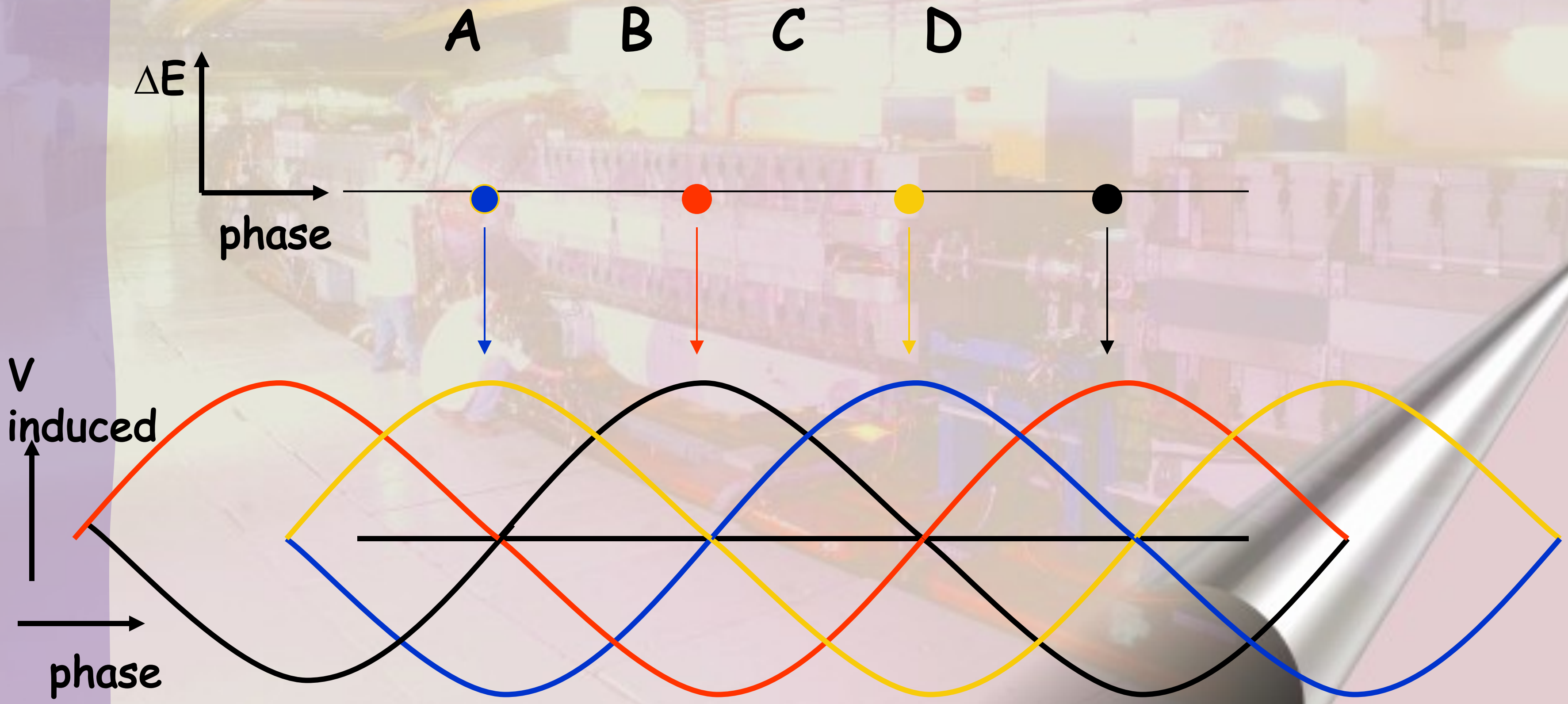
Multi-bunch instabilities (14)

Bunch C



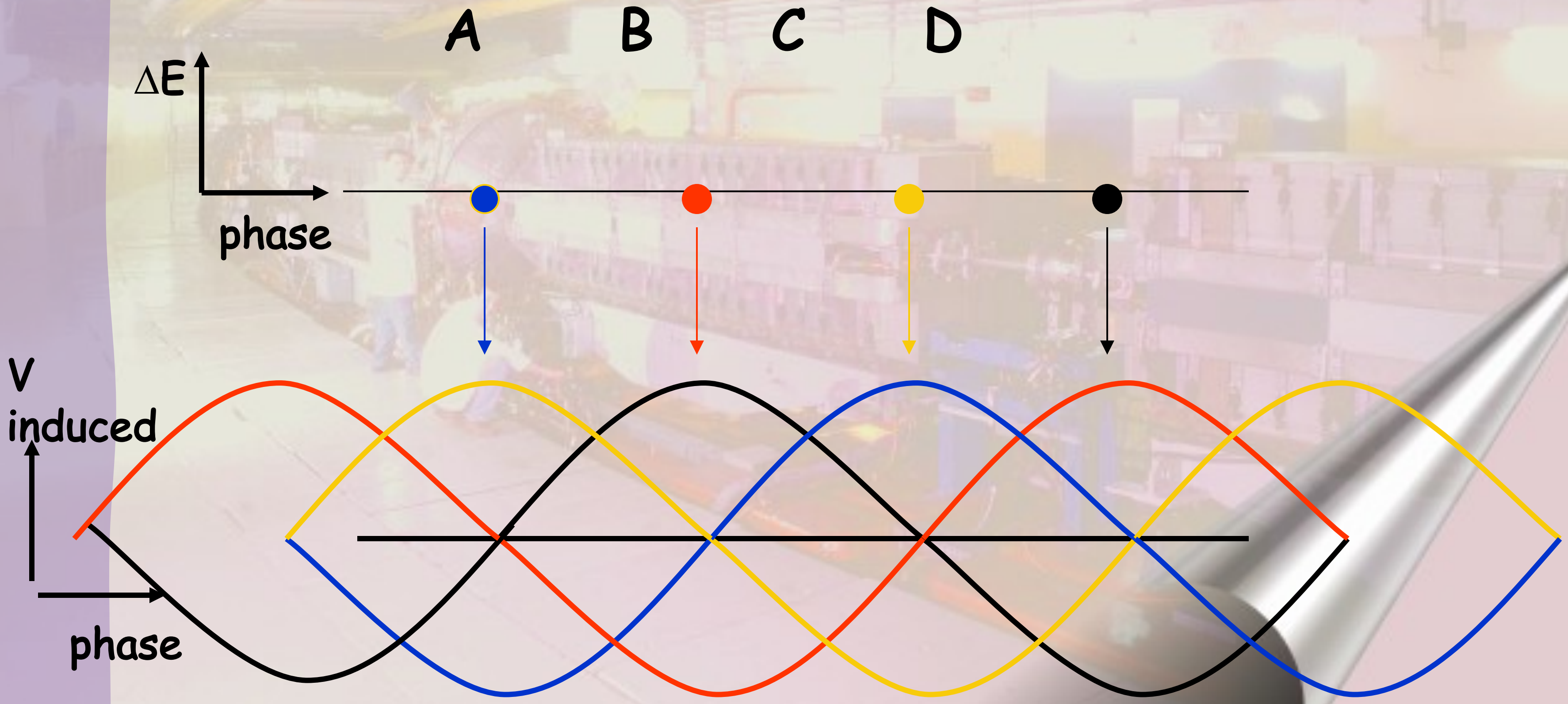
Multi-bunch instabilities (15)

Bunch D



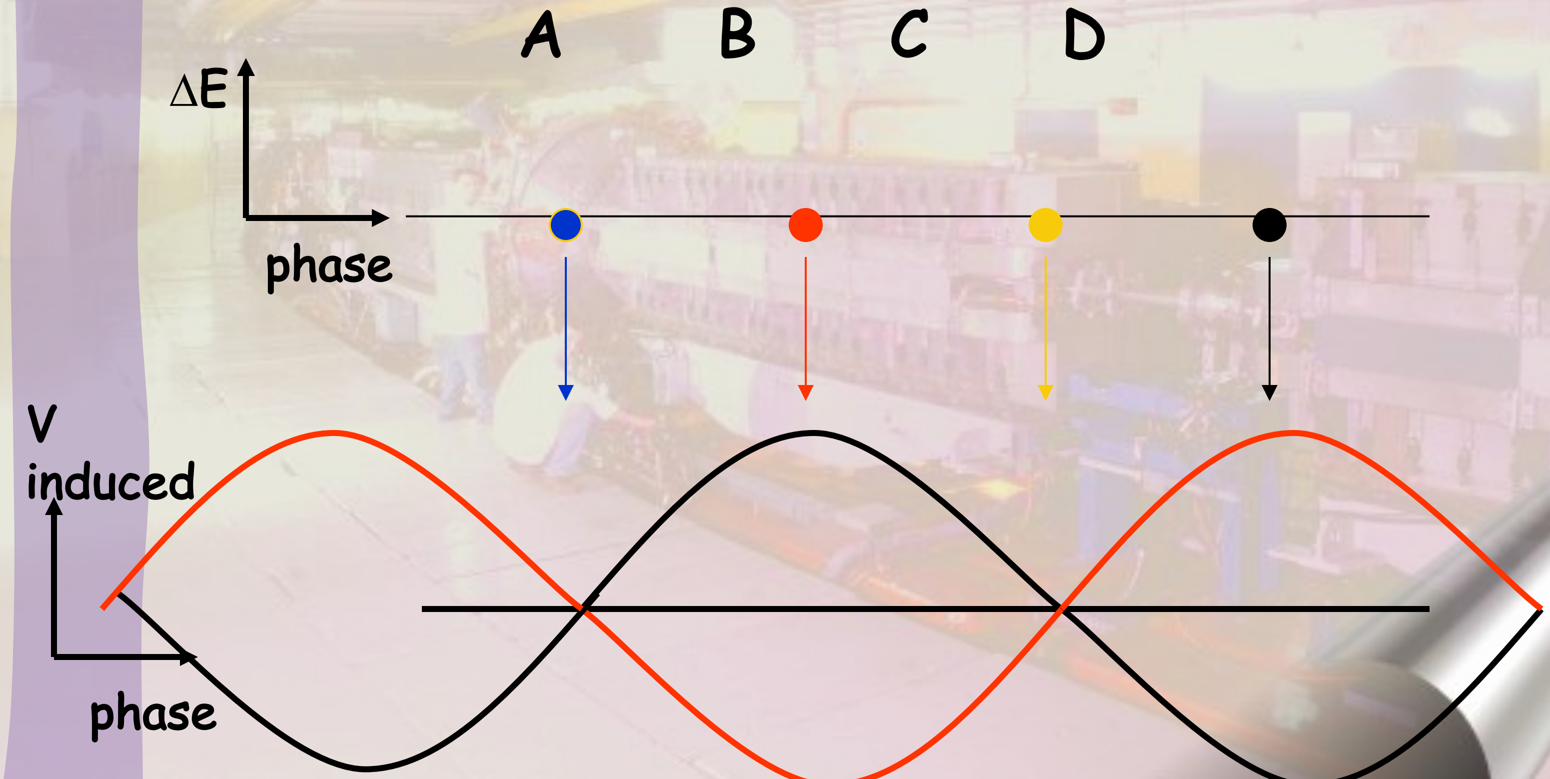
Multi-bunch instabilities (16)

A & C induced voltages cancel



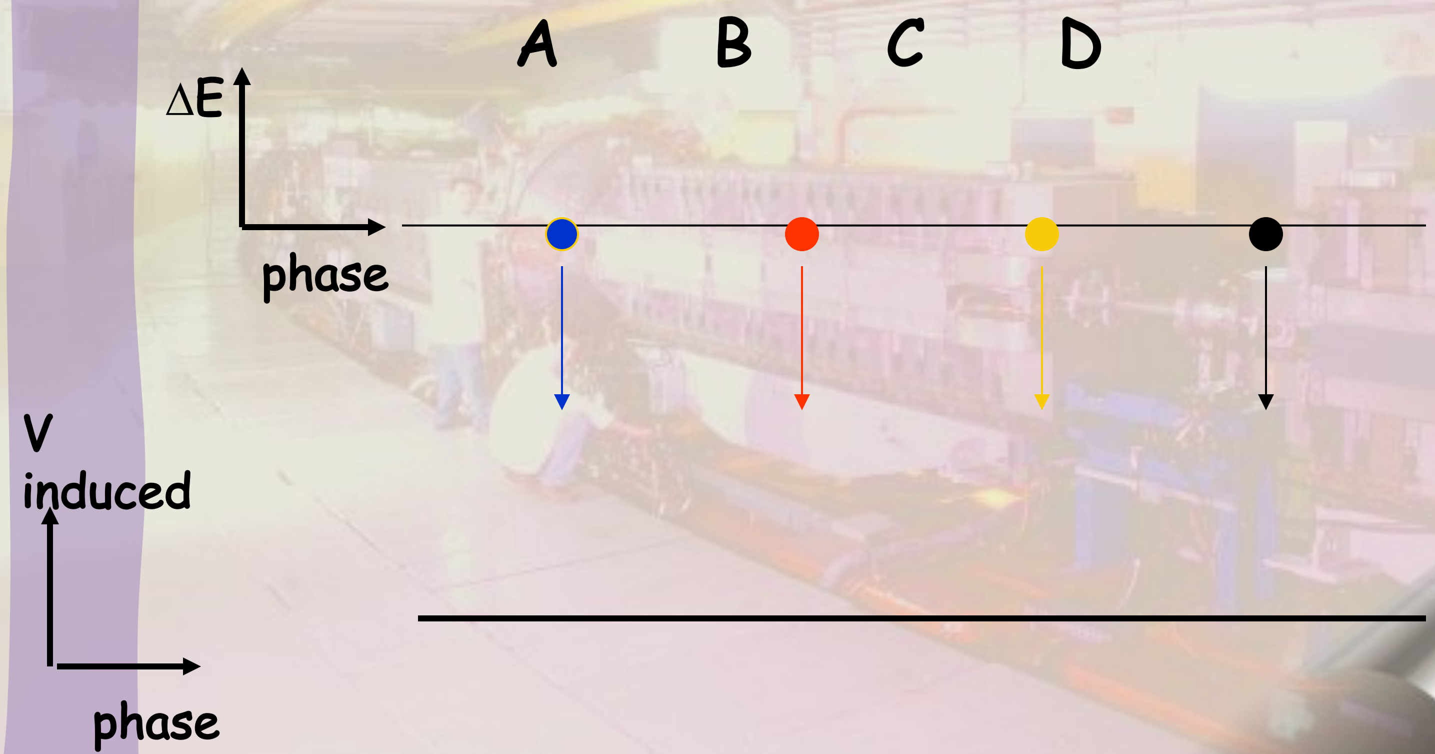
Multi-bunch instabilities (17)

B & D induced voltages cancel

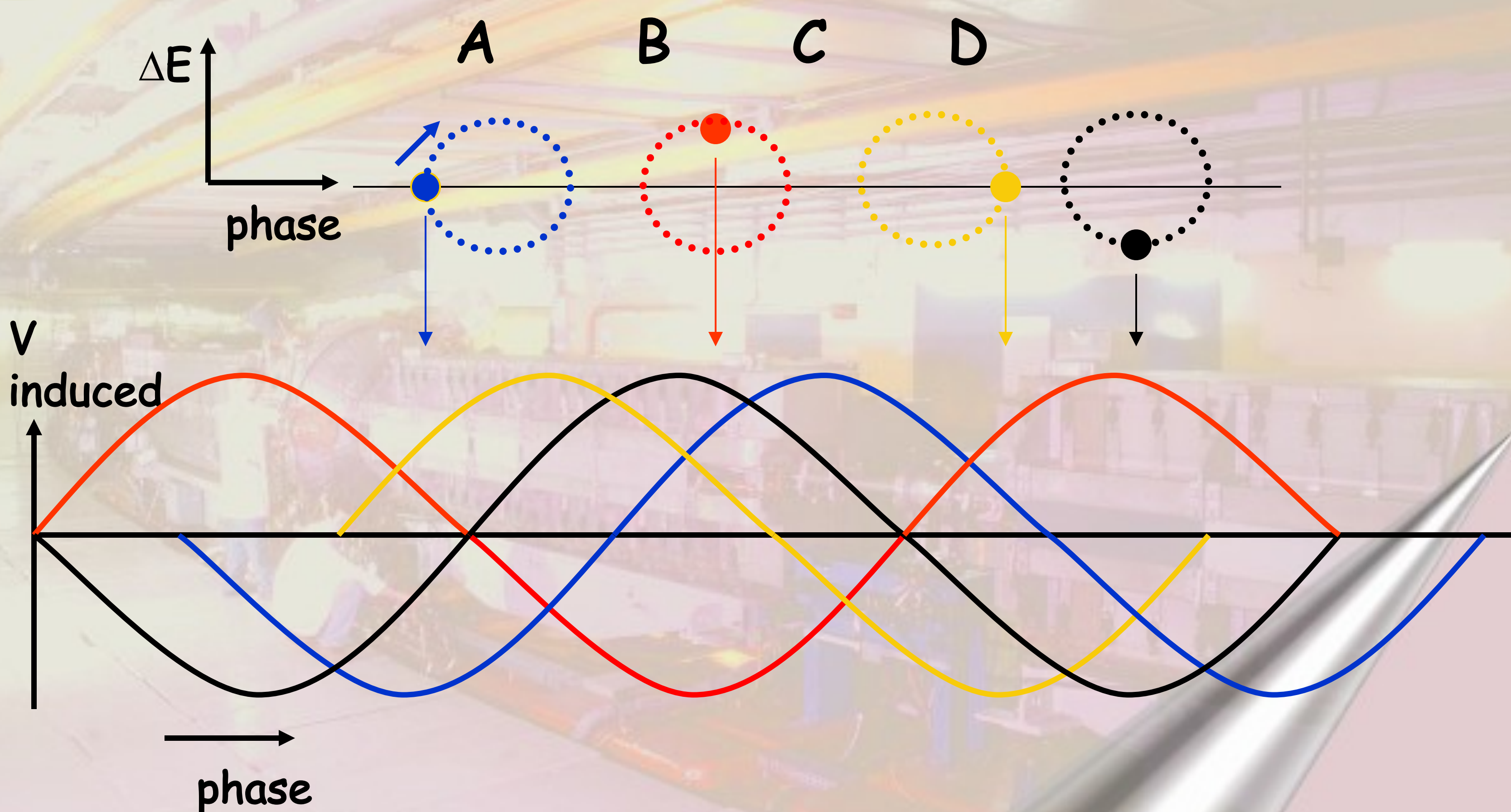


Multi-bunch instabilities (18)

All voltages cancel ⇒ no residual effect



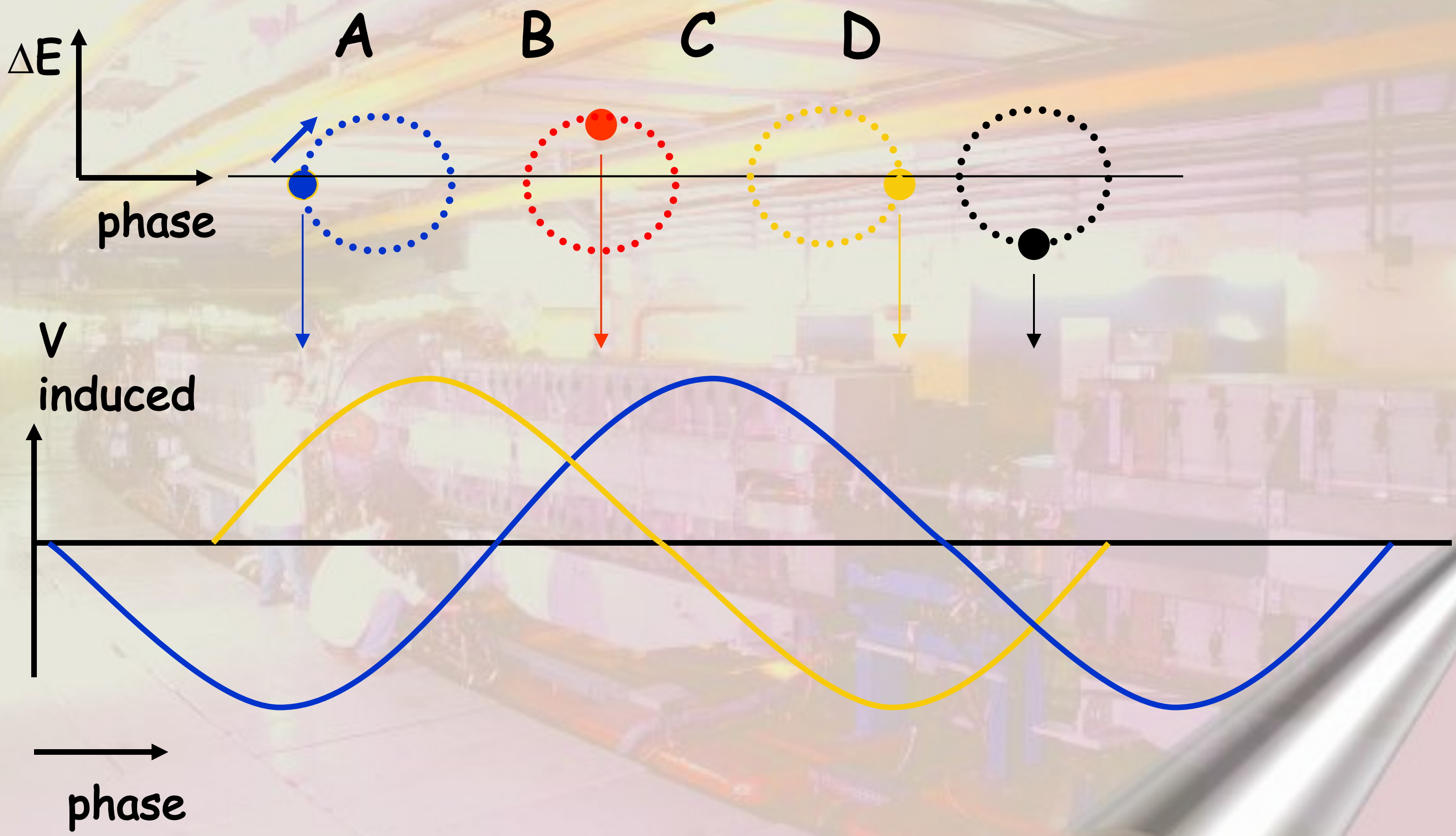
Multi-bunch instabilities (19)



Lets Introduce an **n=1** mode coupled bunch oscillation

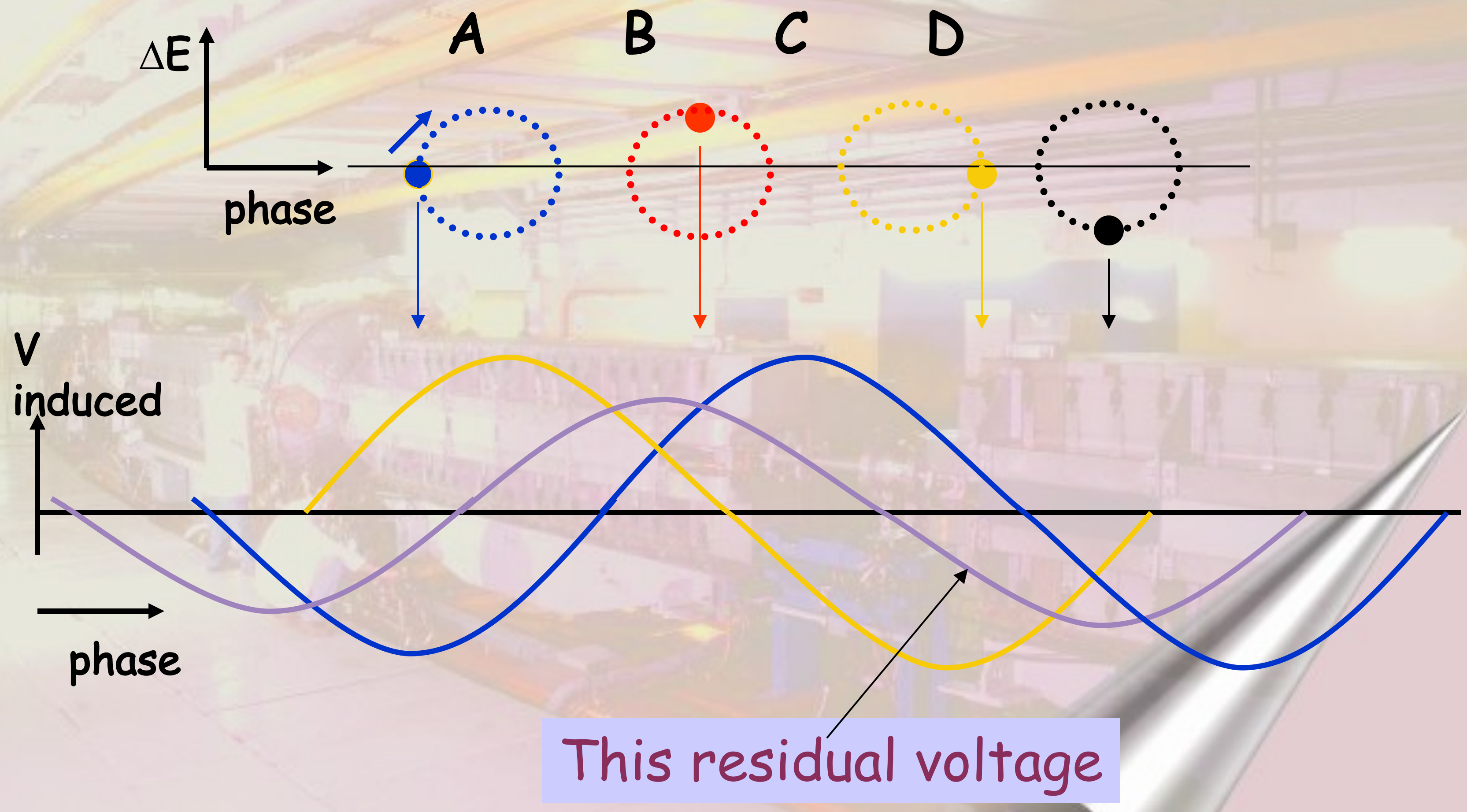
B & D induced voltages cancel

Multi-bunch instabilities (20)

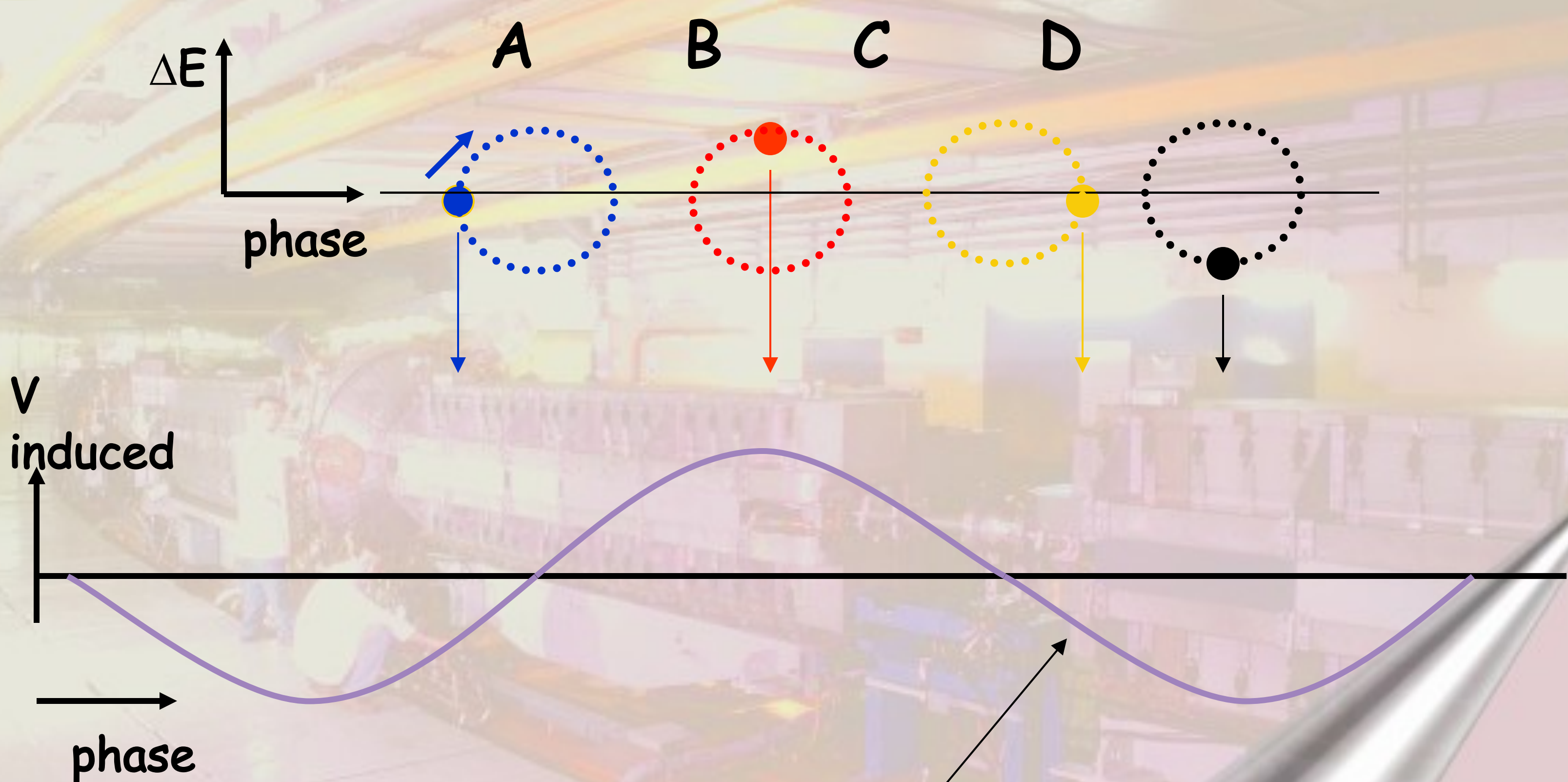


A & C induced voltages do not cancel

Multi-bunch instabilities (21)



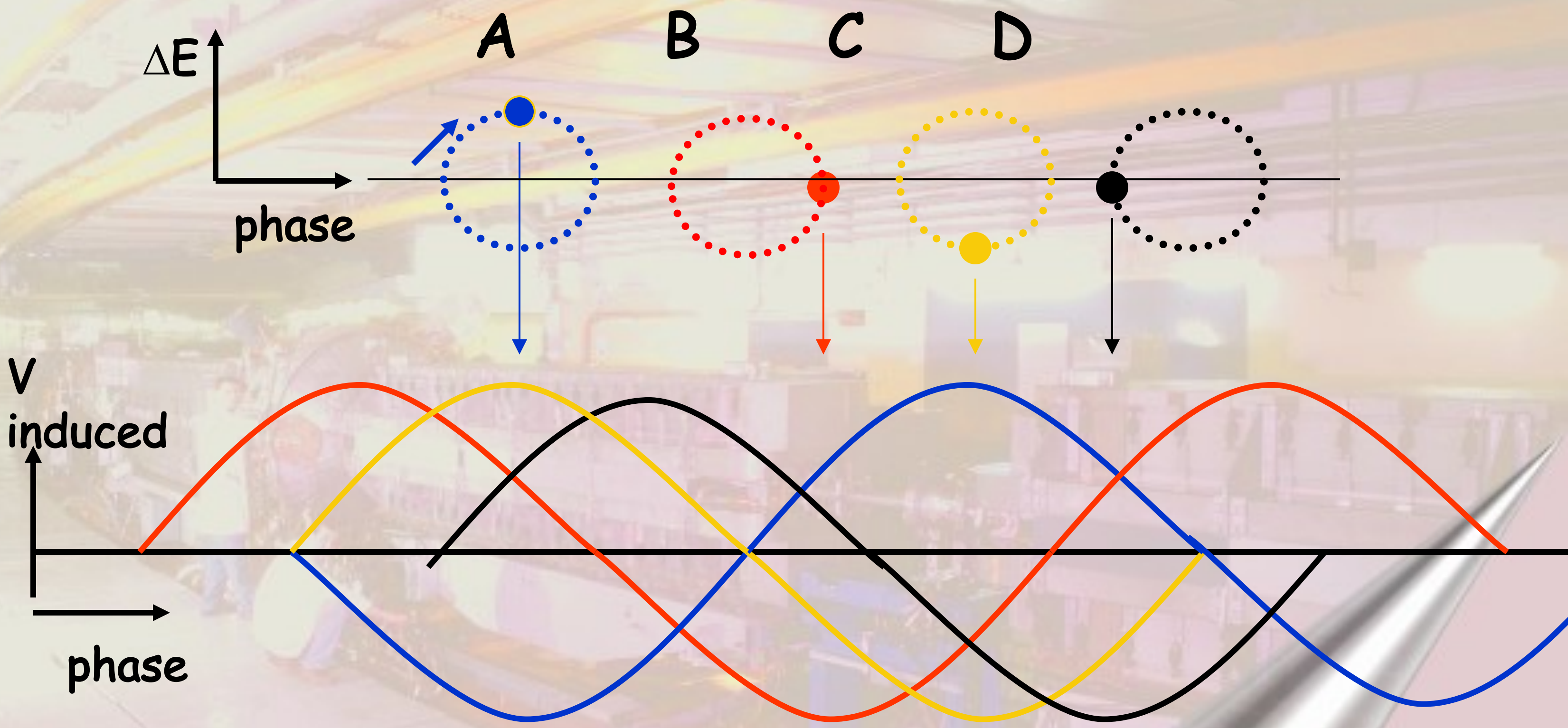
Multi-bunch instabilities (22)



This residual voltage will accelerate B and decelerate D

This increase the oscillation amplitude

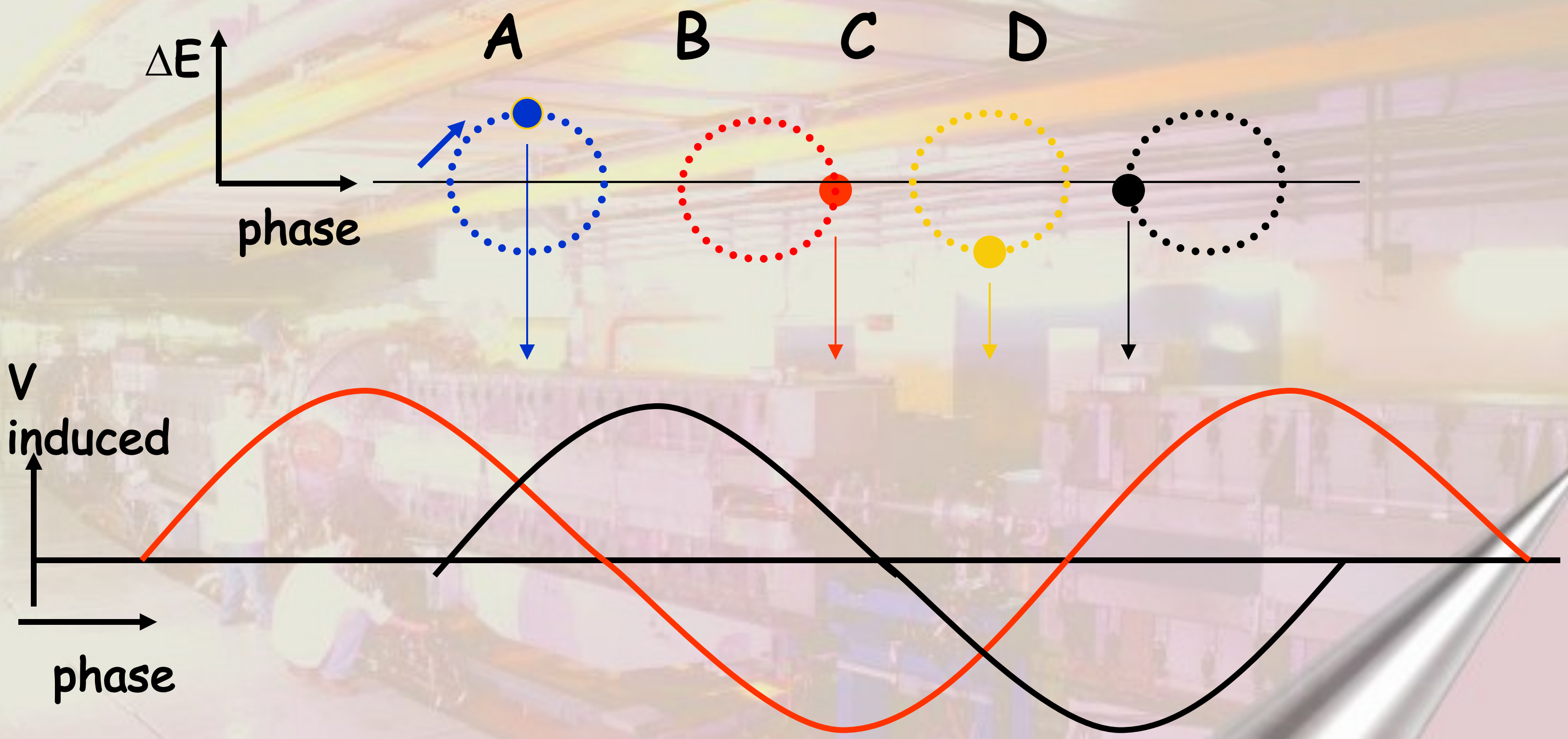
Multi-bunch instabilities (23)



1/4 of a synchrotron
period later

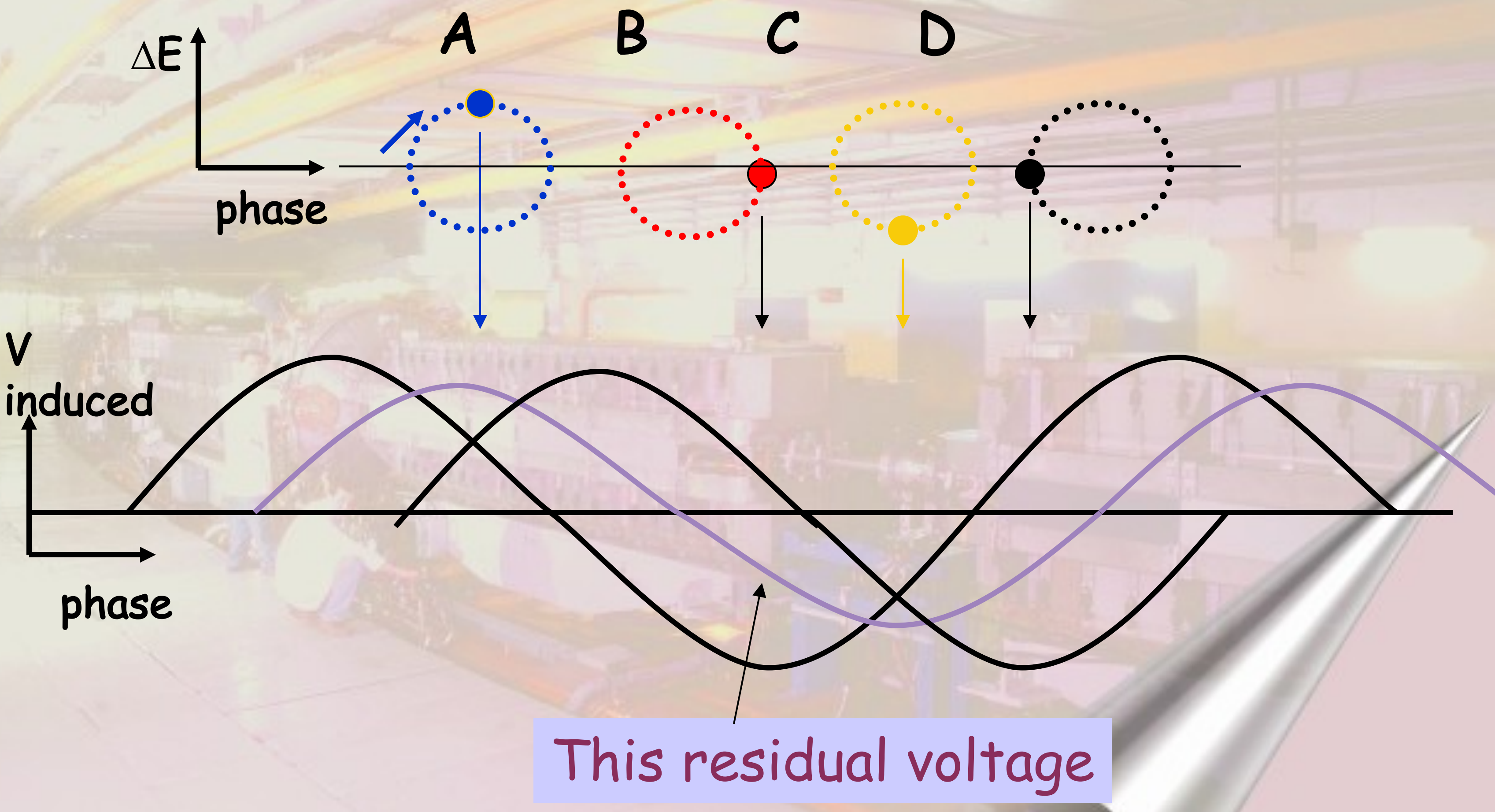
A & C induced voltages now cancel

Multi-bunch instabilities (24)

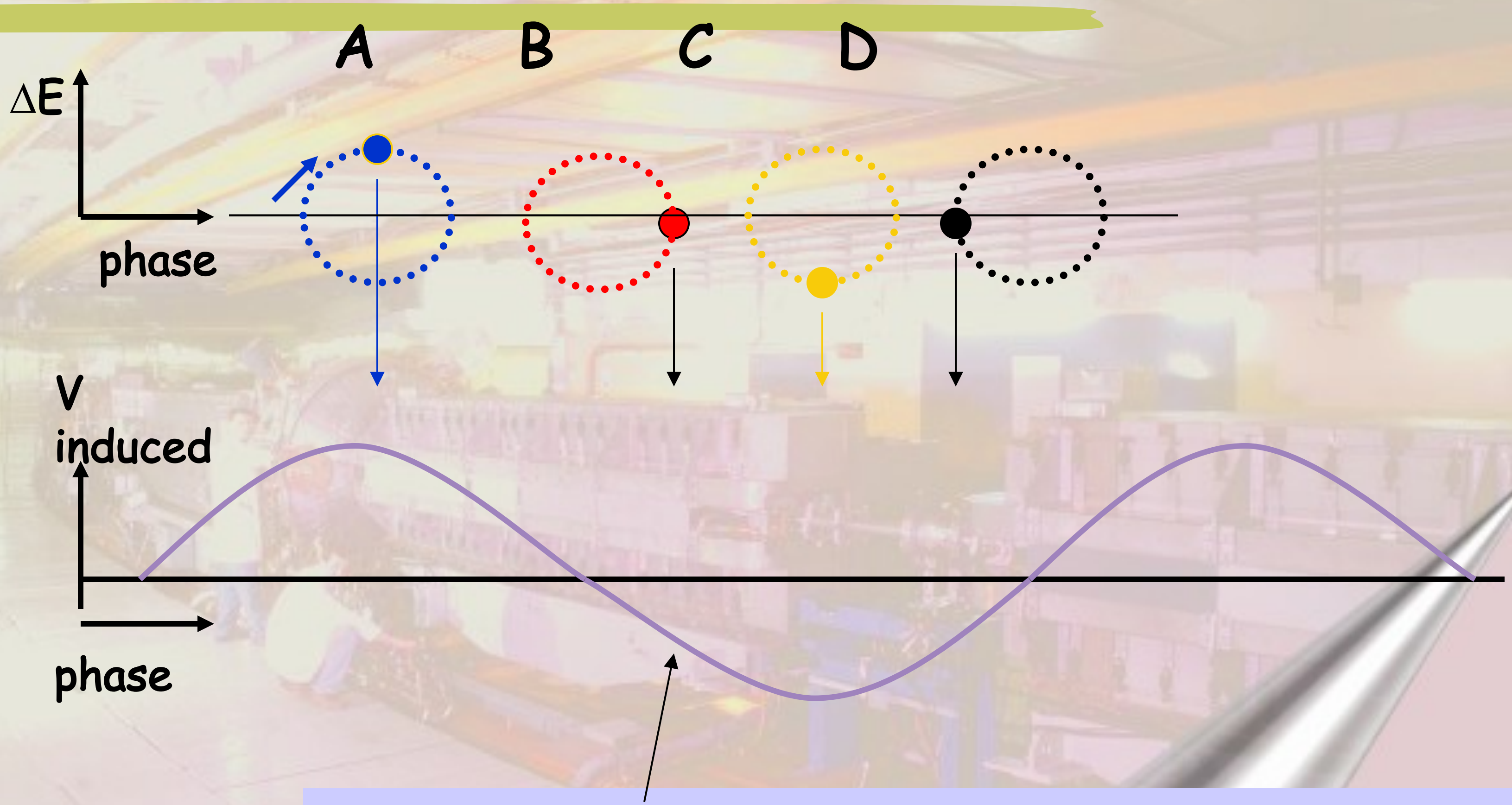


B & D induced voltages do not cancel

Multi-bunch instabilities (25)



Multi-bunch instabilities (26)



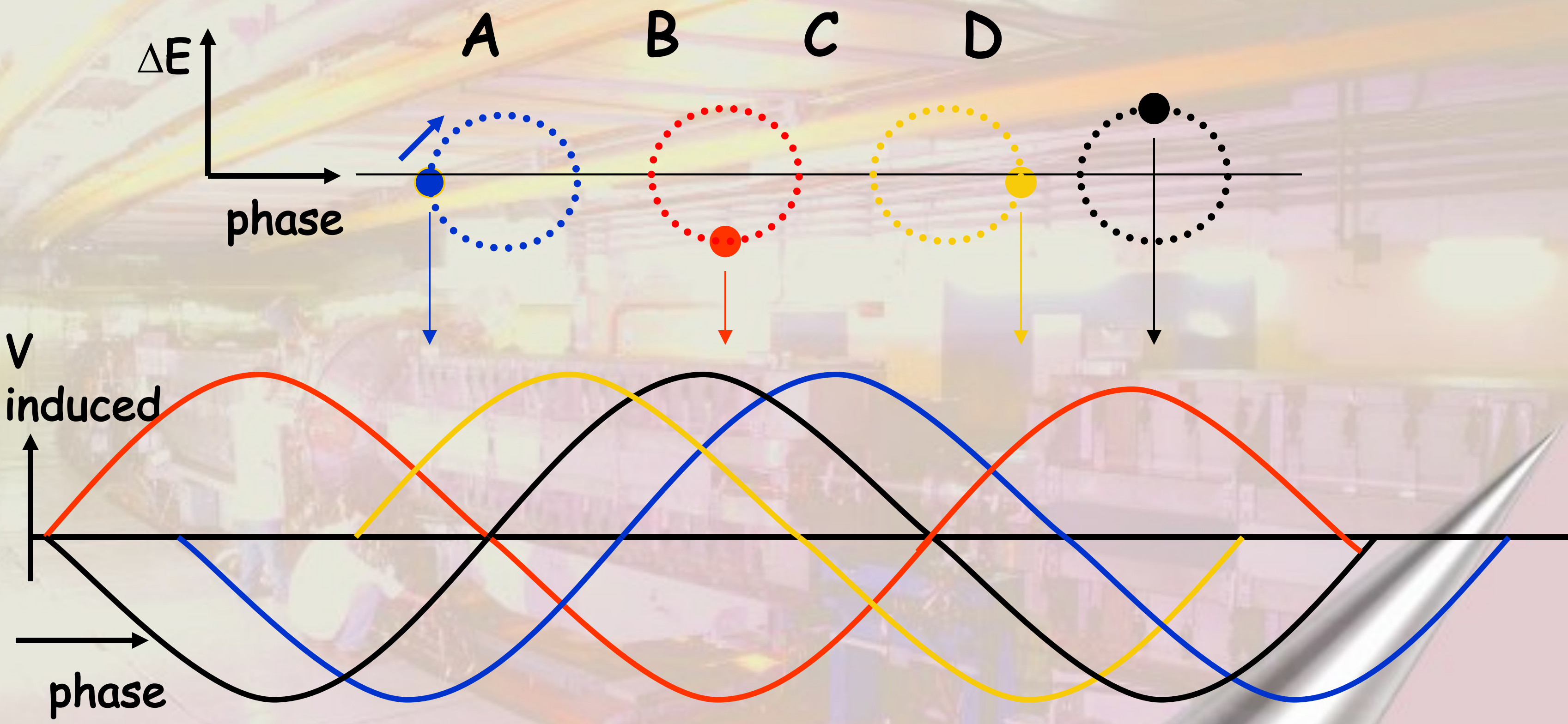
This residual voltage will accelerate A and decelerate C

Again ⇒ increase the oscillation amplitude

Multi-bunch instabilities (27)

- Hence the n=1 mode coupled bunch oscillation is unstable
- Not all modes are unstable look at n=3

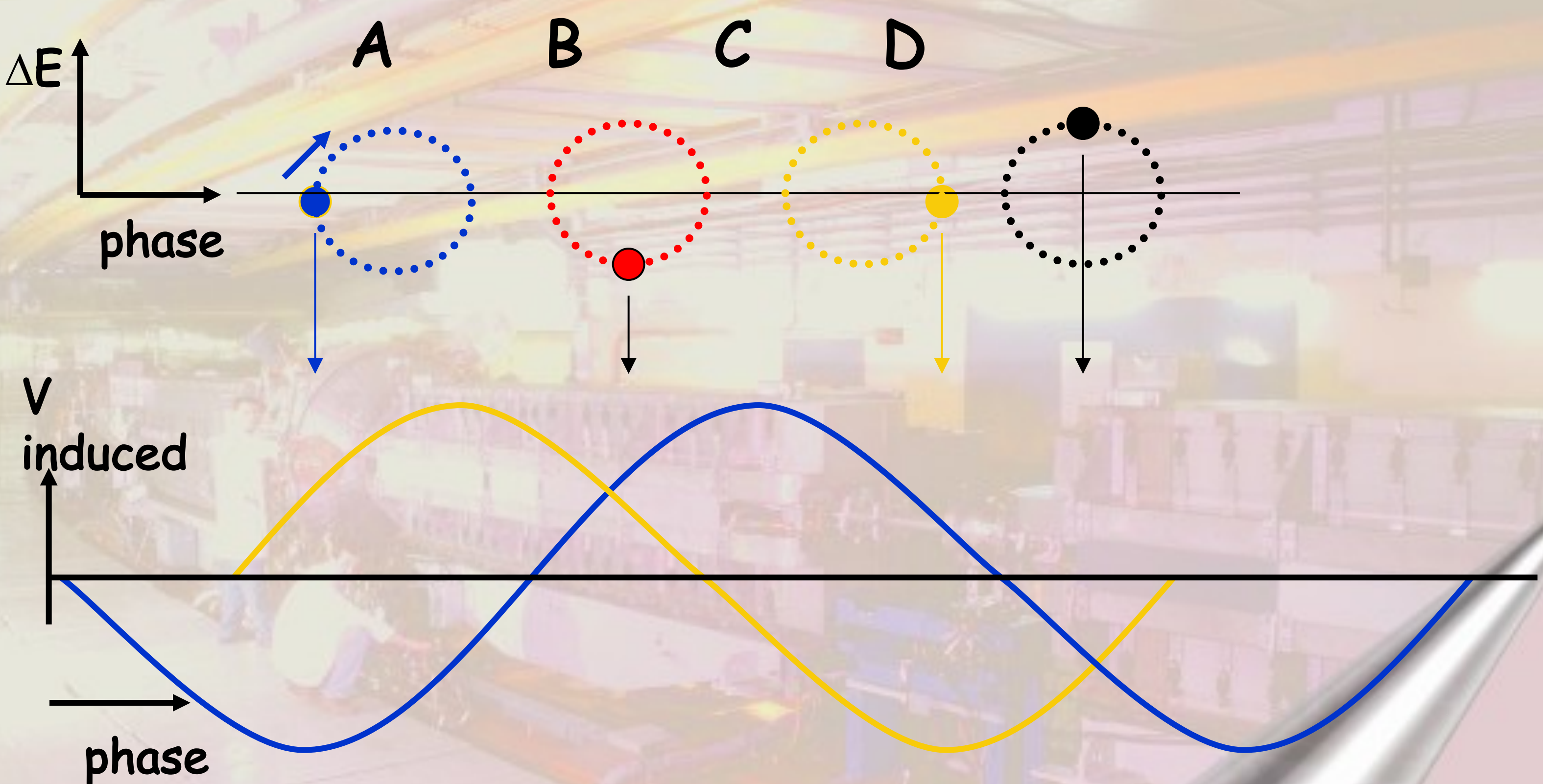
Multi-bunch instabilities (28)



Introduce an **n=3** mode coupled bunch oscillation

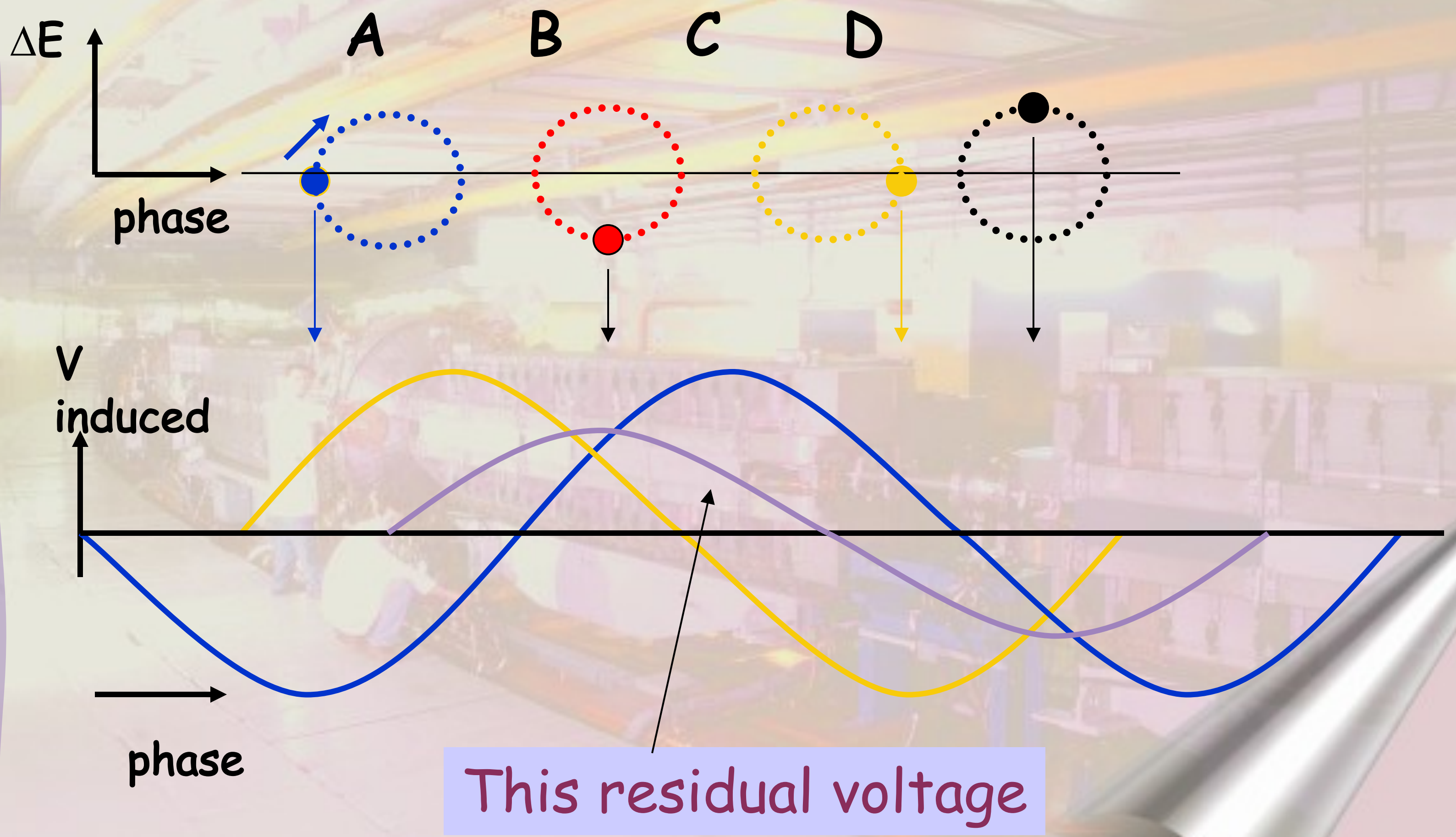
B & D induced voltages cancel

Multi-bunch instabilities (29)

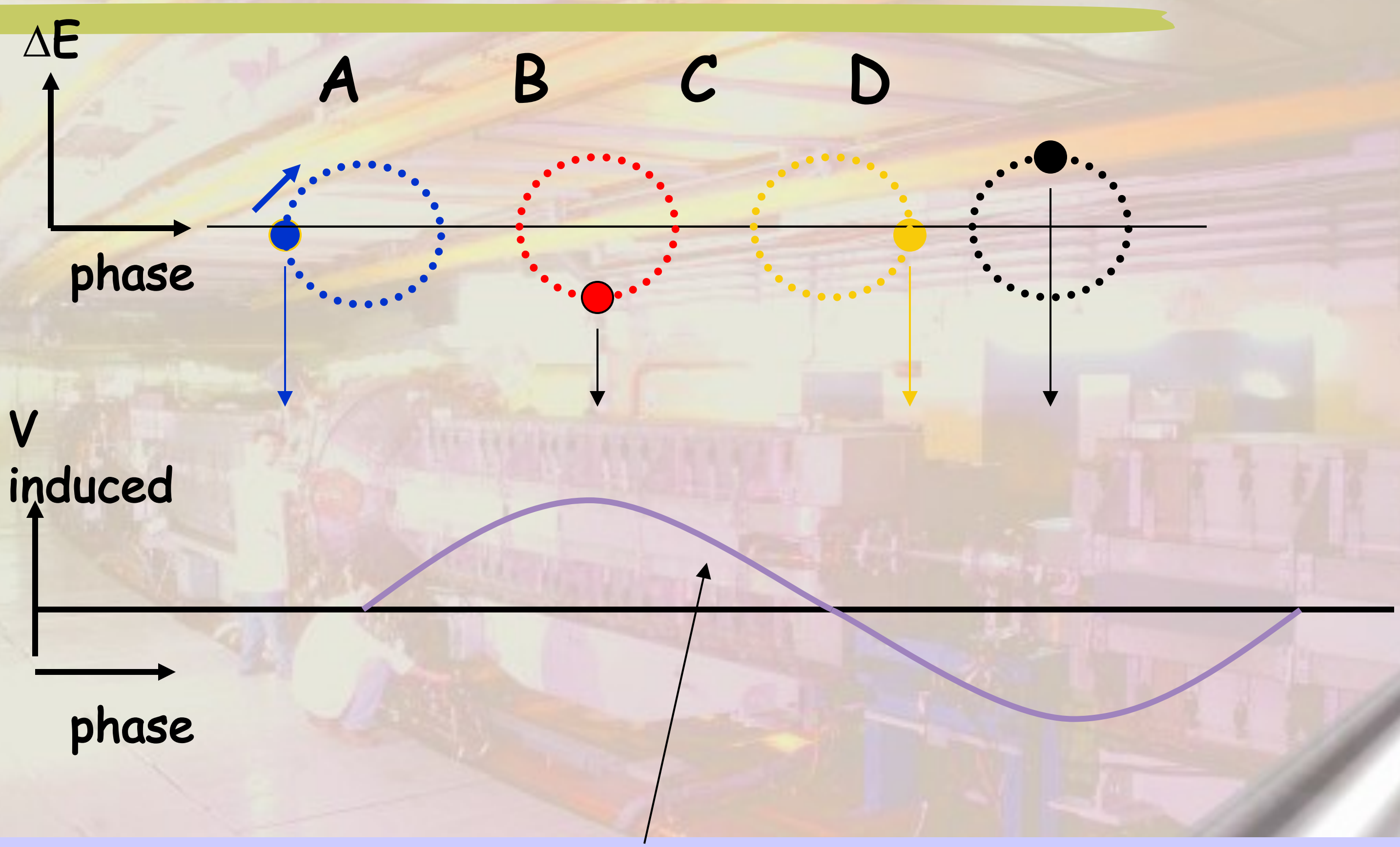


A & C induced voltages do not cancel

Multi-bunch instabilities (30)



Multi-bunch instabilities (31)



This residual voltage will accelerate B and decelerate D
⇒decrease the oscillation amplitude

Multi-bunch instabilities on a ‘scope (1)

Turn “1”

“Mountain range display”

Multi-bunch instabilities on a ‘scope (2)

Add snapshot images some turns later

Multi-bunch instabilities on a ‘scope (3)

Multi-bunch instabilities on a ‘scope (4)

Multi-bunch instabilities on a ‘scope (5)

Multi-bunch instabilities on a ‘scope (6)

Multi-bunch instabilities on a ‘scope (7)

Multi-bunch instabilities on a 'scope (8)

Multi-bunch instabilities on a 'scope (9)

Multi-bunch instabilities on a ‘scope (10)

Multi-bunch instabilities on a 'scope (11)

Multi-bunch instabilities on a ‘scope (12)

Multi-bunch instabilities on a 'scope (13)

Multi-bunch instabilities on a 'scope (14)

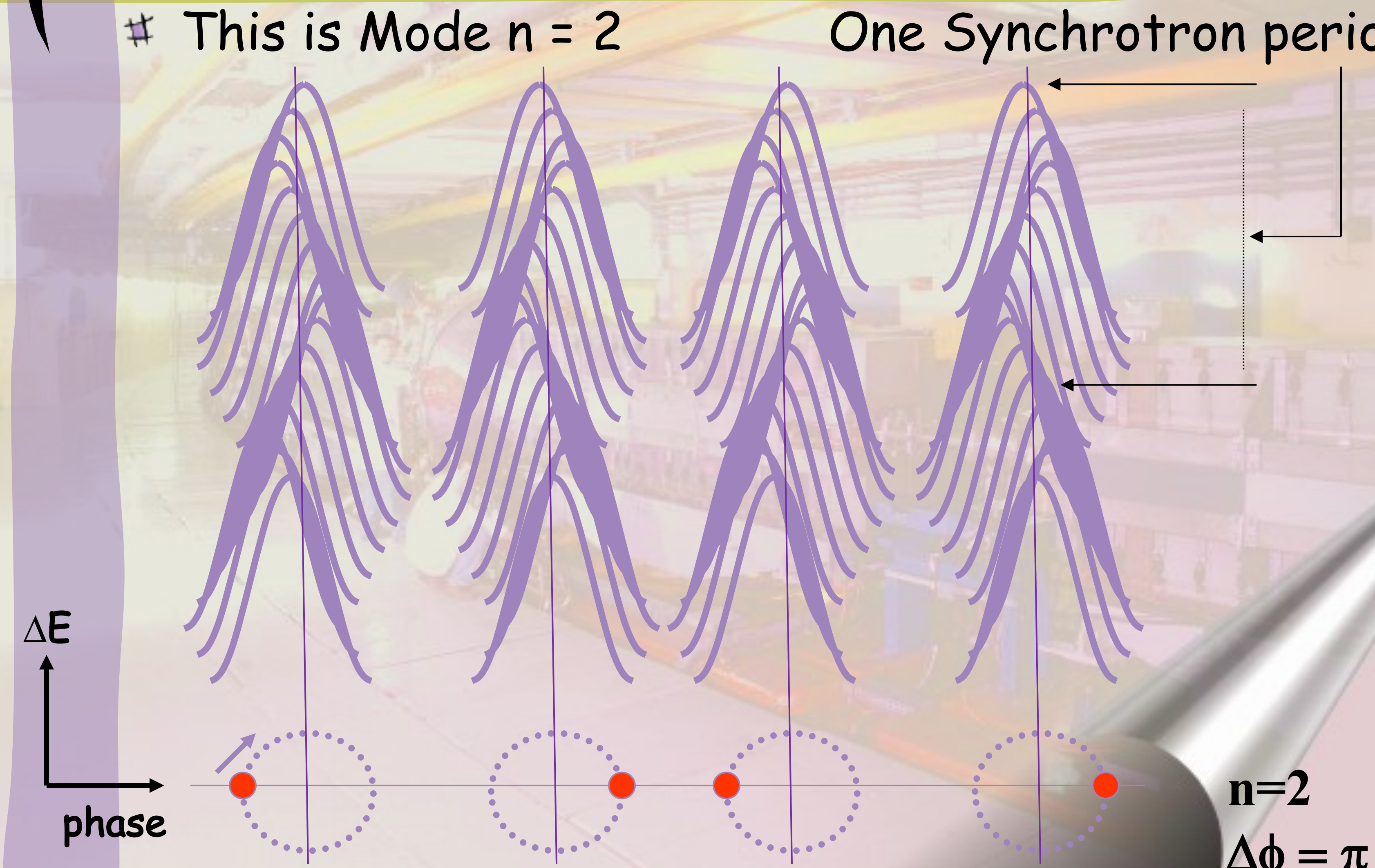
Multi-bunch instabilities on a 'scope (15)

Multi-bunch instabilities on a 'scope (16)

- What mode is this ?
- What is the synchrotron period?

Multi-bunch instabilities on a 'scope (17)

- This is Mode n = 2



Possible cures for single bunch modes

- Tune the RF cavities correctly in order to avoid the Robinson Instability
- Have a phase lock system, this is a feedback on phase difference between RF and bunch
- Have correct Longitudinal matching
- Radiation damping (Leptons)
- Damp higher order resonant modes in cavities
- Reduce machine impedance as much as possible

Possible cures for multi-bunch modes

- Reduce machine impedance as far as possible
- Feedback systems - correct bunch phase errors with high frequency RF system
- Radiation damping
- Damp higher order resonant modes in cavities

Bunch lengthening (1)

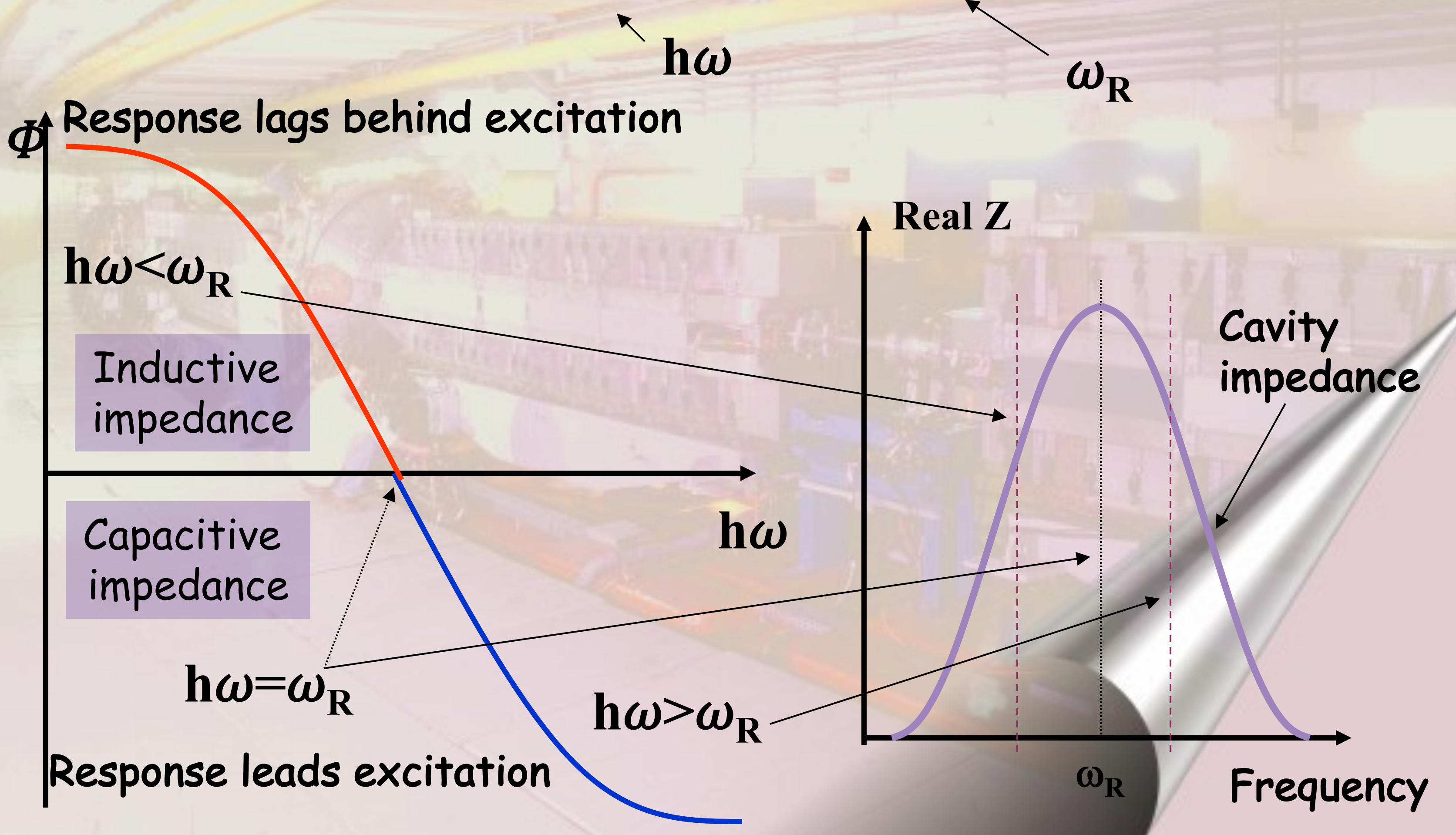
- Now we controlled all longitudinal instabilities, but .....
- It seems that we are unable to increase peak bunch current above a certain level
- The bunch gets longer as we add more particles.

- Why..?
- What happens....?

- Lets look at the behaviour of a cavity resonator as we change the driving frequency.

Bunch lengthening (2)

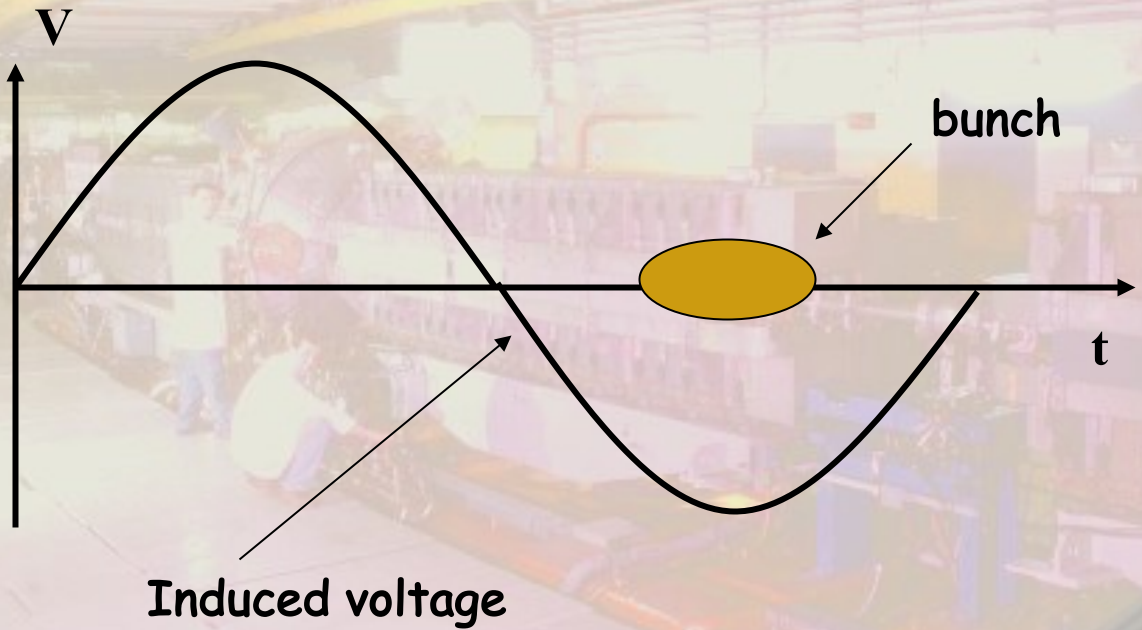
The **phase** of the response of a resonator depends on the difference between the **driving** and the **resonant** frequencies



Bunch lengthening (3)

Cavity driven on resonance

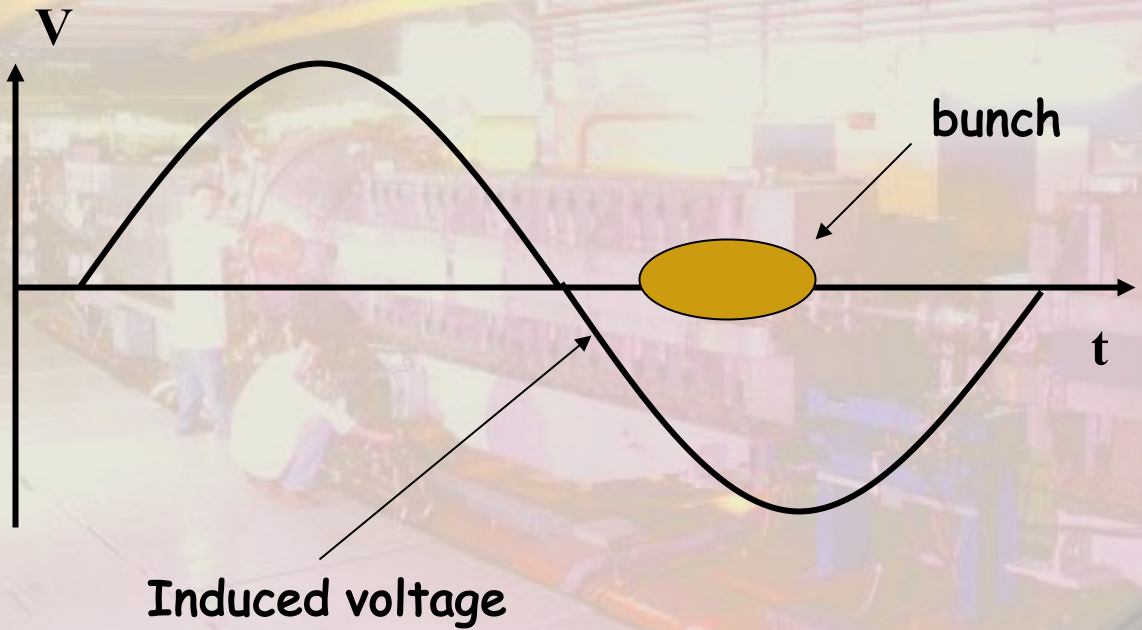
$h\omega = \omega_R \Rightarrow$ resistive impedance



Bunch lengthening (4)

Cavity driven above resonance

$h\omega > \omega_R \Rightarrow$ **capacitive impedance**

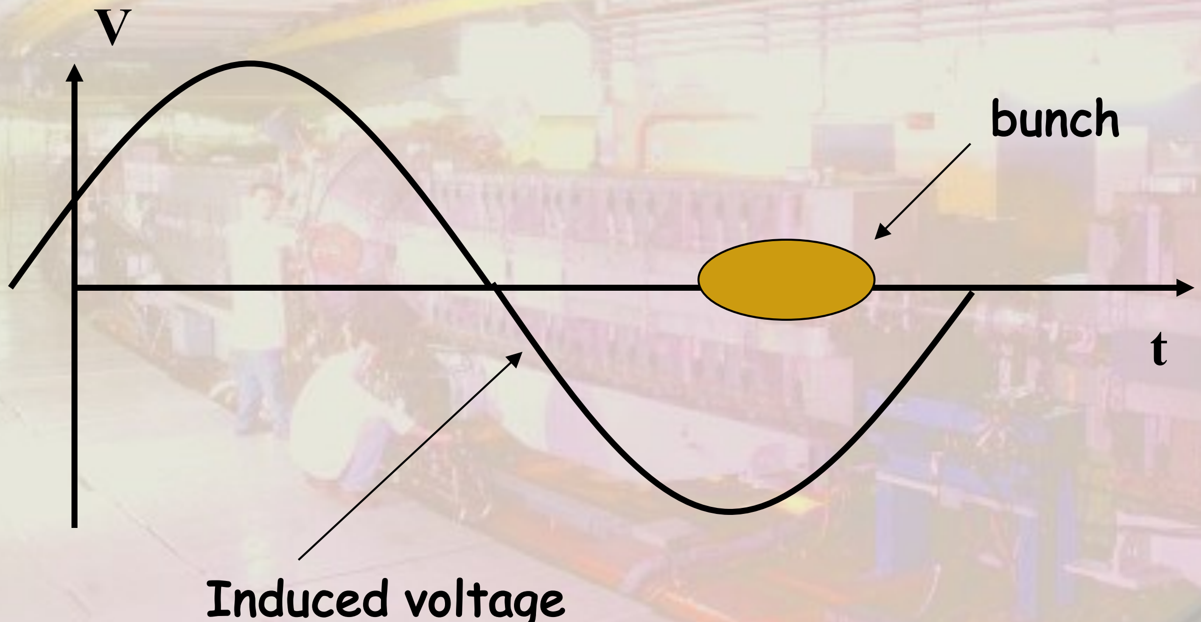


Response leads excitation

Bunch lengthening (5)

Cavity driven below resonance

$h\omega < \omega_R \Rightarrow$ inductive impedance



Response lags behind excitation

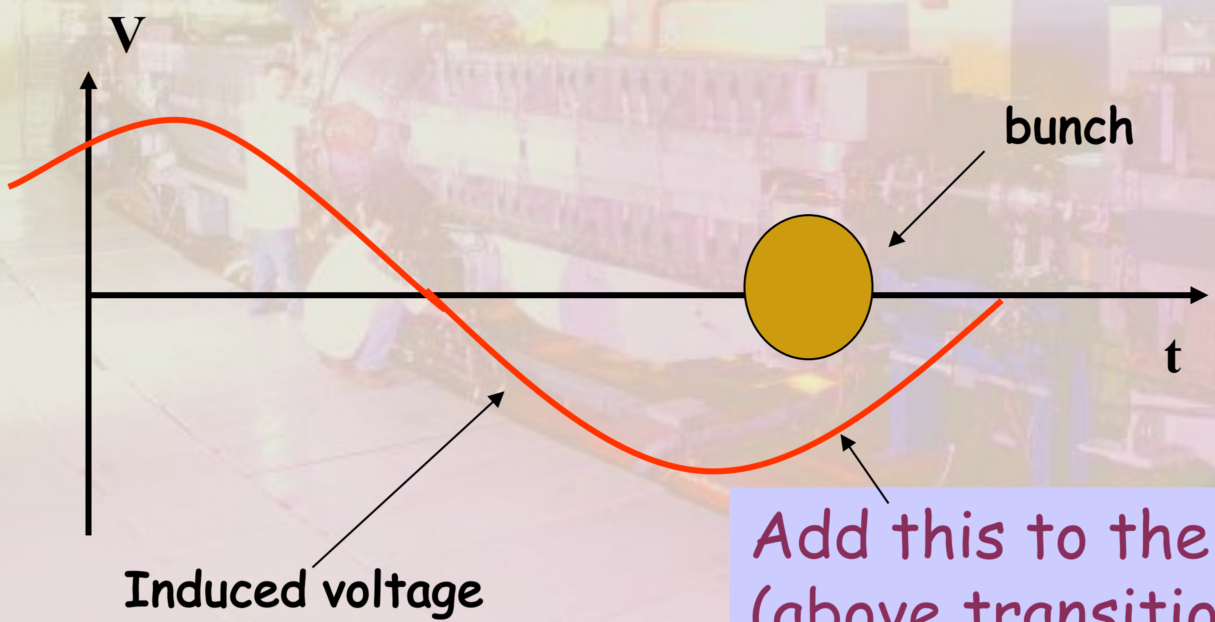
Bunch lengthening (6)

- In general the Broad Band impedance of the machine, vacuum pipe etc (other than the cavities) is inductive

- The bellows etc. represent very high frequency resonators, which resonate mostly at frequencies above the bunch spectrum

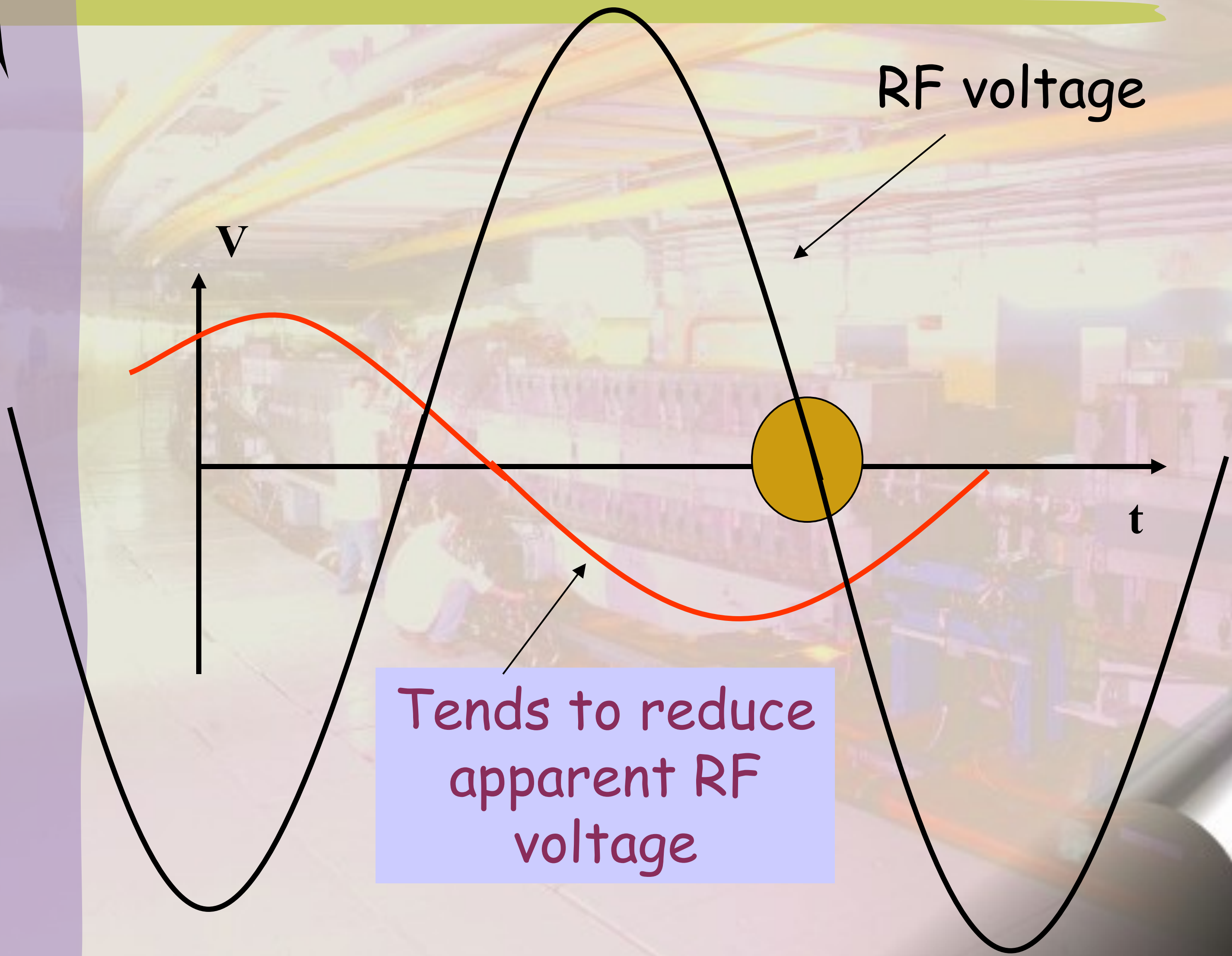
Bunch lengthening (7)

- Since the Broad Band impedance of the machine is predominantly inductive , the response lags behind excitation

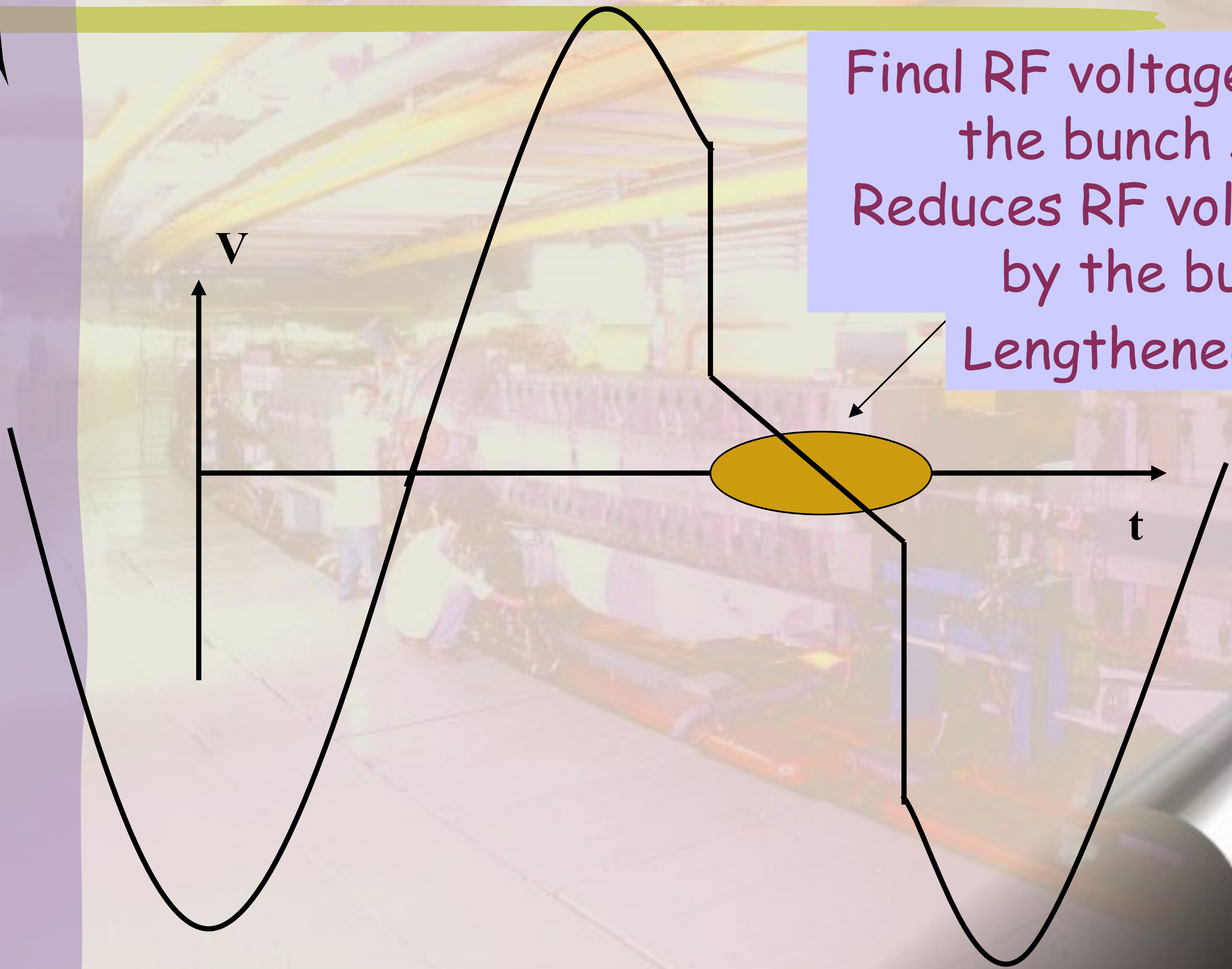


Add this to the RF voltage (above transition)

Bunch lengthening (8)



Bunch lengthening (10)



Final RF voltage modifies the bunch shape
Reduces RF voltage seen by the bunch

Lengthened bunch

Questions….,Remarks…?