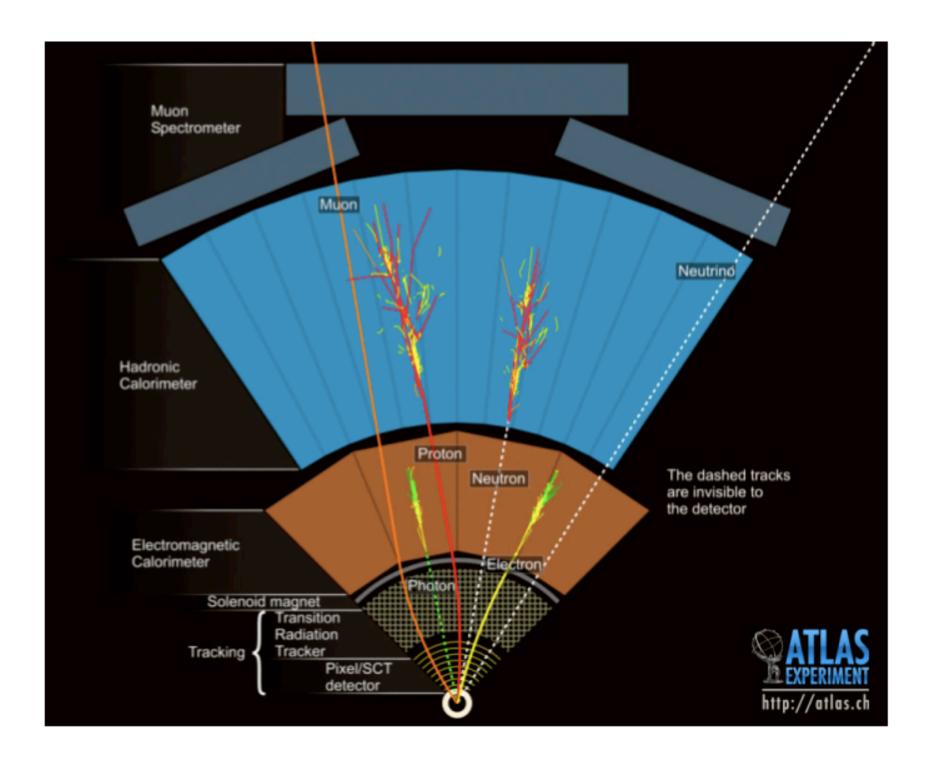


Inhoud

- Introduction, Kinematics
- Interaction of Charged Particles
 - Ionization, scintillation, Cherenkov and Transition Radiation
 - Bremsstrahlung and nuclear interactions
- Interaction of Neutral Particles
 - photons: photoelectric effect, Compton scattering, pair production
 - neutrons
 - neutrinos
- Electromagnetic Showers
- Hadron Showers

Introduction



Particles for Detection

Particle	Mass	Lifetime	Charge	Main interactions
Electron	0.511 MeV	Stable	-1	Electromagnetic
Muon	105 MeV	2.2 10 ⁻⁶ s	-1	Electromagnetic
Photon	0	Stable	0	Electromagnetic
Mesons	140-500MeV	≈ 10 ⁻⁸ s	+1, 0, -1	Strong
p, n	940 MeV	Stable/20min	1, 0	Strong
Nuclei	1–240 protons	Many stable	Z (1–92)	Strong
Neutrino	< eV	Stable	0	Weak

Table 1.3: List of the most common directly observable particles.

A Few Units

- 1 proton charge = 1.602 10⁻¹⁹ C,
- $1 \text{ eV} = 1.602 \ 10^{-19} \ \text{Joule}$
- c = 299792458 m/s.
- mass of the proton is 1.672 10^{-27} kg= 938.272 MeV/ c^2
- mass of the electron = 511 keV/ c^2
- hbar = $1.054 \ 10^{-34} \ Js$
- hbarc= 197.6 10⁻¹⁵ MeV m
- Fine structure constant $\alpha = 1/137.035$
- Classical electron radius $r_e = 2.818 \cdot 10^{-15} m$ or 2.818 fermi
- 1 barn = 10^{-28} m²

http://pdg.lbl.gov/2006/reviews/consrpp.pdf

Introduction: Particle Detection

- Detection and Identification of Particles and Nuclei important in
 - high-energy physics
 - cosmic ray physics
 - nuclear physics

Basic Idea

Every effect of particles or radiation can be used as a working principle for a detector

Main Purpose of particle detectors:

Detection and identification of particles with mass m and charge z

In particle physics: Usually $z=0,\pm 1$, but not in nuclear, heavy ion physics, or cosmic rays

Examples

• Charged particle (charge z) deflected in magnetic field \rightarrow momentum p

$$ho \propto rac{p}{z} = rac{\gamma m eta c}{z}$$

Time of flight determines particle velocity

$$\beta = \frac{v}{c} \propto \frac{1}{t}$$

Cherenkov angle determines particle velocity

$$\theta_C = \frac{1}{\beta n}$$

Calorimeter measurement provides energy measurement

• Charge measurement: Ionization Energy loss

$$rac{dE}{dx} \propto rac{z^2}{eta^2} \ln(aeta\gamma)$$

$$\gamma \equiv \frac{c}{\sqrt{c^2-v^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

With all the information together one can determine the quadri-vector of the particle.

- Basic detection techniques work mostly for charged particles only.
- Neutral particles usually detected "indirectly" via production of charged particles.

Literature

General Detector concepts

- Claus Grupen: Particle Detectors. Cambridge University Press, 2000.
 The best book I know about detectors.
- Konrad Kleinknecht: Detectors for particle radiation. Cambridge University Press, 2nd edition 1998.
 Also a good book.
- Richard C. Fernow: Introduction to experimental Particle Physics. Cambridge University Press, 1986.
- Richard Wigmans: Calorimetry. Oxford Science Publishing, 2000.
 THE book about calorimeters. Written by the expert with all details.
- Lecture notes and Proceedings of ICFA Instrumentation Schools (since 1987 every two years).
- Particle Data Book. short summaries of important things.
- + specialized works (→tasks)

Introduction: Detectors

Design of Instrumentation and Detectors requires knowledge of

- Basic physics for interaction of charged and neutral particles with matter
- Mechanical Engineering
- Electrical Engineering (high voltage)
- Electronic Engineering
- Interfaces to Trigger, Data Acquisition and Computing (Lecture by Nick Elis)
- Software Engineering (calibration)
- Operation (stability)
- To know any one of them is not sufficient
- You have to apply all together to build, operate and use an instrument for your physics measurement
- Always keep in mind what you want to measure, and what precision (resolution) you need

Charged Particles

Energy Loss of Charged Particles

A charged particle passing through matter suffers

- 1. energy loss
- 2. deflection from incident direction

Main type of reactions:

- Inelastic collisions with atomic electrons of the material.
- 2. Elastic scattering from nuclei.

Less important reactions are:

- Emission of Cherenkov radiation
- Nuclear reactions
- 5. Bremsstrahlung

Classification of charged particles with respect to interaction with matter:

- 1. Low mass: electrons and positrons
- 2. High mass: muons, pions, protons, light nuclei.

Energy loss:

- mainly due to inelastic collisions with atomic electrons.
- cross section σ≅ 10⁻¹⁷ 10⁻¹⁶ cm²!
- small energy loss in each collision, but many collisions in dense material. Thus one can work with average energy loss.
- Example: a proton with E_{kin}=10 MeV looses all its energy after 0.25 mm of copper.

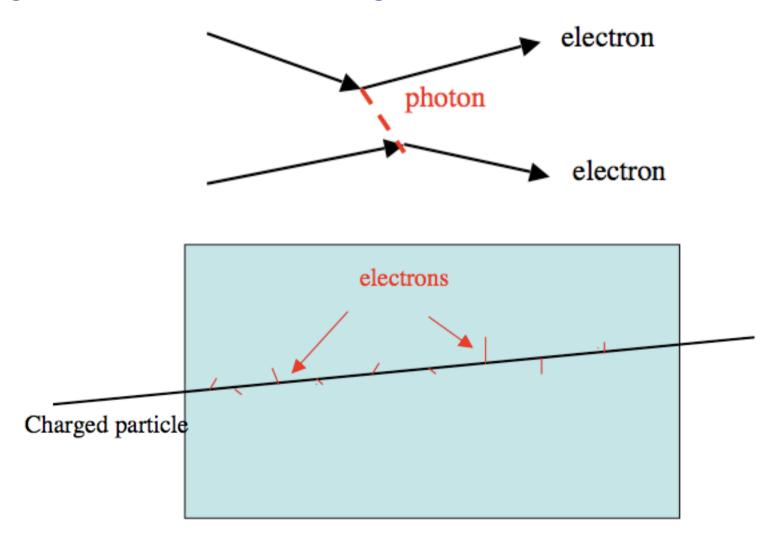
Two groups of inelastic atomic collisions:

- soft collisions: only excitation of atom.
- hard collisions: ionization of atom. In some of the hard collisions the atomic electron get such a large energy that it causes secondary ionisation (δelectrons).

Elastic collisions from nuclei cause very small energy loss. They are the main cause for deflection.

Interaction with Matter

Eg electron-electron scattering



Charged Particles Basic Kinematics

• Conservation of Momentum and Energy: Max. Energy for particle with mass m, velocity $v = \beta c$, collision with electron:

$$E_{\mathrm{kin}}^{\mathrm{max}} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{m} + \left(\frac{m_e}{m}\right)^2} = \frac{2m_e p^2}{m^2 + m_e^2 + 2m_e E/c^2}$$

- Limits:
 - Low energy, heavier than electron $(2\gamma \frac{m_e}{m} \ll 1, m \gg m_e)$

$$E_{\rm kin}^{
m max} = 2m_e c^2 \beta^2 \gamma^2$$

– relativistic ($E_{\rm kin} \approx E, pc \approx E$):

$$E^{\text{max}} = \frac{E^2}{E + m^2 c^2 / 2m_e}$$

- Electron - Electron Collisions ($m = m_e$):

$$E_{
m kin}^{
m max} = rac{p^2}{m_e + E/c^2} = rac{E^2 - m_e^2 c^4}{E + m_e c^2} = E - m_e c^2$$

 Scattering angle (on nucleus Z, impact parameter b)

$$\Theta = \frac{2z \cdot Z \cdot e^2}{\beta cb} \cdot \frac{1}{p}$$

Cross Section (Rutherford):

$$\frac{d\sigma}{d\Omega} = \frac{z^2 Z^2 r_e^2}{4} \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4 \Theta/2}$$

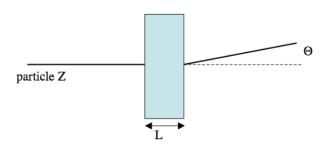
Multiple Scattering

$$\sqrt{\langle\Theta^2\rangle} = \Theta_{\rm plane} = \frac{13.6\,{\rm MeV}}{\beta cp}z \cdot \sqrt{\frac{x}{X_0}} \left\{1 + 0.038\ln\left(\frac{x}{X_0}\right)\right\}$$

$$\Theta_{\rm space} = \sqrt{2}\,\Theta_{\rm plane}$$

Approximation by Gaussian:

$$P(\Theta)d\Theta = \frac{1}{\sqrt{2\pi}\Theta_{\mathrm{plane}}} \mathrm{exp} \left\{ -\frac{\Theta^2}{2\Theta_{\mathrm{plane}}^2} \right\} \, d\Theta$$



Energy Loss for Charged Particles

- Interaction via exchange of Photons (electromagnetic interaction)
- Virtual Photons: Absorbed by atoms in material → Ionization, Excitation
- Real Photons: Radiation is emitted by a charge particle if:
 - 1. v > c/n: Cherenkov radiation
 - 2. $\vec{v}/c_{\rm ph} = \vec{v} \cdot n/c$ changes
 - (a) $|\vec{v}|$ changes: Bremsstrahlung
 - (b) direction of \vec{v} changes: Synchrotron radiation
 - (c) n changes: Transition Radiation

Bethe-Bloch Formula

Bethe-Bloch formula gives the mean rate of energy loss (stopping power) of a heavy charged particle.

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} [\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta \gamma)}{2}]$$

PDG 2008

with

A: atomic mass of absorber

$$\frac{K}{A} = 4\pi N_A r_e^2 m_e c^2 / A = 0.307075 \text{ MeV g}^{-1} \text{cm}^2, \text{ for A} = 1 \text{g mol}^{-1}$$

z: atomic number of incident particle

I = mean excitation energy

Z: atomic number of absorber

 T_{\max} : Maximum energy transfer in a single collision $T_{\max} = \frac{2m_ec^2\beta^2\gamma^2}{1+2\gamma m_e/M+(m_e/M)^2}$

 $\delta(\beta\gamma)$: density effect correction to ionization loss.

 $x = \rho$ s, surface density or mass thickness, with unit g/cm², where s is the length.

dE/dx has the units MeV cm²/g

Project: derive the Bethe-Bloch formula

History of Energy Loss: dE/dx

1915: Niels Bohr, classical formula, Nobel prize 1922. 1930: Non-relativistic formula found by Hans Bethe

1932: Relativistic formula by Hans Bethe

Bethe's calculation is leading order in pertubation theory, thus only z² terms are included.

Additional corrections:

- z³ corrections calculated by Barkas-Andersen
- z⁴ correction calculated by Felix Bloch (Nobel prize 1952, for nuclear magnetic resonance). Although the formula is called Bethe-Bloch formula the z⁴ term is usually not included.
- Shell corrections: atomic electrons are not stationary
- Density corrections: by Enrico Fermi (Nobel prize 1938, for discovery of nuclear reaction induced by slow neutrons).



Hans Bethe 1906-2005

Born in Strasbourg, emigrated to US in 1933. Professor at Cornell U. Nobel prize 1967 for theory of nuclear processes in stars.

$$p_b = \frac{2r_e m_e c}{b\beta}z$$
 Momentum transfer per target electron b : impact parameter

Energy transfer (classical approximation)

$$\epsilon = \frac{p_b^2}{2m_e} = \frac{2r_e^2 m_e c^2}{b^2 \beta^2} z^2$$

Interaction rate per (g/cm²), atomic cross section σ

$$\Phi[g^{-1}cm^2] = \frac{N_A}{A} \sigma[cm^2/atom] \qquad N_A \text{ Avogadro's number, } A \text{ atomic mass}$$

$$\Phi(\epsilon) d\epsilon = \frac{N_A}{A} 2\pi b \, db \, Z$$

 $2\pi b \, db$ area of annulus Z Number of electrons per atom

$$\epsilon = \epsilon(b) \Rightarrow b^2 = \frac{2r_e^2 m_e c^2}{\beta^2} z^2 \frac{1}{\epsilon}$$

$$\Phi(\epsilon) d\epsilon = \frac{N_A}{A} \pi \frac{2r_e^2 m_e c^2}{\beta^2} z^2 Z \frac{d\epsilon}{\epsilon^2}$$

energy loss

$$-dE = \int_{0}^{\infty} \Phi(\epsilon) \epsilon \, dx = \int_{0}^{\infty} \frac{N_A}{A} 2\pi \, bdb \, Z \epsilon \, dx$$
$$-\frac{dE}{dx} = \frac{2\pi N_A}{A} Z \int_{0}^{\infty} \epsilon \, b \, db = 2\pi \, \frac{Z \, N_A}{A} \frac{2r_e^2 m_e c^2}{\beta^2} z^2 \int_{0}^{\infty} \frac{db}{b}$$

Problem: Integral is divergent at $b \to 0$ and $b \to \infty$

$$b \to 0$$
? assume $b_{\min} = \frac{h}{2p} = \frac{h}{2\gamma m_e \beta c}$ half the de Broglie wavelength $0 \to \infty$? if revolution time τ_R of electron in target

 $b \to \infty$? if revolution time τ_R of electron in target atom is smaller than interaction time τ_i target looks neutral

$$\tau_i = \frac{b_{\max}}{v} \sqrt{1 - \beta^2}$$
 Lorentz contraction

$$\tau_R = \frac{1}{v_Z \cdot Z} = \frac{h}{I}$$

mean excitation energy $I \approx 10 \,\mathrm{eV} \cdot Z$

$$au_i = au_R \qquad \Rightarrow b_{ ext{max}} = rac{\gamma h \beta c}{I}$$

$$\begin{split} \tau_i &= \tau_R & \Rightarrow b_{\max} = \frac{\gamma h \beta c}{I} \\ -\frac{dE}{dx} &= 2\pi \ \frac{Z \, N_A}{A} \frac{2 r_e^2 m_e c^2}{\beta^2} z^2 [\ln \frac{2 \gamma^2 \beta^2 m_e c^2}{I} - \eta] \quad \eta \text{: screening effect} \end{split}$$

Exact treatment: Bethe-Bloch formula (summary in PDG)

$$-\frac{dE}{dx} = 2\pi \frac{Z N_A}{A} \frac{2 r_e^2 m_e c^2}{\beta^2} z^2 \left[\frac{1}{2} \ln \frac{2 m_e c^2 \gamma^2 \beta^2}{I^2} \beta^2 - \frac{\delta}{2} \right]$$

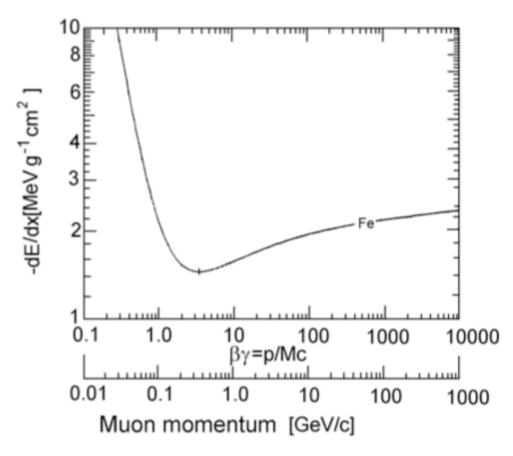
density correction

$$rac{\delta}{2} = \ln rac{\hbar \omega_p}{I} + \ln eta \gamma - rac{1}{2}$$

$$\hbar \omega_p = \sqrt{4\pi N_e r_e^2} rac{m_e \, c^2}{lpha} \quad ext{plasma energy}$$

 N_e electron density of absorbing material α fine structure constant

Example: Energy loss of muons in iron



- Minimum at $3 \le \beta \gamma \le 4$
- Minimum ionizing particles:
 - helium: $-dE/dx = 1.94 \,\text{MeV/(g/cm}^2)$
 - uranium: $-dE/dx = 1.08 \,\mathrm{MeV/(g/cm^2)}$
 - hydrogen: exceptionally large (Z/A = 1)
- $\ln \gamma$ term: relativistic (logarithmic) rise
- Fermi-Plateau due to density effect
- in gases: Plateau ≈ 60 % higher as min. ion.

Examples of Mean Energy Loss

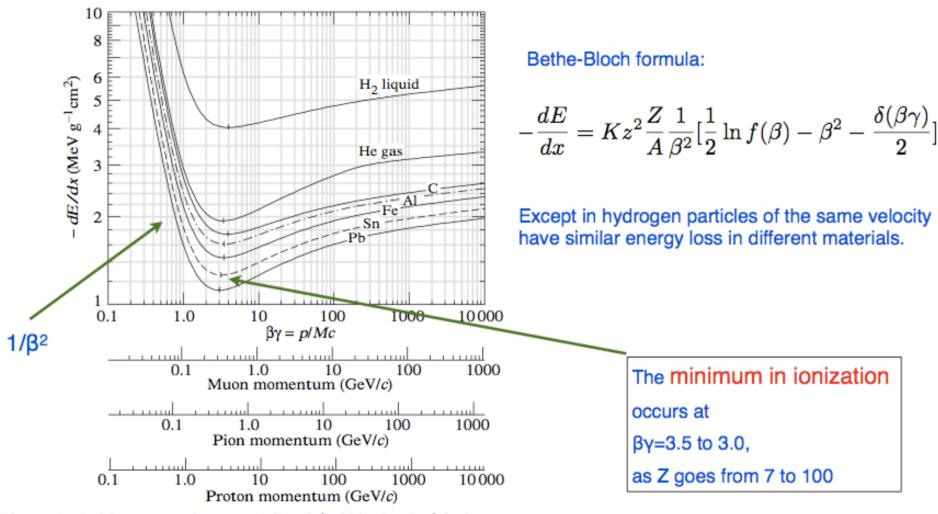
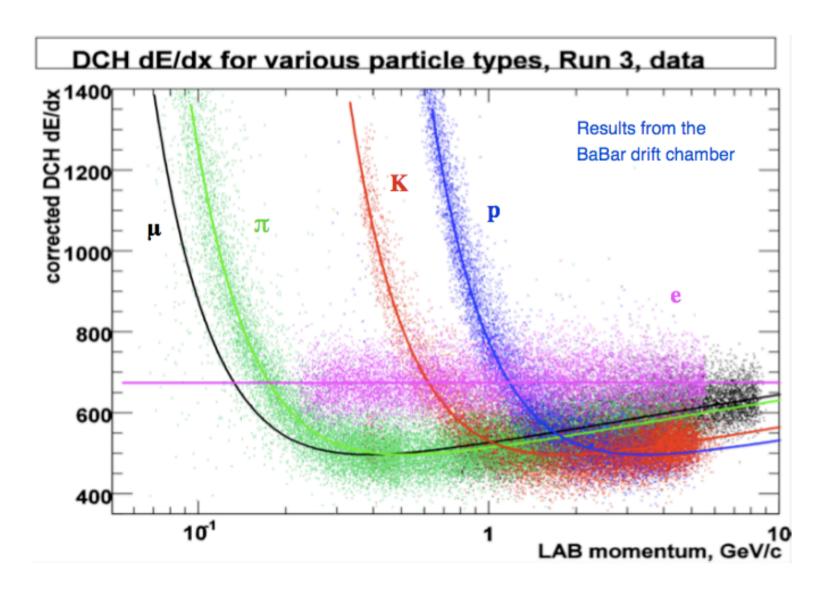


Figure 27.3: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma\gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 27.21.

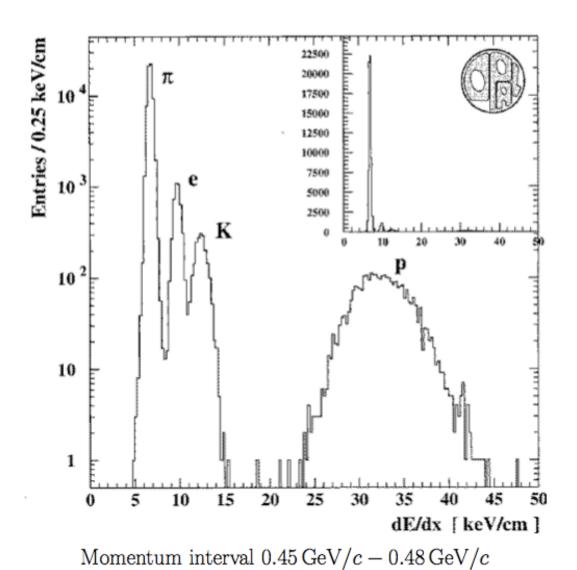
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Particle Identification from dE/dx



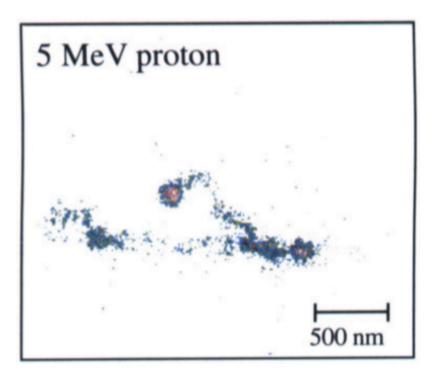
A simultaneous measurement of dE/dx and momentum can provide particle identification.

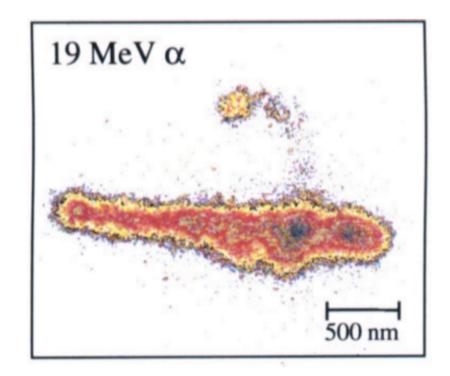
Particle Identification with dE/dx



25

Energy Loss in an optical micro-dosimeter

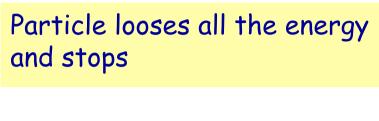


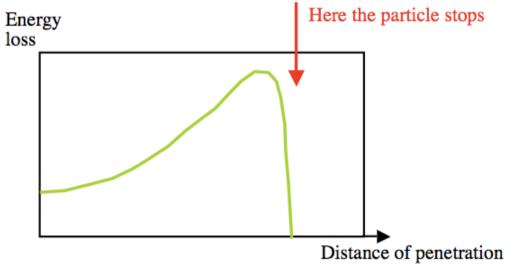


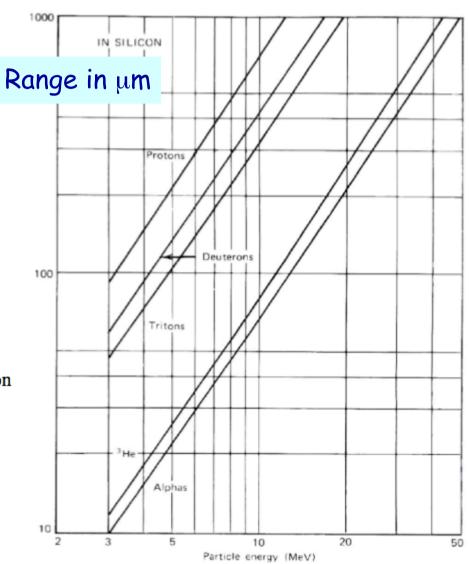
$$rac{dE}{dx}|_{lpha}\ggrac{dE}{dx}|_{p} \ rac{dE}{dx}\propto z^{2}$$

note increase of ionization at end of track, also the δ -rays

Stopping Power







Distance a 10 MeV particle can cross

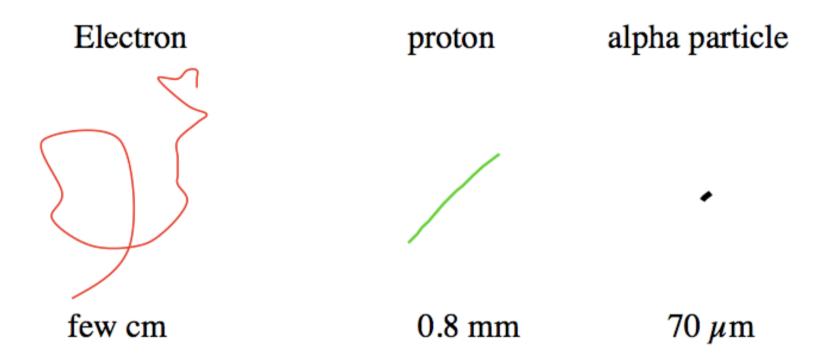
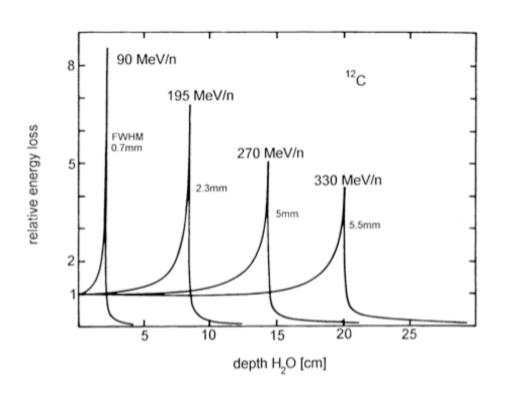
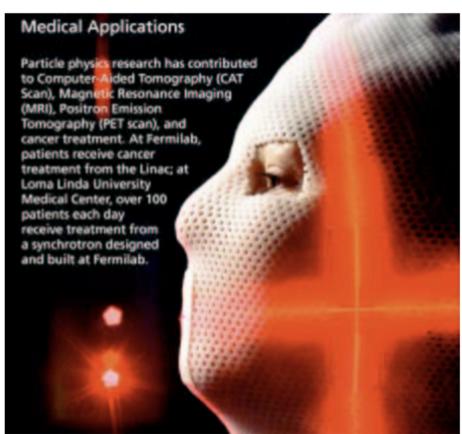


Figure 2.2.2: A typical trajectory for an electron, a proton and an alpha particle of 10 MeV in silicon. The electron trajectory is drawn on a scale 10 times smaller than the trajectory of the proton and the alpha particle.

Application: Bragg Peak





¹²C ions in water. Treatment of deep-seated tumors.

Proton Cancer Therapy

Fluctuations and Energy Loss

Real detector (limited granularity) can not measure $\langle dE/dx \rangle$!

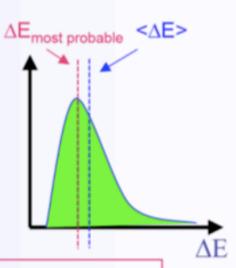
It measures the energy ΔE deposited in a layer of finite thickness δx .

For thin layers or low density materials:

→ Few collisions, some with high energy transfer.



→ Energy loss distributions show large fluctuations towards high losses: "Landau tails"



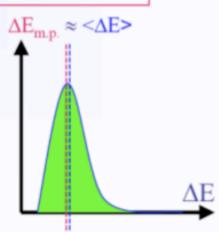
Example: Si sensor: 300 μ m thick. $\Delta E_{m,p} \sim 82 \text{ keV}$

<∆E> ~ 115 keV

For thick layers and high density materials:

- → Many collisions.
- → Central Limit Theorem → Gaussian shaped distributions.





Landau Distribution

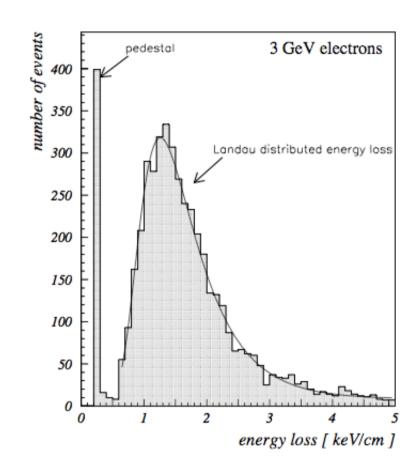
- Bethe-Bloch describes mean energy loss
- Energy loss is distributes asymmetrically
- approximated by

$$\Omega(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$

$$\lambda = \frac{\left(\frac{dE}{dx}\right) - \left(\frac{dE}{dx}\right)^{\text{m.p.}}}{0.123 \text{ keV}}$$

 $\left(\frac{dE}{dx}\right)^{\text{m.p.}}$ most probable energy loss

- important in gases, thin absorbers
- Argon, $\beta \gamma = 4$: $(\frac{dE}{dx})^{\text{m.p.}} = 1.2 \,\text{keV/cm}; \, \langle \frac{dE}{dx} \rangle = 2.69 \,\text{keV/cm}$
- For Particle Identification:
 - Measure often (typ. 160) to get distribution
 - Use "Truncated Mean"



Electrons in Ar:CH₄ (80:20)

Scintillation

```
* C_{24}H_{16}N_2O_2: 1.4-Bis-[2-(5-phenyloxazolyl)]-benzene
```

$C_{27}H_{19}NO$: 2.5-di-(4-biphenyl)-oxazole

+ C₅H₈O₂: polymethylmetacralate

Organic Plastic Scintillator



Non-Linear light yield to Energy Loss:

$$N = N_0 \frac{dE/dx}{1 + k_B \cdot (dE/dx)}$$

 $k_B \approx 0.01 \,\mathrm{g/MeV} \,\mathrm{cm}^2$ Birk's density Parameter Recently better approximation, by A. Menchaca et al.

Typically 100 eV energy loss for one photon wavelength shifting necessary to avoid self-absorption

Cherenkov Radiation

A charged particle with a velocity v larger than the velocity of light in a medium emits light (Pavel A. Cherenkov, Ilja M. Frank, Igor Y. Tamm, Nobel Price 1958)

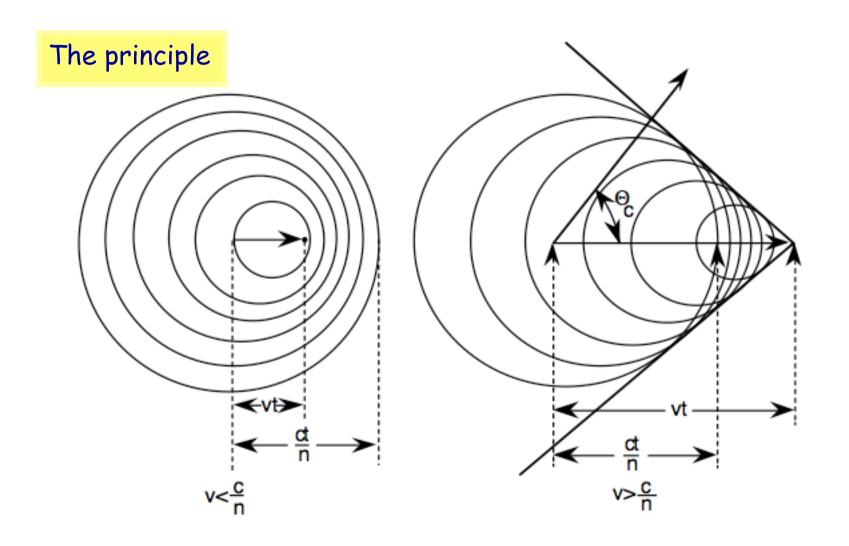
Threshold:
$$\beta_{\rm thres} = \frac{v_{\rm thres}}{c} \geq \frac{1}{n} \qquad \gamma_{\rm thres} = \frac{n}{\sqrt{n^2-1}}$$
 Angle of emission:
$$\cos\theta_c = \frac{1}{\beta \, n} = \frac{1}{\frac{v}{c} \, n}$$

$$\theta_c^{\rm max} = \arccos\frac{1}{n} \quad {\rm Water:} \ \theta_c^{\rm max} = 42^\circ \quad {\rm Neon} \ ({\rm 1atm}): \ \theta_c^{\rm max} = 11 \, {\rm mrad}$$

$$\frac{d^2N}{dEdl} = \frac{\alpha z^2}{\hbar c} \left(1 - \frac{1}{(\beta n)^2}\right) = \frac{\alpha z^2}{\hbar c} \sin^2\theta_c$$

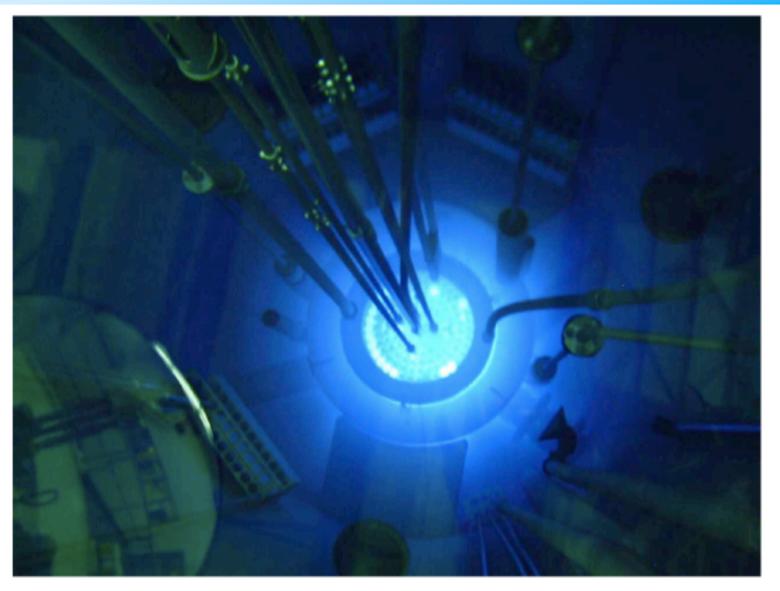
$$\frac{d^2N}{d\lambda dl} = \frac{2\pi\alpha z^2}{\lambda^2} \sin^2\theta_c$$

Cherenkov Radiation

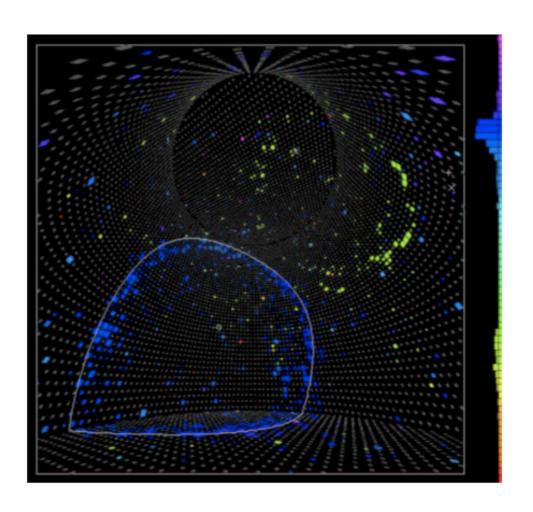


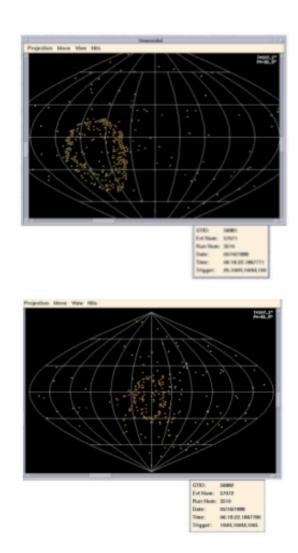
Project: calculate the Cherenkov radiation characteristics

Cherenkov Radiation in a Nuclear Reactor



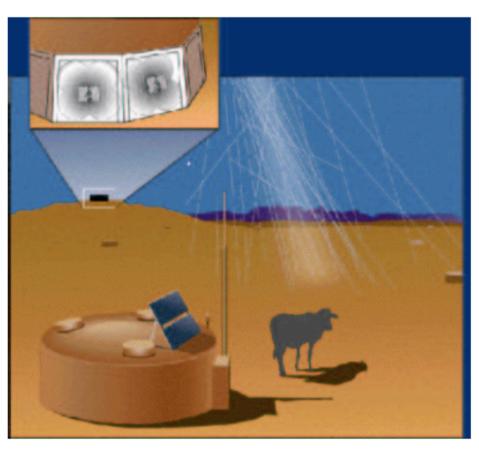
Water Cherenkov

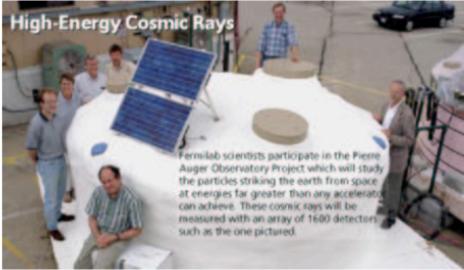




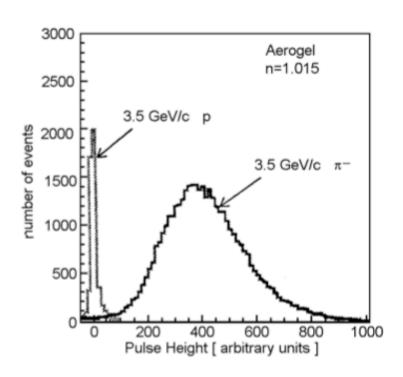
neutrino induced muon (top) and electron (bottom) in SNO $\,$

Water Cherenkov (Auger Experiment)





Threshold Cherenkov Detector



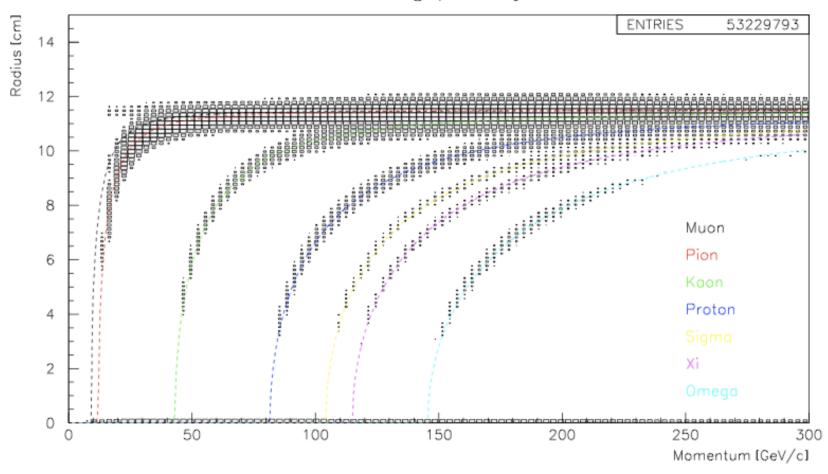
Aerogel:
$$n = 1.015 \Rightarrow \gamma_{\text{thres}} = 5.84$$

$$3.5\,\mathrm{GeV/c} \Rightarrow \gamma_{\pi} = 24.2, \quad \gamma_{p} = 2.86$$

To identify more than 2 particles and/or to cover wider momentum range: Several counters at different thresholds

Ring Imaging Cherenkov Detector (RICH)

Measure Cherenkov angle, not only threshold



SELEX RICH, 53 Million single negative track events

Transition Radiation Detectors

Transition Radiation: Reformation of particle field while traveling from medium with $\epsilon = \epsilon_1$ to medium with $\epsilon = \epsilon_2$.

Energy of radiation emitted at a single interface

$$S = \frac{\alpha \hbar z^2}{3} \frac{(\omega_1 - \omega_2)^2}{\omega_1 + \omega_2} \gamma$$

 $\alpha=1/137,\,\omega_1,\,\omega_2$ plasma frequencies, $\gamma=E/mc^2.$

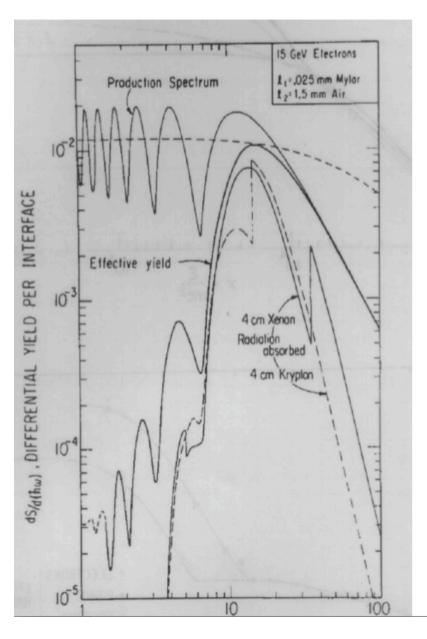
Typical values: Air $\omega_1 = 0.7 \, \text{eV}$, polypropylene $\omega_2 = 20 \, \text{eV}$

Spectral and angular dependence of Transition Radiation:

$$\frac{d^2}{d\vartheta d\omega} = \frac{2e^2}{\pi c} \left(\frac{\vartheta}{\gamma^{-2} + \vartheta^2 + \omega_1^2/\omega^2} - \frac{\vartheta}{\gamma^{-2} + \vartheta^2 + \omega_2^2/\omega^2} \right)^2$$

 \implies Most of radiation in cone with half angle $1/\gamma$: forward in particle direction.

Transition Radiation Detector



- Large photon energies $\omega > \gamma \omega_2 \approx 20 30 \text{ KeV}$: large drop of intensity $\propto \gamma^4/\omega^4$
- Medium energies $\gamma \omega_1 < \omega < \gamma \omega_2$: Logarithmic decrease with ω
- Small energies $\omega < \gamma \omega_1 \approx 1 \text{ KeV}$: intensity almost constant

Probability to emit a KeV photon: $\approx 10^{-2} \Longrightarrow$ Need a lot of interfaces: stack of radiator foils. Consequences:

- Need minimum foil thickness so particle field reaches new equilibrium
- Transition $\omega_1 \to \omega_2$ and $\omega_2 \to \omega_1$ equal \Longrightarrow Interference effects (min and max in fig)
- Equally spaced foils: Interference between amplitude of different foils
- Finite thickness of foils: re-absorption of radiation ($\propto Z^5$): Low Z materials.

Typical values used in TRDs: Thickness: $30 \,\mu\text{m}$, distance: $300 \,\mu\text{m}$, materials: mylar, CH₂, carbon fibers, lithium.

Detection of Transition Radiation

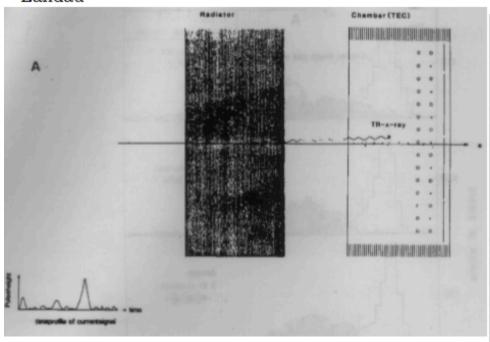
X-rays emitted under small angle to particle track

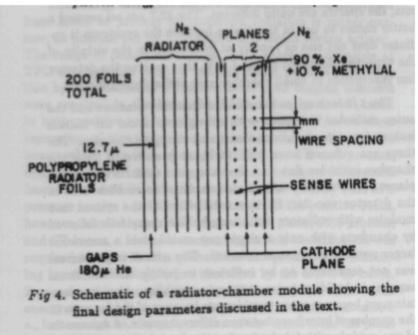
 \implies X-ray detector sees X-rays and particle dE/dx together.

Typical dE/dx in gas detectors: some KeV/cm and Landau distributed

 \implies Signals from dE/dx and X-ray similar

Detector: Use "thin" MWPC, with Xenon or Krypton, several (10) radiator / chamber units to beat Landau

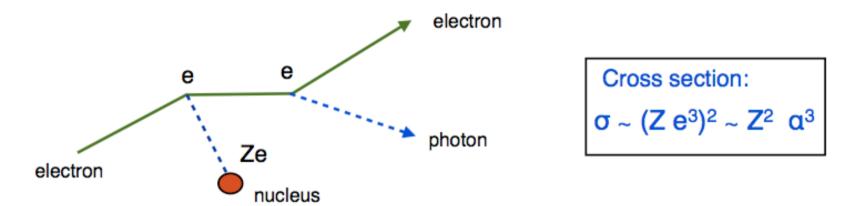




Two identification methods: Charge integration, Cluster counting

Bremsstrahlung

High energy electrons loose their energy predominantly through radiation (bremsstrahlung).



The electron is decelerated (accelerated) in the field of the nucleus. Accelerated charges radiate photons. Thus the bremsstrahlung is strong for light charged particles (electrons), because its acceleration is large for a given force. For heavier particles like muons bremsstrahlung effects are only important at energies of a few hundred GeV.

The presence of a nucleus is required to restore energy-momentum conservation. Thus the cross section is proportional to \mathbb{Z}^2 and α^3 (α = fine structure constant).

The characteristic length which a electron travels in material until a bremsstrahlung happens is the radiation length X₀.

Radiation Length

The radiation length is the characteristic length scale to describe electromagnetic showers in material. It is usually measured in g/cm².

The radiation length is:

- the mean distance over which a high energy electron looses all but 1/e of its energy.
- 7/9 of the mean free path for pair production by a high energy photon.

The radiation length is give by:

$$I=I_0 \exp[-x/X_0]$$

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \Big\{ Z^2 \big[L_{\rm rad} - f(Z) \big] + Z \, L_{\rm rad}' \Big\}$$

For
$$A=1~{\rm g~mol^{-1}},~4\alpha r_e^2 N_A/A=(716.408~{\rm g~cm^{-2}})^{-1}.$$

$$f(Z) = a^{2} [(1 + a^{2})^{-1} + 0.20206$$
$$-0.0369 a^{2} + 0.0083 a^{4} - 0.002 a^{6}]$$

where
$$a = \alpha Z$$

Element	Z	$L_{ m rad}$	$L'_{ m rad}$	
H	1	5.31	6.144	
$_{ m He}$	2	4.79	5.621	
Li	3	4.74	5.805	
Be	4	4.71	5.924	
Others	> 4	$\ln(184.15Z^{-1/3})$	$\ln(1194Z^{-2/3})$	

Bremsstrahlung

$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} z^2 \left(\frac{e^2}{mc^2}\right)^2 E \ln \frac{183}{Z^{1/3}}$$

$$-\frac{dE}{dx} = \frac{E}{X_0}$$
 PDG: $X_0 = \frac{716.4 \,A}{Z(Z+1) \,\ln(287/\sqrt{Z})} \,[\text{g/cm}^2]$

$$\frac{dE}{dx} \propto E \Rightarrow$$
 Critical Energy $\frac{dE}{dx}|_{\text{ion}} = \frac{dE}{dx}|_{\text{brems}}$

$$E_{\mathrm{crit}}^e = \begin{cases} \frac{610\,\mathrm{MeV}}{Z+1.24} & \text{for solids and liquids} \\ \frac{710\,\mathrm{MeV}}{Z+0.92} & \text{for gases} \end{cases}$$

Bremsstrahlung

Material

$$X_0$$
 [g/cm²]
 X_0 [cm]
 E_{crit}^e [MeV]

 air
 37
 30000
 84

 iron
 13.9
 1.76
 22

 lead
 6.4
 0.56
 7.3

$$-\frac{dE}{dx} \propto \frac{1}{m^2} \Rightarrow$$
 Electron Bremsstrahlung dominates

But: Muons in iron:
$$E_{\rm crit}^{\mu} = E_{\rm crit}^{e} \left(\frac{m_{\mu}}{m_{e}}\right)^{2} = 960 \, {\rm GeV}$$
 \Rightarrow Muon Calorimetry at TeV Energies

Bremsstrahlung important for electromagnetic cascades (Calorimetry)

Direct Electron Pair Production

in Coulomb field of Nucleus via virtual photons

$$-\frac{dE}{dx}|_{\mathrm{pair}} \propto E$$
 for large E

$$-\frac{dE}{dx} = a(E) + b(E)E$$

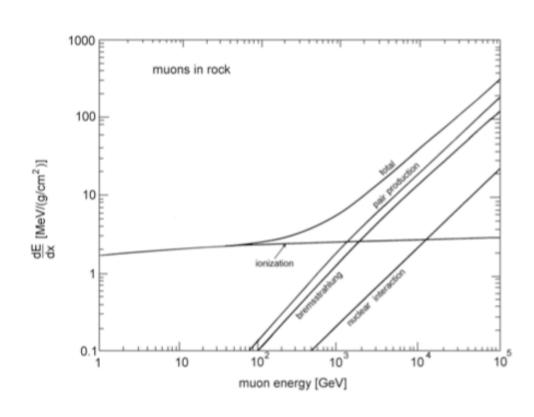
$$a(E)$$
 – Ionization energy loss

$$b(E) = b_{\text{brems}}(E) + b_{\text{pair}}(E) + b_{\text{nucl.int.}}(E)$$

Range of muons

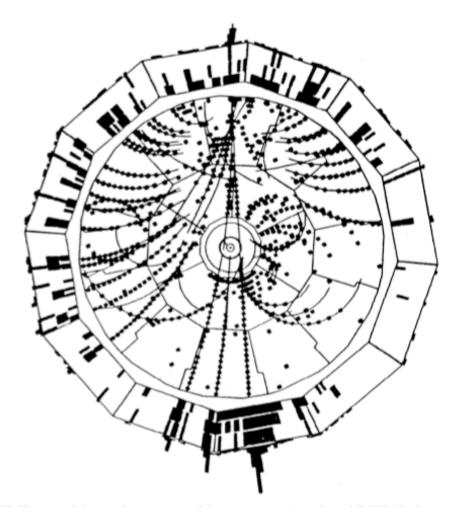
$$R = \int_{E}^{0} \frac{dE}{-dE/dx} = \frac{1}{b} \ln(1 + \frac{b}{a}E)$$

$$R = \begin{cases} 140\,\mathrm{m} & \mathrm{rock~for~} E = 100\,\mathrm{GeV} \\ 800\,\mathrm{m} & \mathrm{rock~for~} E = 1\,\mathrm{TeV} \\ 2300\,\mathrm{m} & \mathrm{rock~for~} E = 10\,\mathrm{TeV} \end{cases}$$



Energy loss of muons in Rock (
$$Z=11,\,A=22;\,\rho=3\,\mathrm{g/cm^2}$$
)

High Energy Muon Bremsstrahlung



Bremsstrahlung off muons starts to be important for momenta of a few hundred GeV.

Cosmic muon bremsstrahlung event in the ALEPH detector at LEP.

Nuclear Interactions

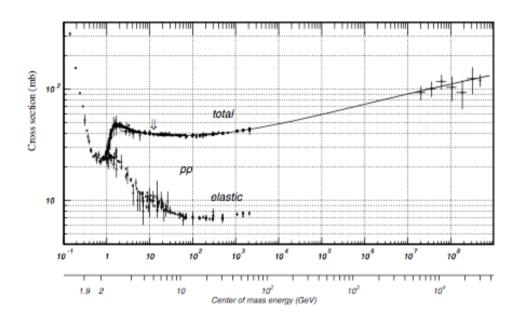
Important for the detection of neutral particles $\sigma_{\rm tot} \approx 50\,{\rm mbarn}$ $\sigma_{\rm inel} \propto A^{\alpha}, \alpha = 0.71$

$$\mbox{Nuclear Interaction length} \quad \lambda_I = \frac{A}{N_A \, \rho \, \sigma_{\rm tot}} \label{eq:lambdaI}$$

Nuclear Absorption length
$$\lambda_a = \frac{A}{N_A \, \rho \, \sigma_{\rm inel}}$$

for most material $\lambda_I, \lambda_a > X_0$ Al Fe Pb air

$$\lambda_I$$
 [cm] 26.2 10.6 10.4 48000 λ_I [g/cm] 70.6 82.8 116.2 62.0



Multiplicity grows logarithmically with E, Average $p_T = 350 \,\mathrm{MeV}/c$

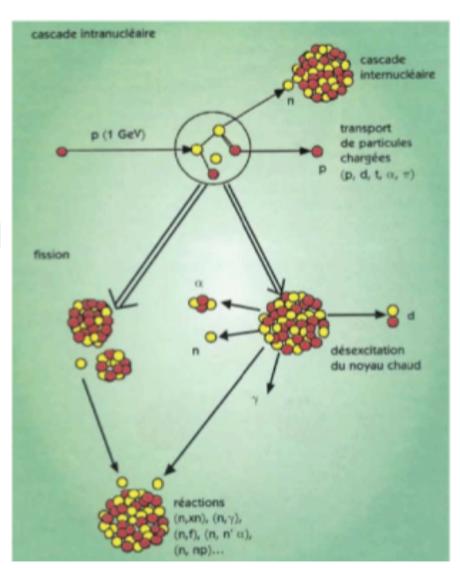
)

Interaction Lenght

Completely different!

Hadron calorimetry is not for the weak at heart

Most notably, neutrons are abundantly produced



Material Characteristics

Material	Z	Density [g/cm³]	X ₀ [cm]	λ _{int} [cm]	dE/dx _{mip} [MeV/cm]
Fe	26	7.9	1.8	17	11
Cu	29	9.0	1.4	15	13
Pb	82	11	0.6	17	13
w	74	19	0.4	9.6	22
²³⁸ U	92	19	0.3	11	21
Plastic Scint.	-	1.0	42	80	2.0
LAr	18	1.4	14	84	2.1
Quartz	-	2.3	12	43	3.9
Si	14	2.3	9.4	46	3.9
Al	13	2.7	8.9	39	4.4

Interaction of Neutral Particles

Introduction: Interactions of Photons

Photons are attenuated in matter.

$$I = I_0 e^{-\mu x}$$

 μ Mass Attenuation Coefficient

$$\mu = \frac{N_A}{A} \sum_{i=1}^{3} \sigma_i$$

$$\sigma_i = egin{cases} i = 1 \ i = 1 \ \text{Photoelectric Effect} \ i = 2 \ \text{Compton Scattering} \ i = 3 \ \text{Pair Production} \end{cases}$$

Photoelectric Effect

$$\gamma + \text{atom} \rightarrow \text{atom}^+ + e^-$$
 predominantly in K-shell

Complicated energy and Z dependence

$$\sigma_{\rm photo}^{K} = \left(\frac{32}{\epsilon^7}\right)^{1/2} \alpha^4 Z^5 \sigma_{\rm Thomson} \ [{\rm cm}^2/{\rm atom}]$$

$$\epsilon = \frac{E_{\gamma}}{m_e c^2}$$
 $\sigma_{\text{Thomson}} = \frac{8}{3}\pi r_e^2 = 665 \,\text{mbarn}$

For high energies:

$$\sigma_{\mathrm{photo}}^{K} = 4\pi \, r_e^2 \, Z^5 \, \alpha^4 \, \frac{1}{\epsilon}$$

Compton Scattering

$$\gamma + e \rightarrow \gamma' + e'$$

at high energies:
$$\sigma_C \propto \frac{\ln \epsilon}{\epsilon} \cdot Z$$

$$\frac{E_{\gamma}'}{E_{\gamma}} = \frac{1}{1 + \epsilon(1 - \cos\theta_{\gamma})}$$

$$E_{\rm kin}^{\rm max}(\theta_{\gamma}=\pi) = \frac{2\epsilon^2}{1+2\epsilon} m_e c^2$$
 for $\epsilon \gg 1$ $E_{\rm kin}^{\rm max}(\theta_{\gamma}=\pi) \to E_{\gamma}$

Pair Production

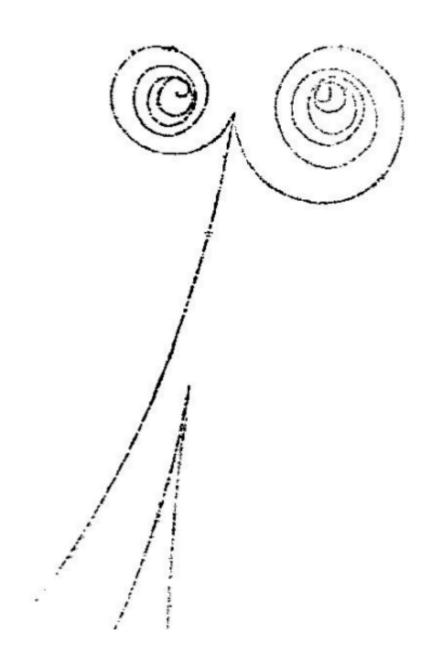
$$\gamma + \text{nucleus} \rightarrow \text{nucleus}' + e^+ + e^-$$

threshold energy:

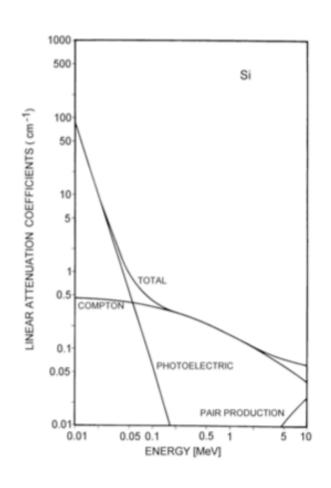
$$E_{\gamma} \geq 2 \, m_e c^2 + rac{2 \, m_e c^2}{m_{
m target}} = \left\{ egin{array}{l} pprox 2 \, m_e c^2 \, {
m on \, \, a \, \, nucleus} \\ 4 \, m_e c^2 \, {
m \, on \, \, an \, \, electron} \end{array}
ight.$$

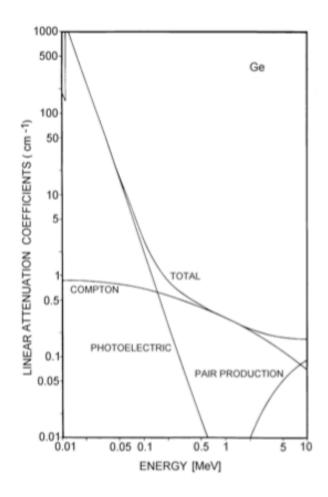
Cross section for $E_{\gamma} \gg 20 \,\mathrm{MeV}$

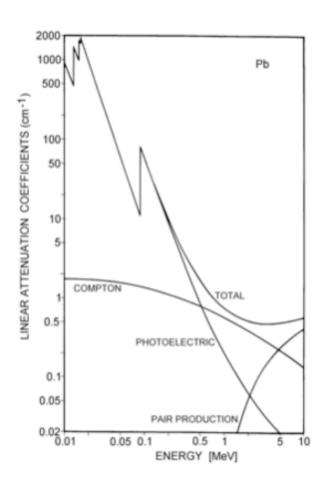
$$\sigma_{\mathrm{pair}} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \text{ [cm}^2/\text{atom]}$$
 $\approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$



Mass Attenuation Coefficients



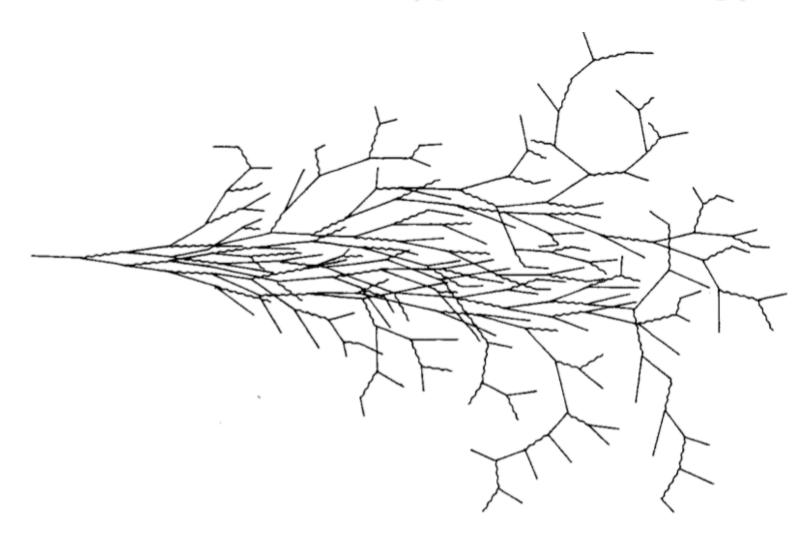




An Electromagnetic Shower

Photon → Pair Production

Electron / positron → Bremsstrahlung (Photon)



Electromagnetic Showers

$$\mu^-$$
 + nucleus $\rightarrow \mu^-$ + nucleus' + γ
 $\gamma \rightarrow$ electromagnetic shower

Multiplate Cloud Chamber

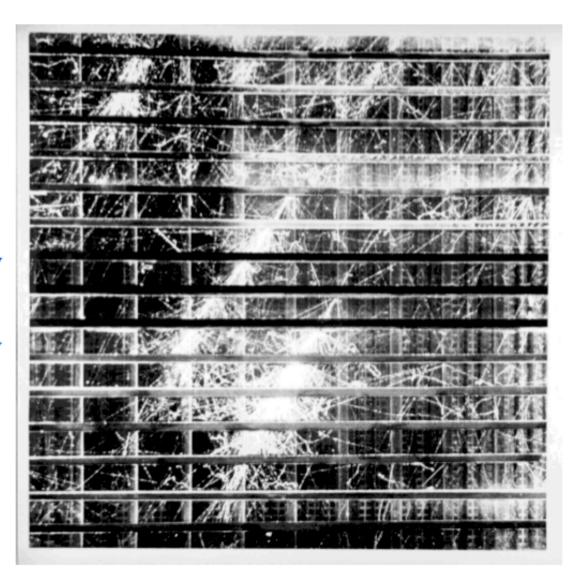


Electromagnetic Showers

Air shower experiment

Multiplate Cloud Chamber below 3 m of concrete

Electromagnetic showers initiated by muon Bremsstrahlung



Hadronic Showers

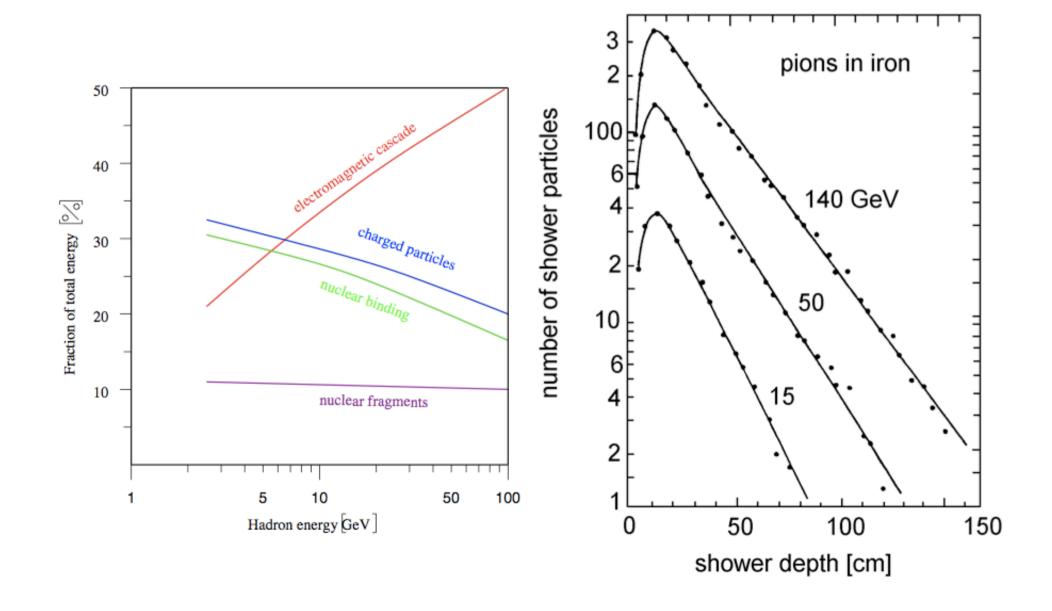
Longitudinal development: governed by nuclear interaction length λ_I Lateral development: transverse momentum p_T of particles since $\lambda_I > X_0$ and $\langle p_T \rangle \gg \langle p_T \rangle^{\text{mult. scatt.}}$

⇒ hadron showers are wider and longer than electromagnetic showers

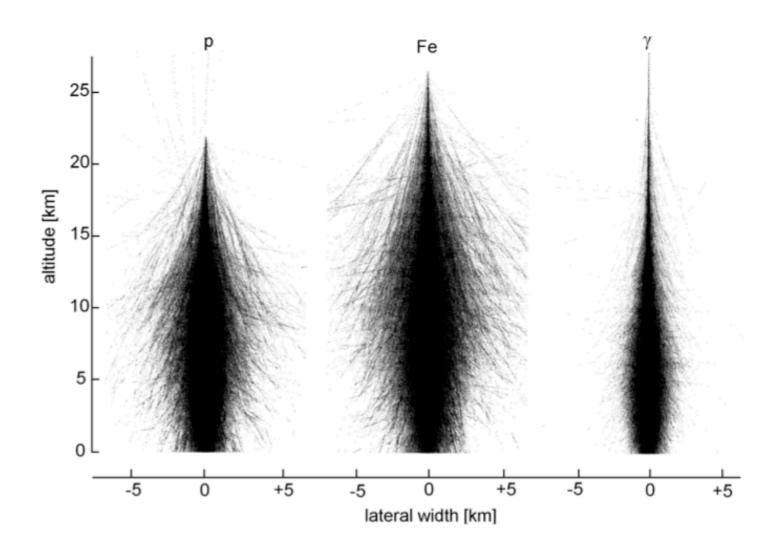
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Hadron Energy \Rightarrow \begin{cases} \text{charged particles } (\mu \text{'s are lost}) \\ \text{electromagnetic showers via } \pi^0 (e, \gamma; \text{ contained}) \\ \text{nuclear binding energy (can be partially recovered)} \\ \text{nuclear fragments (partially lost)} \end{cases}
```

- ⇒ Visible energy systematically lower
- ⇒ Due to fluctuations in energy losses Energy resolution is worse than for electromagnetic calorimeters

Hadronic Showers



Extensive Air Showers



Interaction of Neutrons

indirect technique: neutrons interact and produce charged particles

• Low Energies ($< 20 \,\text{MeV}$): $n + ^6 \,\text{Li} \to \alpha + ^3 \,\text{H} \implies \text{LiI}(\text{Tl}) \text{ scintillators}$ $n + ^{10} \,\text{B} \to \alpha + ^7 \,\text{Li} \implies \text{BF}_3 \text{ gas counters}$ $n + ^3 \,\text{He} \to p + ^3 \,\text{H} \implies ^3 \,\text{He-filled proportional counters}$ $n + p \to n + p \implies \text{proportional chambers with for example CH}_4$

• High Energies $(E > 1 \,\text{GeV})$ $n + ^{235}\text{U} \rightarrow \text{fission products} \Rightarrow \text{coated proportional counters}$ $n + \text{nucleus} \rightarrow \text{hadron cascade} \Rightarrow \text{calorimeters}$

Interaction of Neutrinos

$$egin{array}{lll}
u_e + n &
ightarrow & p + e^- & ar{
u_e} + p &
ightarrow & n + e^+ \
u_\mu + n &
ightarrow & p + \mu^- & ar{
u_\mu} + p &
ightarrow & n + \mu^+ \
u_ au + n &
ightarrow & p + au^- & ar{
u_ au} + p &
ightarrow & n + \mu^+ \
align* & ar{
u_ au} + p &
ightarrow & n + au^+ \
\end{array}$$

Small cross section: (MeV range):
$$\sigma(\nu_e N) = \frac{4}{\pi} \cdot 10^{-10} \left(\frac{\hbar p}{(m_p c)^2} \right)^2 = 1.6 \cdot 10^{-44} \, \text{cm}^2 \text{ for } 0.5 \, \text{MeV}$$

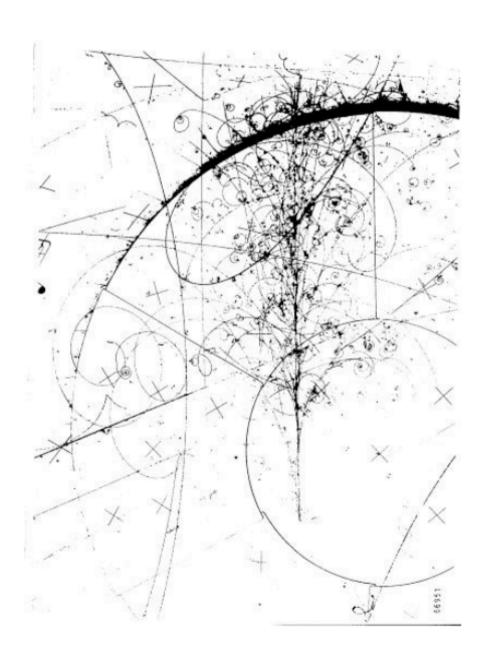
for high Energies (GeV range):

$$\sigma(\nu_{\mu}N) = 0.67 \cdot 10^{-38} E_{\nu} \text{ cm}^2/(\text{nucleon GeV})$$

 $\sigma(\bar{\nu}_{\mu}N) = 0.34 \cdot 10^{-38} E_{\nu} \text{ cm}^2/(\text{nucleon GeV})$

Indirect Measurement by missing momentum and missing energy technique

Interaction of Neutrinos



 $u_e + {
m nucleon}
ightarrow e^- + {
m hadron}$ Electromagnetic shower

Radiation Length

For a material with N atoms/cm³ there will be n collisions for a path length L

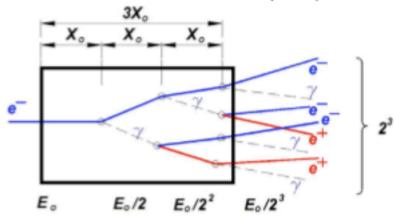
$$n = N\sigma_{radiative}L$$

Setting n=1 and writing L=X₀, a radiation length is given by

$$\frac{1}{X_0} = N\sigma_{radiative} \approx 4N\alpha \left(\frac{\alpha}{m_e}\right)^2 Z^2 \log\left(\frac{183}{Z^{1/3}}\right)$$

Electron Impinging on a Target

FNAL-CERN Summer School 2009 Calorimetry Lecture 1



For photons

$$X_0^{\gamma} = \frac{9}{7}X_0$$

6

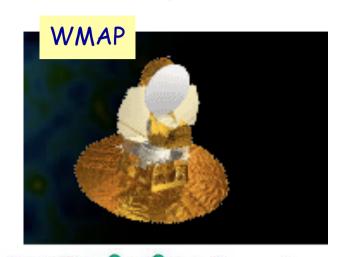
End of Lecture 2

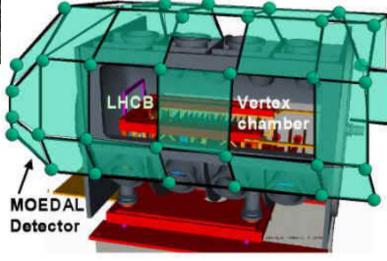
Projects

- Simple Projects (*)
 - Derive the Bethe Bloch Formula (Grupen)
 - Cherenkov light radiation (Grupen)
- Other key experiments (**)
 - The Auger cosmic ray experiment. Ultra High Cosmic Rays
 - The WMAP experiment (satellite, dark matter/dark energy)
 - The Planck experiment (satellite, dark mater dark energy)
 - Ligo: gravitational waves / laser interferometry
 - The MOEDAL experiment at CERN (monopole search)
- Detection techniques (**)
 - General: EM shower calorimeters (Grupen)
 - Had shower calorimeters (Grupen)
 - Compensating calormeters (Wigmans lectures)
 - Dual readout calorimeters (Wigmans lectures)
 - Silicon detectors (Kleinknecht)
 - MSGC gas detectors
- Difficult projects (***)
 - CMS papers with real data (calibration, alignment, efficiency of the CMS detector

Other Experiments











Timeline & Projects

```
Report (5-10 pages) + Presentation of ~ 15 minutes
When?
⇒ Agenda
2 oct : ADR Particles through matter
– 9 oct : ADR Detectors and CMS
– 16 oct : JD (Brussels)
 - 23 oct : -- no lectures

    30 oct : ADR Other LHC experiments/ other applications

                     (medicine/other areas)
 – 6 nov : JD
 - 13 nov : JD
 – 20 nov : ADR?
 27 nov : JD
 – 4 Dec: ADR Projects <---</p>
 - 11 Dec: JD?
 - 18 Dec: ---
```

Drift and Diffusion in Gases

- Electrons and Ions lose Energy by multiple collisions: → Thermalization
- Ionization diffuses (Gaussian) Width $\propto \sqrt(t)$
- With electric field:

 Drift with constant velocity v_{drift} In argon-isobutane: Typical $v_{\text{drift}}^{\text{electron}} \approx 5 \text{ cm}/\mu\text{sec}$ Longitudinal and transverse diffusion different $(\sigma = \sigma(E_e))$
- $v_{\rm drift} \propto 1/m$: Ions > 10^3 times slower!
- With additional magnetic field: drift under angle (Lorentz force)

