Extensions of the Standard Model

(part 2)

Prof. Jorgen D'Hondt
Vrije Universiteit Brussel
Inter-university Institute for High Energies

Content:

- **The Higgs sector of the Standard Model and extensions**
- **Theoretical constraints on the Higgs boson mass**
- **Searching for the Higgs boson**
- **The hierarchy problem in the Standard Model**
- **Introduction to the phenomenology of Supersymmetry**

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

Extensions of the Standard Model
(part 2)

Prof. Jorgen D'Hondt
Vrije Universiteit Brussel
Inter-university Institute for High Energies

Lecture 1

Important note: whenever we note "Higgs boson" (or mechanism), we mean the well-known Brout-Englert-Higgs boson or mechanism.

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

How far can we stretch our theory?

Prof. Jorgen D'Hondt
Vrije Universiteit Brussel
Inter-university Institute for High Energies

Content:

- **Short reminder of the spontaneous ElectroWeak symmetry breaking**
- **One missing piece: the mass of the Brout-Englert-Higgs scalar**
- **Theoretical constraints on the Higgs boson mass**
    - **Pertubativity or unitarity constraint**
    - **Triviality bound and stability bound**
    - **Fine-tuning**
- **Methods can be applied to models beyond the Standard Model**
- **What about more then one Higgs doublet…**

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

Spontaneously broken QED theory

Let us illustrate the "Higgs" mechanism with a massive U(1) theory before going to the symmetry group $SU(2)_L \times U(1)_Y$. The Lagrangian of QED is:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi$$

This is invariant under the U(1) gauge transformation

$$\psi \to e^{-i\alpha(x)}\psi$$

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

Now we wish to give the photon a mass by adding the term

$$\mathcal{L}_{mass} = \frac{m_A^2}{2}A_\mu A^\mu$$

Which breaks the initial U(1) gauge symmetry. Hence need to invoke a mechanism which introduces a mass without breaking the symmetry.

Spontaneously broken QED theory

Introduce a complex scalar field Φ as

$$\mathcal{L} = \mathcal{L}_{QED} + (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi)$$

with the potential V defined as $V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$ which is symmetric under the transformation $\Phi \to -\Phi$

We can choose a parametrization as

$$\Phi = \frac{1}{\sqrt{2}} \phi(x) e^{i\xi(x)}$$

where both fields ϕ and ξ are real fields.

The potential becomes

$$V(\Phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

where the Higgs self-coupling term should be positive (λ>0) to get a potential bound from below. When $\mu^2 < 0$ a non-zero vacuum expectation value is obtained.

$$< 0|\phi^2|0 >= \phi_0^2 = \frac{-\mu^2}{\lambda} = v^2$$

Spontaneously broken QED theory

Therefore we can normalize the field $\xi(x)$ as $\frac{\xi(x)}{\phi_0}$.

We can choose the unitary gauge transformation

$$\alpha(x) = -\frac{\xi(x)}{\phi_0}$$

and then Φ becomes real-valued everywhere. The kinetic term in the Lagrangian becomes

$$(D_\mu\Phi)^*(D^\mu\Phi) \rightarrow \frac{1}{2}\partial_\mu\psi\partial^\mu\psi + \frac{e^2}{2}A_\mu A^\mu\psi^2$$

The Lagrangian can be expanded around its minimum ϕ_0 by introducing a degree of freedom h (a new field). The potential becomes

$$V(\phi \rightarrow \phi_0 + h) = + \frac{m_h^2}{2}h^2 + \frac{\mu'}{3!}h^3 + \frac{\eta}{4!}h^4$$

with $m_h^2 = 2\lambda\phi_0^2$ and $\mu' = \frac{3m_h^2}{\phi_0}$ and $\eta = 6\lambda = 3\frac{m_h^2}{\phi_0^2}$

Spontaneously broken QED theory

The kinetic term becomes

$$\frac{1}{2}\partial_\mu(\phi_0 + h)\partial^\mu(\phi_0 + h) + \frac{e^2}{2}A_\mu A^\mu(\phi_0 + h)^2$$

and with $\partial_\mu \phi_0 = 0$ this becomes

$$\frac{e^2}{2}A_\mu A^\mu \phi_0^2 + e^2 A_\mu A^\mu \phi_0 h + \frac{e^2}{2}A_\mu A^\mu h^2 + \frac{1}{2}(\partial_\mu h)(\partial^\mu h)$$

where the first term provides a mass to the photon $m_A^2 = e^2\phi_0^2$,
the second term gives the interaction strength of the coupling A-A-h,
the third term the interaction strength of the coupling A-A-h-h

In the new potential term $V(\phi_0 + h)$ also cubic terms appear which break the reflexion symmetry $\phi \rightarrow -\phi$.

This U(1) example is the most trivial example of a spontaneous broken symmetry.

Spontaneously broken SU(2)xU(1) theory

The bosonic part of the Lagrangian is

$$\mathcal{L}_{bosonic} = |D_\mu \Phi|^2 - \mu^2 |\Phi|^2 - \lambda |\Phi|^4 - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}$$

with Φ a doublet field consisting out of two complex scalar fields or components

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$$

We need at least 3 massive gauge bosons, hence need at least 2 complex fields (cfr. Goldstone theorem).

$$\begin{aligned} D_\mu \Phi &= \left(\partial_\mu + ig\frac{\tau^a}{2} W^a_\mu + ig' \frac{Y}{2} B_\mu\right) \Phi \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^a_{\mu\nu} &= \partial_\nu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu \end{aligned}$$

with τ^a the Pauli matrices and f^{abc} the structure constants of the $SU(2)_L$ group.

Spontaneously broken SU(2)xU(1) theory

The B_μ field corresponds to the generator Y of the $U(1)_Y$ group and the three W_μ^a fields to the generators T^a of the $SU(2)_L$ group.

$$T^a = \frac{1}{2}\tau^a$$
$$\left[T^a, T^b\right] = if^{abc}T^c$$
$$Tr\left[T^a T^b\right] = \frac{\delta_{ab}}{2}$$

When $\mu^2<0$ the vacuum expectation value of Φ is non-zero.

$$< 0|\Phi|0 >= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad with \qquad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

The VEV will carry the hypercharge and the weak charge into the vacuum, but the electric charge remains unbroken, hence $Q = T^3 + \frac{Y}{2}$ and we break $SU(2)_L xU(1)_Y$ to $U(1)_Q$ with only one generator. Expending the terms in the Lagrangian around the minimum of the potential gives

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Spontaneously broken SU(2)xU(1) theory

We obtain

$$|D_\mu\Phi|^2 \to \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{8}g^2(v+h)^2|W_\mu^{(1)} + iW_\mu^{(2)}|^2 + \frac{1}{8}(v+h)^2|gW_\mu^{(3)} - g'B_\mu|^2$$

and define the following fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^{(1)} \mp iW_\mu^{(2)}\right)$$

$$Z_\mu = \frac{gW_\mu^{(3)} - g'B_\mu}{\sqrt{g^2+g'^2}}$$

$$A_\mu = \frac{gW_\mu^{(3)} + g'B_\mu}{\sqrt{g^2+g'^2}}$$

which we can transform into expressions for $B_\mu \; and \; W_\mu^{(i)}$ **and put this in the above equation for** $|D_\mu\Phi|^2$ **and isolate the Higgs boson interaction terms**

$$\mathcal{L}_{Higgs\ int} = \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2}Z_\mu Z^\mu\right)\left(1+\frac{h}{v}\right)^2 - \frac{m_h^2}{2} - \frac{\xi}{3!}h^3 - \frac{\eta}{4!}h^4$$

Spontaneously broken SU(2)xU(1) theory

...

$$\mathcal{L}_{Higgs\ int} = \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)\left(1 + \frac{h}{v}\right)^2 - \frac{m_h^2}{2} - \frac{\xi}{3!}h^3 - \frac{\eta}{4!}h^4$$

with

$$\begin{aligned} m_W^2 &= \tfrac{1}{4} g^2 v^2 \\ m_Z^2 &= \tfrac{1}{4}(g^2 + g'^2) v^2 \\ m_h^2 &= 2\lambda v^2 \\ \xi &= 3\frac{m_h^2}{v} \\ \eta &= 6\lambda = 3\frac{m_h^2}{v^2} \end{aligned}$$

where it is convenient to define the Weinberg mixing angle θ_W

$$tan\theta_W = \frac{g'}{g}$$

and therefore

$$\frac{m_W^2}{m_Z^2} = 1 - sin^2\theta_W$$

Spontaneously broken SU(2)xU(1) theory

From experiment we know

$$
\begin{aligned}
m_W &\simeq 80GeV \\
m_Z &\simeq 91GeV \\
g &\simeq 0.65 \\
g' &\simeq 0.35
\end{aligned}
$$

Hence we obtain $v \simeq 246\ GeV$

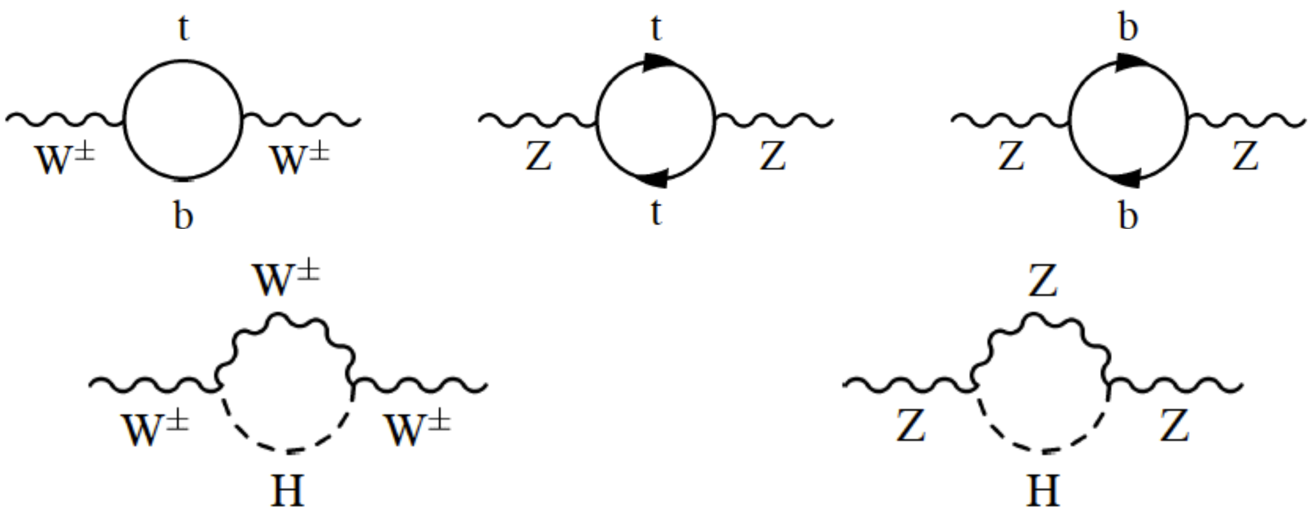
And for the couplings between V=W/Z bosons and the Higgs boson

$$
\begin{aligned}
g_{hVV} &= 2\frac{m_V^2}{v} \\
g_{hhVV} &= 2\frac{m_V^2}{v^2} \\
g_{hhh} &= 3\frac{m_h^2}{v} \\
g_{hhhh} &= 3\frac{m_h^2}{v^2}
\end{aligned}
$$

We observe that the Higgs sector in the Standard Model is completely determined from the mass of the Higgs boson.

Experimental constraints on the Higgs boson mass

Radiative corrections on the propagators of bosons in the theory



References:

- "A combination of preliminary Electroweak Measurements and Constraints on the Standard Model", hep-ex/0612034 (and recent updates)
- "Precision Electroweak measurements on the Z boson resonance", hep-ex/0509008

The free parameters in the fit

- The Standard Model gives a unified description of Electro-Magnetic & Weak interactions, hence the weak coupling is related to the EM coupling → only 2 coupling constants remain independent
  1) α : EM interactions (fine structure constant)
  2) α_s : strong interactions

- Among the fermion masses only the top quark mass plays an important role (all others are well enough determined and can be assumed fixed) as they have $m_f \ll m_Z$ and do not influence the observations at high energies significant: m_t

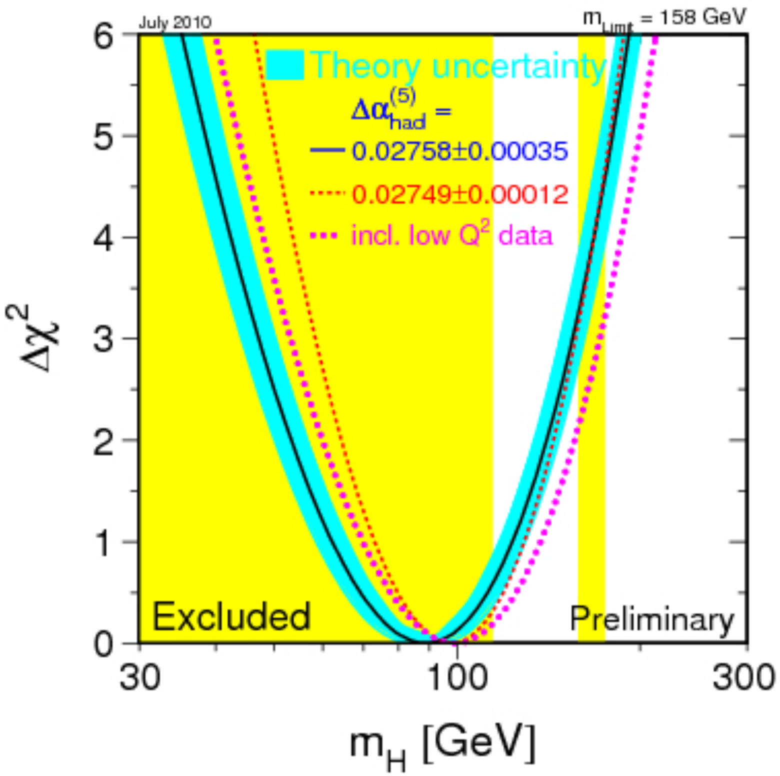
- Among the boson masses the Z boson mass (m_Z) is very well measured while the W boson mass not that presice. The free parameter m_W has been replaced by G_F, hence m_W becomes a quantity derived from the SM relations or the EW fit.

- The Higgs boson mass (m_H).

→ the free parameters are $\alpha_s(m_Z^2)$, $\alpha(m_Z^2)$, m_Z, m_t, m_H, G_F

The ElectroWeak fit: the result

- **Five relevant input parameters of the Standard Model relations**
  $\alpha_s(m_Z^2), \alpha(m_Z^2), m_Z, m_t, m_H, G_F$
- **Given these parameters we can obtain indirect measurements of the observables measured directly by LEP, SLC, Tevatron.**
- **These predictions go through radiative corrections calculated to some precision**
  → **blueband in the plot**
  (eg. 2-loop fermionic and bosonic corrections for the calculation of m_W)
- **On each of the input parameters there is some uncertainty, hence we derive a confidence interval where the observed quantity should have its value give the SM relations**
  → **reflected in the $\Delta\chi^2(m_H)$**



Theoretical constraints on the Higgs boson mass

Aim:

- Get a feeling how one can test if a theory is consistent
- How far can we stretch the EW theory until it does not make sense anymore?
- Example for the yet unobserved Higgs sector in the Standard Model, but techniques can be applied elsewhere

Content:

- Perturbativity & unitarity
- The triviality bound
- The vacuum stability bound
- The fine tuning constraints

Perturbative constraint & unitarity

The scattering of vector bosons at high energies is divergent due to their longitudinal polarization. Take V = W or Z traveling in the z-direction with 3-momentum magnitude k.

$$k^\mu = (E_k; \vec{k}) = (E_k; 0, 0, k)$$

with

$$E_k^2 = k^2 - m_V^2$$

The three polarization vectors are (resp. right handed, left handed and longitudinal):

$$\begin{aligned}
\epsilon_+^\mu(\vec{k}) &= \frac{1}{\sqrt{2}}(0; 1, i, 0) \\
\epsilon_-^\mu(\vec{k}) &= \frac{1}{\sqrt{2}}(0; 1, -i, 0) \\
\epsilon_L^\mu(\vec{k}) &= \frac{1}{m_V}(k; 0, 0, E_k)
\end{aligned}$$

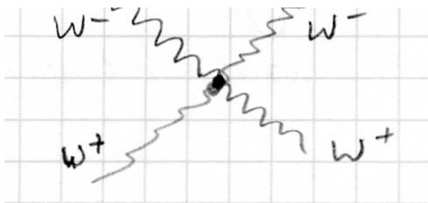
which satisfy (a,b = +, - , L)

$$\begin{aligned}
k_\mu \epsilon_a^\mu(\vec{k}) &= 0 \\
\epsilon_a^\mu(\vec{k}) \epsilon_{b\mu}^*(\vec{k}) &= -\delta_{ab}
\end{aligned}$$

Perturbative constraint & unitarity

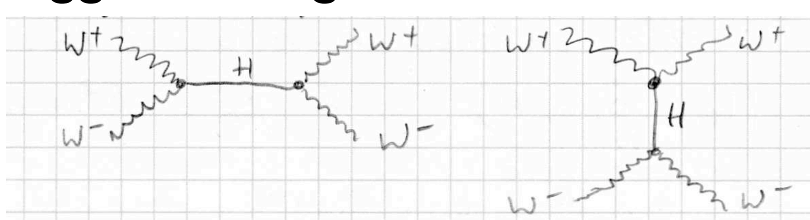
When $E_k >> m_V$ the longitudinal polarization is divergent. Diagrams with external vector bosons have divergent cross sections. Consider the process $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

(i) Four point interaction



(ii) Gauge exchange of photon/Z in the s- and t-channel

(iii) Higgs exchange in the s- and t-channel



Prof. Jorgen D'Hondt (Jorgen.DHondt@vub.ac.be)
Vrije Universiteit Brussel & Inter-university Institute for High Energies (IIHE)

Perturbative constraint & unitarity

The amplitude can be written as (S.Weinberg, Vol.1, sec 3.7)

$$\mathcal{A} = \mathcal{A}^{(2)} s^2 + \mathcal{A}^{(1)} s + \mathcal{A}^{(0)}$$

s = Mandelstam variable (square sum of initial momenta)
From computations we learn that (when $s, t >> m_V^2, m_h^2$)

$$\begin{array}{lcl} \mathcal{A}^{(2)} & \longrightarrow & 0 \\ \mathcal{A}^{(1)} & \longrightarrow & 0 \\ \mathcal{A}^{(0)} & \longrightarrow & -\frac{2m_h^2}{v^2} \simeq -4\lambda \end{array}$$

Perfect cancellation between the diagrams. But the amplitude remains proportional to the Higgs boson mass. If the Higgs boson mass is too large the theory becomes strongly interacting and we cannot perform expansions versus λ.

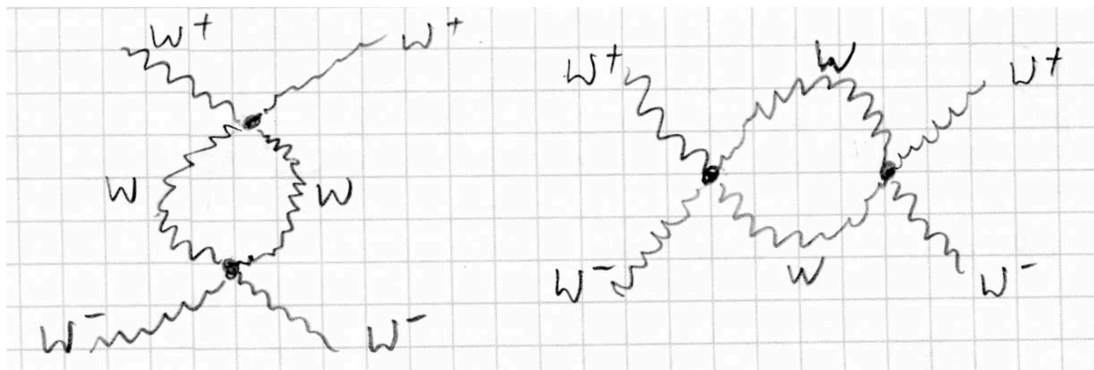
Perturbative constraint & unitarity

At the loop level the process

$$W^+W^- \rightarrow (WW)_{loop} \rightarrow W^+W^-$$

has an amplitude of

$$\frac{2\lambda^2}{16\pi^2}$$



The one-loop amplitude becomes equal to the tree-level amplitude when $\lambda \sim 32\pi^2$, hence the Electro-Weak theory should break down when $m_h > 6$ TeV.

More rigorous via partial wave analysis: $m_h < 870$ GeV
When taking also the WW→ZZ process into account: $m_h < 710$ GeV

The triviality bound

The couplings should remain finite at all energy scales Q.

$$g_i = (0.41; 0.64; 1.2)$$
$$y_t = \sqrt{2}\frac{m_t}{v} \simeq 1$$
$$\lambda = \frac{m_h^2}{2v^2}$$

Via the renormalization group equations we can evolve the couplings to higher scales Q.

$$t = ln\left(\frac{Q}{Q_0}\right)$$

$$\begin{aligned}
\frac{dg_1}{dt} &= \frac{41}{10}\frac{g_1^3}{16\pi^2} \\
\frac{dg_2}{dt} &= -\frac{19}{6}\frac{g_2^3}{16\pi^2} \\
\frac{dg_3}{dt} &= -7\frac{g_3^3}{16\pi^2} \\
\frac{dy_t}{dt} &= \frac{y_t}{16\pi^2}\left(-\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_t^2\right) \\
\frac{d\lambda}{dt} &= \frac{1}{16\pi^2}\left(24\lambda^2 - \lambda\left(\frac{9}{5}g_1^2 + 9g_2^2 + 12y_t^2\right) + \frac{9}{8}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) - 6y_t^4\right)
\end{aligned}$$

For large Higgs boson masses the term λ^2 dominates and after integration one obtains Landau pole or a limit on the value of Q for which the theory is still valid.

$$Q_{LP} = m_h exp\left(\frac{4\pi^2 v^2}{3m_h^2}\right)$$

Prof. Jorgen D'Hondt (Jorgen.DHondt@vub.ac.be)
Vrije Universiteit Brussel & Inter-university Institute for High Energies (IIHE)

The vacuum stability bound

When the Higgs boson mass is light the term $-6y_t^4$ will dominate:

$$\frac{d\lambda}{dt} \simeq -\frac{1}{16\pi^2} 6y_t^4$$

hence for higher scales Q the value of λ could become negative, hence the vacuum instable (V<0). With the constraint $\lambda(Q)>0$ for all values of Q we obtain a lower limit on the Higgs boson mass. After integrating the part of the RGE which is λ independent from Q_0 to Q we obtain:

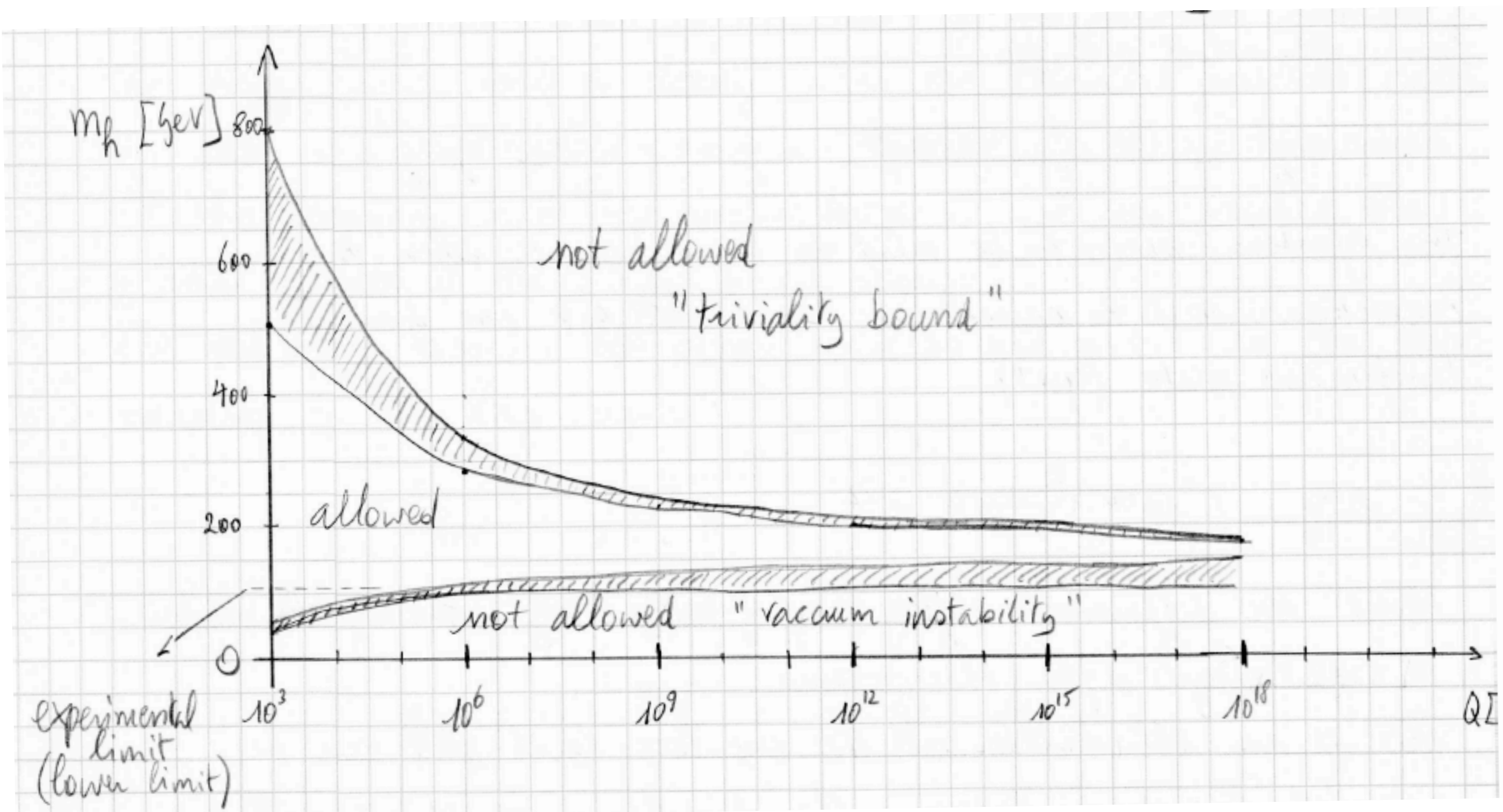
$$m_h^2 > \frac{v^2}{8\pi^2}\left(\frac{9}{8}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) - 6y_t^4\right) ln\left(\frac{Q}{Q_0}\right)$$

Hence a lower limit for the Higgs boson mass for a given Q scale to keep the vacuum stable (without the presence of new physics phenomena beyond the Standard Model).

The full calculations at higher order (more loops) is done.

All together: theoretical bounds on the Higgs boson mass

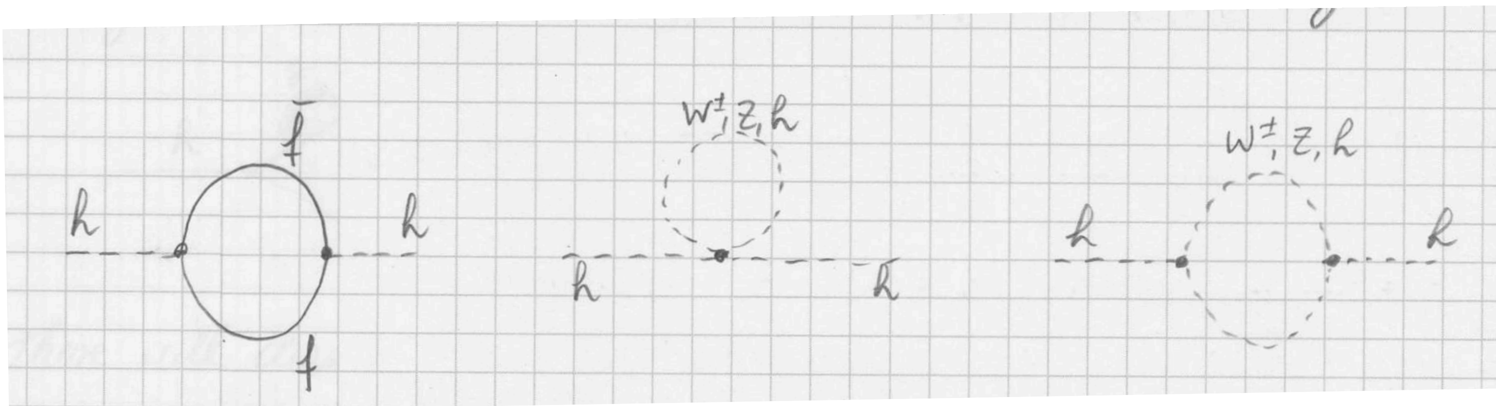
If the Higgs boson is to be found at 60 GeV then this means the vacuum is instable in the absence of new physics. Only when the mass is between 130-180 GeV the vacuum can remain stable up to the Planck scale.



Prof. Jorgen D'Hondt (Jorgen.DHondt@vub.ac.be)
Vrije Universiteit Brussel & Inter-university Institute for High Energies (IIHE)

The fine-tuning constraint

The radiative corrections to the Higgs boson mass induce a fine tuning problem. At one loop



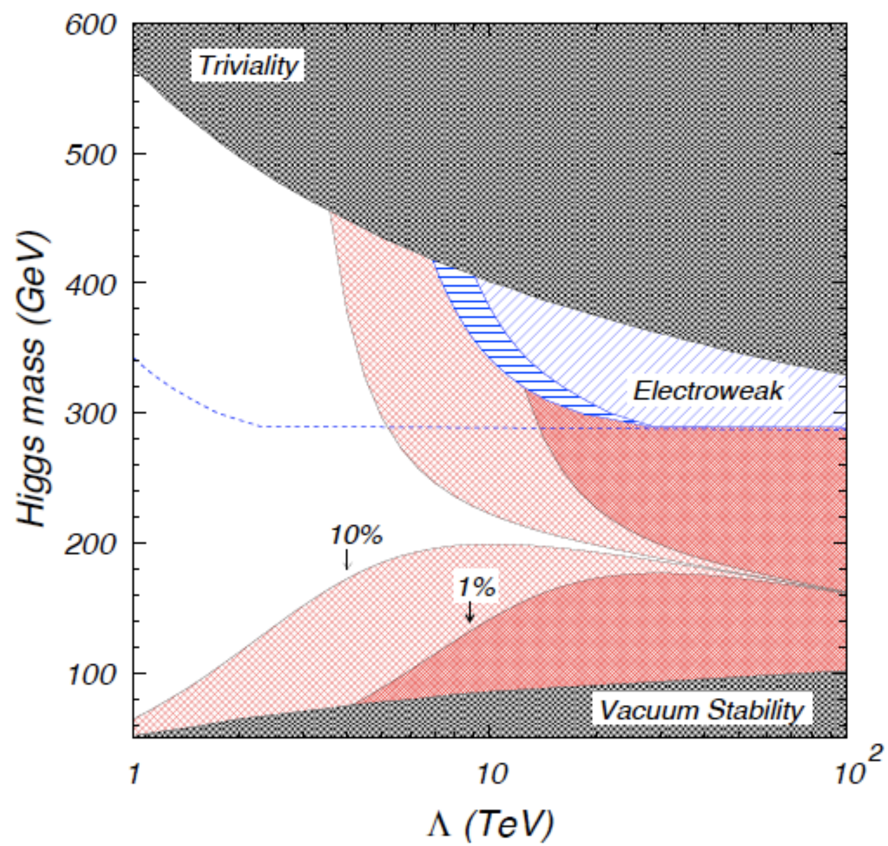
The integral can be cut-off at a momentum scale Λ

$$m_h^2 = (m_h^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)$$

hence to cancel this we need $m_h^2 \sim (320\ \mathrm{GeV})^2$
To cancel the radiative terms up to the GUT scale $\Lambda \sim 10^{16}$ GeV we need to cancel up to 32 digits after the comma.

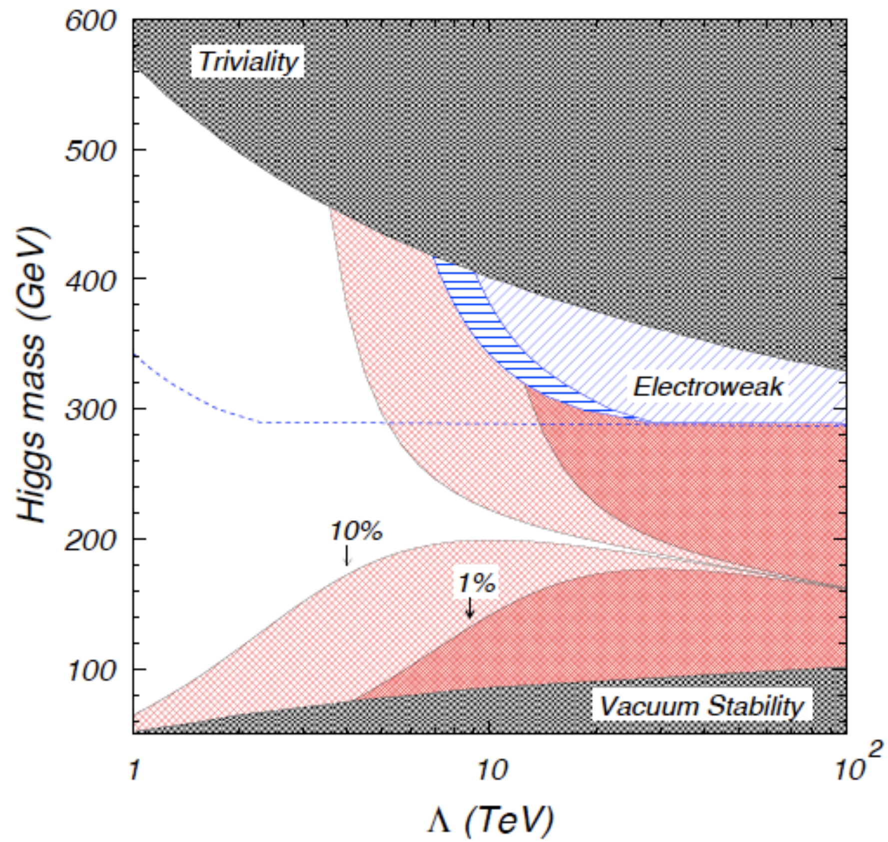
The fine-tuning constraint

Requesting up to 10% (or 1%) fine-tuning the allowed range for the validity of the Standard Model is reduced.



The fine-tuning constraint

Requesting up to 10% (or 1%) fine-tuning the allowed range for the validity of the Standard Model is reduced.



… why should there be only one Higgs doublet ?

Two Higgs Doublet Models (2HDM)

In the Standard Model we have introduced only one complex Higgs doublet resulting into one physical Higgs boson field and masses for 3 vector bosons. There is however no experimental reason why we cannot have more then one Higgs doublet.

Let us introduce two complex Higgs doublet fields ϕ_1 and ϕ_2.
The most general Higgs potential V which spontaneously breaks $SU(2)_L x U(1)_Y$ into $U(1)_{EM}$ is

$$
\begin{aligned}
V(\phi_1, \phi_2) \quad = \quad & \lambda_1(|\phi_1|^2 - v_1^2)^2 + \lambda_2(|\phi_2|^2 - v_2^2)^2 \\
& +\lambda_3 \left[(|\phi_1|^2 - v_1^2) + (|\phi_2|^2 - v_2^2)\right]^2 \\
& +\lambda_4 \left[|\phi_1|^2|\phi_2|^2 - (\phi_1^{*T}\phi_2)(\phi_2^{*T}\phi_1)\right] \\
& +\lambda_5[Re(\phi_1^{*T}\phi_2) - v_1 v_2 cos\xi]^2 \\
& +\lambda_5[Im(\phi_1^{*T}\phi_2) - v_1 v_2 sin\xi]^2
\end{aligned}
$$

where the λ_i values are real and the ϕ_i's are the Higgs fields with a minimum of the potential appearing at

$$
\phi_1 = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}_2 = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}
$$

Two Higgs Doublet Models (2HDM)

When $sin\xi = 0$ there is no CP violation in the Higgs sector, which we will force. The last two terms can be combined when $\lambda_5=\lambda_6$ into

$$\left|\phi_1^{*T}\phi_2 - v_1 v_2 e^{i\xi}\right|^2$$

where the $e^{i\xi}$ term can be rotated away by redefining one of the ϕ fields.

We develop the two doublets around the minimum of the potential. For this we parameterize the fields as

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ v_1 + \eta_1 + i\chi_1 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \eta_2 + i\chi_2 \end{pmatrix}$$

where η_i is the CP-even part and χ_i the CP-odd part. We put these fields in the potential and the mass terms appear.

Two Higgs Doublet Models (2HDM)

This results in the following relevant terms, grouped according to the CP-even, CP-odd and charged Higgs sectors:

$$(\eta_1 \eta_2) \begin{pmatrix} 4(\lambda_1+\lambda_3)v_1^2+\lambda_5 v_2^2 & (4\lambda_3+\lambda_5)v_1 v_2 \\ (4\lambda_3+\lambda_5)v_1 v_2 & 4(\lambda_2+\lambda_3)v_2^2+\lambda_5 v_1^2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

$$\lambda_6\,(\chi_1 \chi_2) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\lambda_4\,(\phi_1^- \phi_2^-) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

where these squared-mass terms can be obtained from

$$\mathcal{M}_{ij}^2 = \frac{1}{2}\frac{\partial^2 V(\phi_1,\phi_2)}{\partial\psi_i \partial\psi_j}$$ **with** $\psi_i \in \{\phi_1^\pm, \phi_2^\pm, \eta_1, \eta_2, \chi_1, \chi_2\}$

Two Higgs Doublet Models (2HDM)

The physical eigenstates of the Higgs fields are obtained from a mixing between these fields $\psi_i \in \{\phi_1^\pm, \phi_2^\pm, \eta_1, \eta_2, \chi_1, \chi_2\}$
With a rotation of the eigenstates the squared-mass matrices can be diagonalized and the masses of the physical eigenstates can be determined. Express the potential in terms of the real fields.

For the CP-odd Higgs (mixing angle β with $\tan\beta = v_2/v_1$)

$$M_A^2 = \lambda_6(v_1^2 + v_2^2) \qquad M_{G^0}^2 = 0$$

For the CP-even Higgs (mixing angle α)

$$M_{H^0,h^0}^2 = \frac{1}{2}\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}$$

For the charged Higgs (mixing angle β)

$$M_{H^\pm}^2 = \lambda_4(v_1^2 + v_2^2) \qquad M_{G^\pm}^2 = 0$$

3 Goldstone bosons to give masses to W and Z bosons

Two Higgs Doublet Models (2HDM)

These relations depend on the values of λ and the mixing angles, hence they can be inverted to write the λ values as a functions of the masses and mixing angles.

These will fully define the potential. Hence this non-CP violating 2HDM Higgs sector has 6 free parameters:

$$M_{H^{\pm}}, M_{H^0}, M_{h^0}, m_{A^0}, tan\beta, \alpha$$

The fermions can couple to these two Higgs doublet field in different ways:

- **Type-I 2HDM**: the field ϕ_2 couples couples to both the up- and down-type fermions
- **Type-II 2HDM**: the field ϕ_1 generates the masses for the down-type fermions, while the field ϕ_2 generates the masses for the up-type quarks (this is the basis of the Higgs sector in the MSSM)

The couplings between the fermions and the neutral Higgs bosons are defined from the mixing angles α and β.

Lecture summary

- Reminder of the mechanism of spontaneous symmetry breaking
- Applied on the EW symmetry of SU(2)xU(1)
- The yet to be observed Higgs sector of the Standard Model depends only on one parameter, the mass of the Higgs boson

- Diverse arguments can be invoked to put theoretically constraints on the Higgs boson mass (m_H < 1 TeV)
- For the theory to be valid up to the Planck scale, the allowed range of the Higgs boson mass is very limited (m_H ~ 130-180 GeV)
- When you do not "believe" that Nature has fine-tuned the parameters of the model, the allowed range is even vanishing or new physics has to appear at scale below Λ ~ 10-100 TeV

- Maybe one Higgs doublet is not enough…
- 2-Higgs Doublet Models are the basis of supersymmetric models
- We have walked through the techniques needed to calculate the mass spectrum of the Higgs sector in a general 2HDM