

# **Electroweak measurements and interpretation**

*(mainly LEP, SLD and Tevatron)*

- key aspects of the Electroweak theory
- important parameters to measure in the theory
- overview of the key experiments and analyses
- combining the measurements (Higgs boson constraints)
- interpretation of the results and outlook for the future

Jorgen D'Hondt  
(Vrije Universiteit Brussel)

*Belgian-Dutch-German Summer School – September, 2007*

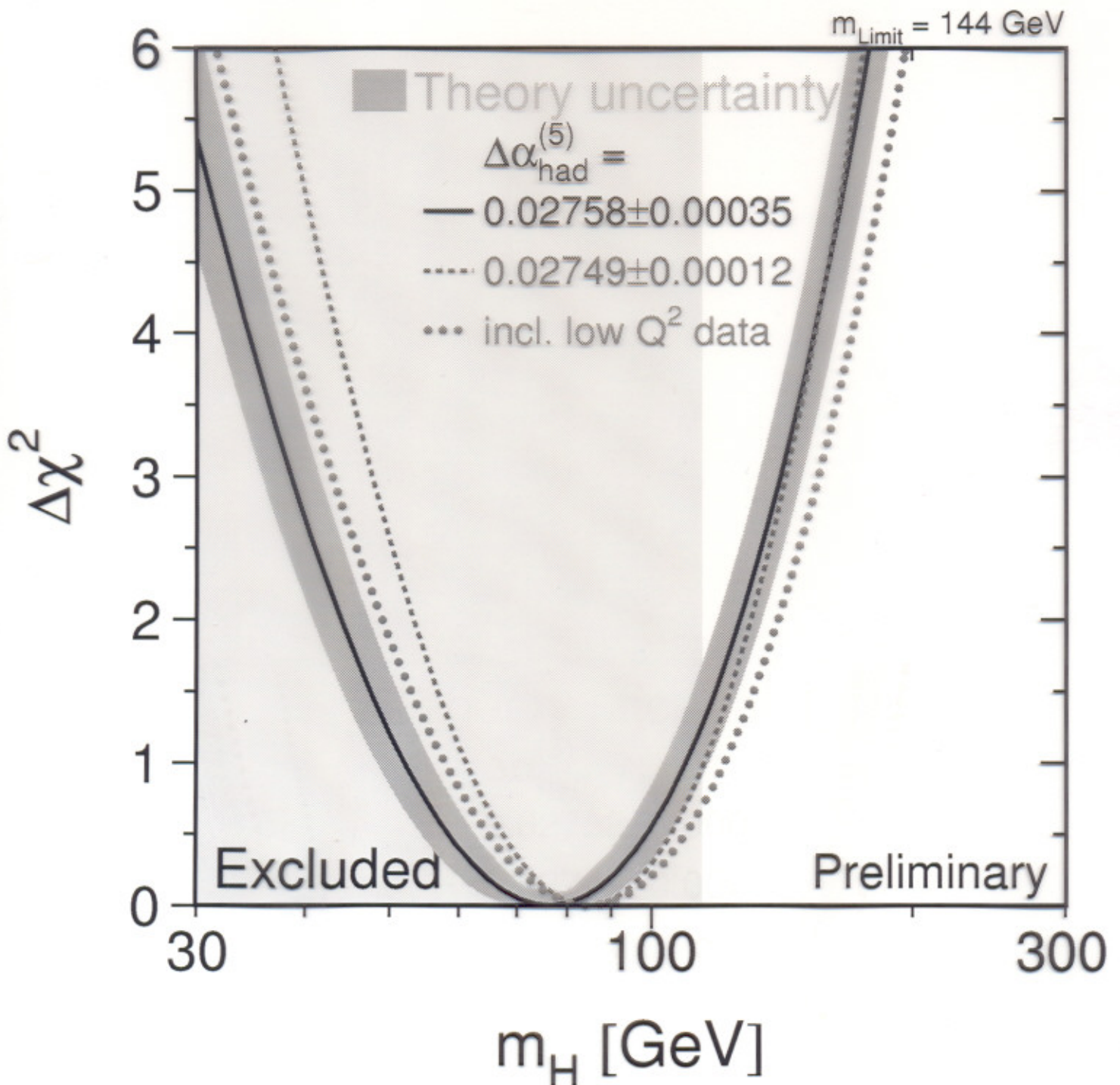


# Who am I ?

- **DELPHI (LEP) on WW physics (as PhD student)**
  - measuring the W boson mass
  - phenomenological effects (soft-QCD)
  - tuning fragmentation parameters (hard versus soft QCD)
- **CMS (LHC) on the building of the Tracker (as post-doc)**
  - constructing and testing the silicon modules of the Tracker
- **CMS (LHC) on top quark physics (as professor)**
  - prepare analyses for top quark measurements at the LHC
  - jet reconstruction and calibration, b-tagging methods
  - currently the convenor of this effort in the CMS experiment
- for the future interest in experiments at the energy frontier (measurements and searches) and the link between cosmology/astro-particle physics and collision physics

# AIM OF THESE LECTURES

WHAT GOES INTO THE HIGGS BLUEBAND PLOT OR TO LEARN SOMETHING ABOUT THE ELECTROWEAK FIT





MAIN REFERENCE  
+ hep-ex/0509008

CERN-PH-EP/2006-042  
LEPEWWG/2006-01  
ALEPH 2006-001 PHYSICS 2006-001  
DELPHI 2006-014 PHYS 948  
L3 Note 2833  
OPAL PR 419  
hep-ex/0612034  
14 December 2006

# A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model

The LEP Collaborations<sup>1</sup> ALEPH, DELPHI, L3, OPAL, and  
the LEP Electroweak Working Group<sup>2</sup>

Prepared from Contributions of the LEP Experiments  
to the 2006 Summer Conferences.

<sup>1</sup>The LEP Collaborations each take responsibility for the preliminary results of their own experiment.

<sup>2</sup>WWW access at <http://www.cern.ch/LEPEWWG>

The members of the LEP Electroweak Working Group who contributed significantly to this note are:

J. Alcaraz, P. Azzurri, A. Bajo-Vaquero, E. Barberio, A. Blondel, D. Bourilkov, P. Checchia, R. Chierici, R. Clare, J. D'Hondt, B. de la Cruz, P. de Jong, G. Della Ricca, M. Dierckxsens, D. Duchesneau, G. Duckeck, M. Elsing, M.W. Grunewald, A. Gurtu, J.B. Hansen, R. Hawkins, St. Jezequel, R.W.L. Jones, T. Kawamoto, E. Lançon, W. Liebig, L. Malgeri, S. Mele, M.N. Minard, K. Mönig, C. Parkes, U. Parzefall, B. Pietrzyk, G. Quast, P. Renton, S. Riemann, K. Sachs, A. Straessner, D. Strom, R. Tenchini, F. Teubert, M.A. Thomson, S. Todorova-Nova, A. Valassi, A. Venturi, H. Voss, C.P. Ward, N.K. Watson, P.S. Wells, St. Wynhoff †.

† deceased.



# ①. THE ELECTROWEAK MODEL

VERY BRIEF REVIEW

FAMILY			<u>T</u> <u>WEAK-ISOSPIN</u>	<u>T<sub>3</sub></u> <u>3<sup>o</sup> COMP.</u>	<u>Q</u> <u>CHARGE</u>
$\Psi_i =$	$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$	$1/2$	$+1/2$	$0$
	$\nu_{eR}$	$\nu_{\mu R}$	$0$	$-1/2$	$-1$
	$e_R$	$\mu_R$	$0$	$0$	$0$
			$0$	$0$	$-1$
$\Psi_i =$	$\begin{pmatrix} u \\ d_L \end{pmatrix}$	$\begin{pmatrix} c \\ s_L \end{pmatrix}$	$1/2$	$+1/2$	$+2/3$
	$u_R$	$c_R$	$0$	$-1/2$	$-1/3$
	$d_R$	$s_R$	$0$	$0$	$+2/3$
			$0$	$0$	$-1/3$

THE WEAK ISOSPIN STRUCTURE OF THE FERMIONS WITH "L" LEFT-HANDED AND "R" RIGHT-HANDED.

↳ TRANSFORM AS DOUBLETS UNDER SU(2)      ↳ AS SINGLETS.

AFTER SPONTANEOUS SYMMETRY BREAKING:

$$\begin{aligned}
 \mathcal{L}_{\text{Fermions}} &= \sum_i \bar{\Psi}_i \left( i\not{\partial} - m_i - \frac{g m_i H}{2M_W} \right) \Psi_i \\
 \text{CHARGED CURRENT} &= -\frac{g}{2\sqrt{2}} \sum_i \bar{\Psi}_i \not{\gamma}^\mu (1-\gamma^5) (T^+ W_\mu^+ + T^- W_\mu^-) \Psi_i \\
 \text{QED} &= -e \sum_i Q_i \bar{\Psi}_i \not{\gamma}^\mu \Psi_i A_\mu \\
 \text{NEUTRAL CURRENT} &= -\frac{g}{2 \cos \theta_W} \sum_i \bar{\Psi}_i \not{\gamma}^\mu (g_V^i - g_A^i \gamma^5) \Psi_i Z_\mu
 \end{aligned}$$

THIS MODEL LEADS TO RELATIONS BETWEEN SOME PARAMETERS (CFR. LECTURES ON EW THEORY)

$$G_F = \frac{\pi \alpha}{\sqrt{2} m_W^2 \sin^2 \theta_W^{\text{tree}}} \quad \text{at tree level}$$

$G_F$ : FERMION CONSTANT FROM MUON DECAY

$\alpha$ : ELECTROMAGNETIC FINE-STRUCTURE CONSTANT

$m_W$ : W BOSON MASS

$\sin^2 \theta_W^{\text{tree}}$ : ELECTROWEAK MIXING ANGLE

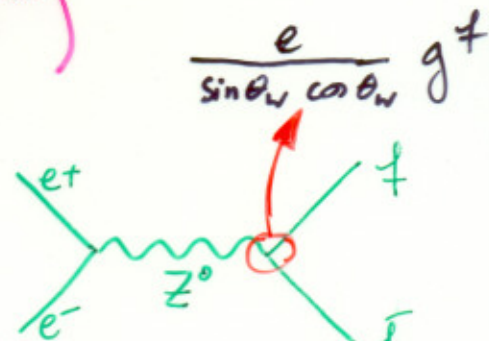
RELATION BETWEEN CHARGE & NEUTRAL CURRENT:

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W^{\text{tree}}} \equiv 1 \quad \text{at tree level}$$

$m_Z$ : Z BOSON MASS

THE INTERACTION OF THE Z BOSON WITH FERMIONS DEPENDS ON CHARGE Q AND  $T_3$ :

$$\begin{cases} g_L^{\text{tree}} = \sqrt{\rho_0} (T_3^f - Q_f \sin^2 \theta_W^{\text{tree}}) \\ g_R^{\text{tree}} = -\sqrt{\rho_0} Q_f \sin^2 \theta_W^{\text{tree}} \end{cases}$$

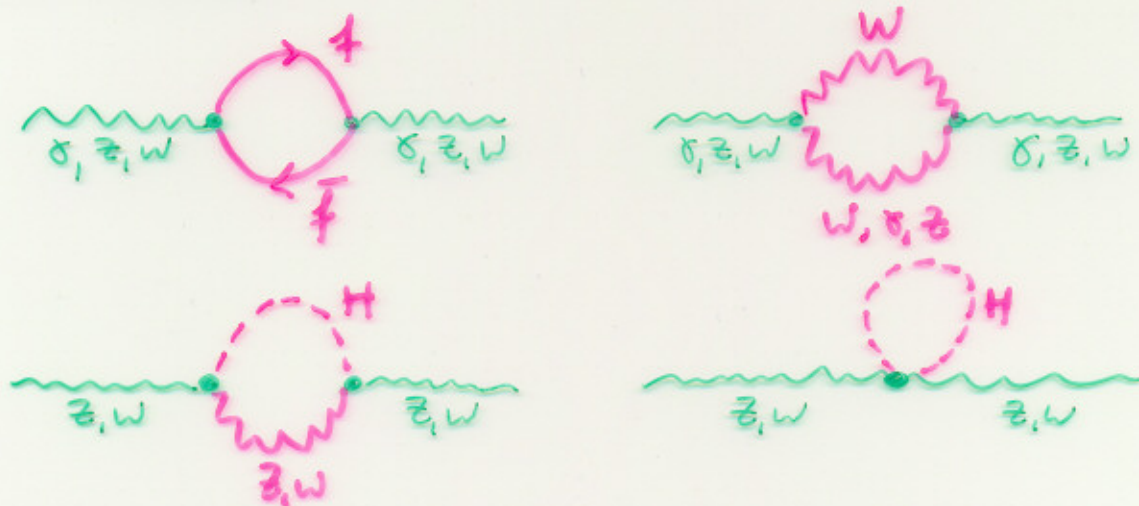


OR VECTOR vs. AXIAL-VECTOR

$$\begin{cases} g_V^{\text{tree}} \equiv g_L^{\text{tree}} + g_R^{\text{tree}} = \sqrt{\rho_0} (T_3^f - 2Q_f \sin^2 \theta_W^{\text{tree}}) \\ g_A^{\text{tree}} \equiv g_L^{\text{tree}} - g_R^{\text{tree}} = \sqrt{\rho_0} T_3^f \end{cases}$$



THESE ARE TREE LEVEL QUANTITIES TO BE MODIFIED BY RADIATIVE CORRECTIONS TO PROPAGATORS AND VERTICES



THE VALUE OF  $\sin^2 \theta_w$  DEPENDS ON THE RENORMALIZATION PROCEDURE TO RENORMALIZE THESE CORRECTIONS  $\rightarrow$  "ON-SHELL" SCHEME

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2 \theta_w} \equiv 1$$

TO ALL ORDERS IN PERTURBATION THEORY

AT TREE LEVEL THE EW THEORY DETERMINED BY THREE "INPUT" PARAMETERS

$$\alpha, G_F, m_Z \quad (\text{with QCD also } \alpha_s)$$

WITH THE ABOVE LOOPS THE EW OBSERVABLES DEPEND ON

$$\alpha, G_F, m_Z, m_{\text{top}}, m_{\text{Higgs}}$$

THE EW CORRECTIONS TO THE COUPLINGS ARE ABSORBED IN COMPLEX FORM FACTORS (AT THE 2-POLE)

$$\Rightarrow \text{effective couplings } \& \sin^2 \theta_{\text{eff}}^f$$

ABSORBING EW CORRECTIONS TO COUPLINGS:

$$\begin{cases} g_{Vf} = \sqrt{R_f} (T_3^f - 2 Q_f \cancel{K_f} \sin^2 \theta_w) \\ g_{Af} = \sqrt{R_f} T_3^f \end{cases}$$

$R$  and  $\cancel{K}_f$  are complex form factors

$R_f$ : overall scale

$\cancel{K}_f$ : for the on-shell EW mixing angle

IN TERMS OF THE REAL PARTS OF THESE FORM FACTORS THE EFFECTIVE EW MIXING ANGLE AND THE REAL EFFECTIVE COUPLINGS ARE DEFINED AS:

$$\begin{cases} \sin^2 \theta_{\text{eff}}^f \equiv \cancel{K}_f \sin^2 \theta_w \\ g_{Vf} \equiv \sqrt{P_f} (T_3^f - 2 Q_f \sin^2 \theta_{\text{eff}}^f) \\ g_{Af} \equiv \sqrt{P_f} T_3^f \end{cases}$$

WITH

$$P_f \equiv R(R_f)$$

$$\cancel{K}_f \equiv R(\cancel{K}_f)$$

HENCE THE RATIO BECOMES

$$\frac{g_{Vf}}{g_{Af}} = R\left(\frac{g_{Vf}}{g_{Af}}\right) = 1 - 4 |Q_f| \sin^2 \theta_{\text{eff}}^f$$



THE CORRECTIONS ARE IN THE DEFINITION OF  $\rho_f$  AND  $K_f$ :

$$\left\{ \begin{array}{l} \rho_f = 1 + \Delta \rho_{se} + \Delta \rho_f \\ K_f = 1 + \Delta K_{se} + \Delta K_f \end{array} \right.$$

↳ flavour specific vertex corrections  
 ↳ universal corrections from the propagator self-energies

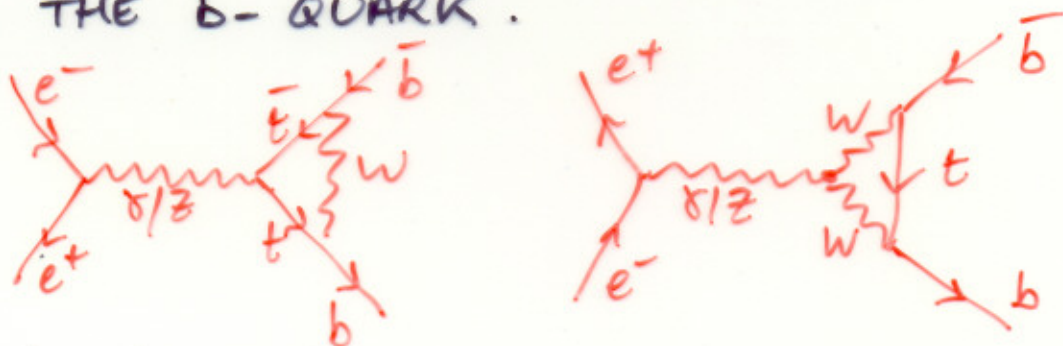
IN LEADING ORDER: for  $m_H \gg m_W$

$$\Delta \rho_{se} = \frac{3 G_F m_W^2}{8 \sqrt{2} \pi^2} \left[ \frac{m_t^2}{m_W^2} - \frac{\sin^2 \theta_W}{\cos^2 \theta_W} \left( \ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right) + \dots \right]$$

$$\Delta K_{se} = \frac{3 G_F m_W^2}{8 \sqrt{2} \pi^2} \left[ \frac{m_t^2}{m_W^2} \frac{\cos^2 \theta_W}{\sin^2 \theta_W} - \frac{10}{9} \left( \ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right) + \dots \right]$$

THE RADIATIVE CORRECTIONS HAVE A QUADRATIC DEPENDENCE ON  $m_t$  AND A WEAKER LOG DEPENDENCE ON  $m_H$ .

THE FLAVOUR DEPENDENCE IS VERY SMALL EXCEPT FOR THE b-QUARK.



AS  $|V_{tb}| \approx 1$  THE TOP QUARK HAS A SIGNIFICANT CONTRIBUTION

$$\Delta K_b = \frac{G_F m_t^2}{4 \sqrt{2} \pi^2} + \dots$$

$$\Delta \rho_b = -2 \Delta K_b + \dots$$

HENCE THE PARAMETER  $\rho$  IS MODIFIED BY LOOPS:

$$\rho = 1 + \Delta\rho$$

FROM THE PREVIOUS RELATIONS WE GET

$$\cos^2 \theta_w \sin^2 \theta_w = \frac{\pi \alpha(0)}{\sqrt{2} m_Z^2 G_F} \cdot \frac{1}{1 - \Delta R} \quad \text{low mom. transfer}$$

$$\cos^2 \theta_{\text{eff}}^+ \sin^2 \theta_{\text{eff}}^+ = \frac{\pi \alpha(0)}{\sqrt{2} m_Z^2 G_F} \cdot \frac{1}{1 - \Delta R^+} \quad \text{at the } z\text{-pole}$$

WITH

$$\begin{cases} \Delta R = \Delta\alpha + \Delta R_w \\ \Delta R^+ = \Delta\alpha + \Delta R_w^+ \end{cases}$$

THE  $\Delta\alpha$  TERM ARISES FROM THE RUNNING OF THE ELECTROMAGNETIC COUPLING DUE TO FERMION LOOPS IN THE PHOTON PROPAGATOR, USUALLY DIVIDED AS:

$$\Delta\alpha(s) = \underbrace{\Delta\alpha_{\text{em}}(s) + \Delta\alpha_{\text{top}}(s)}_{\text{precise calculations}} + \Delta\alpha_{\text{had}}^{(5)}(s)$$

FROM THE ANALYSIS OF LOW ENERGY  $e^+e^-$  DATA USING A DISPERSION RELATION

THE EFFECTS ARE ABSORBED AS

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

$$\alpha(0) = 1/137.036 \quad \rightarrow \quad \alpha(m_Z) = 1/128.945$$



IN THE WEAK PART  $\Delta R_W$  THE VALUE OF  $\Delta\rho$ .  
USUALLY  $g_F$  AND  $m_Z$  ARE BETTER MEASURED  
COMPARED TO  $m_W$ , HENCE ONE USUALLY CALCULATES  
 $m_W$  VIA  $g_F$  AND  $m_Z$ :

$$m_W^2 = \frac{m_Z^2}{2} \left( 1 + 1 - 4 \frac{\pi\alpha}{\sqrt{2} g_F m_Z^2} \cdot \frac{1}{1-\Delta R} \right) + \dots$$

$\Rightarrow$  THIS GIVES YOU A DEPENDENCY BETWEEN  
 $m_W, m_t, m_H, \dots$  AND PREVIOUS EQUATIONS  
BETWEEN  $\sin^2\theta_W, m_t, m_H, \dots$

ALL THE ABOVE IS IN LEADING ORDER TO  
ILLUSTRATE THE MAIN ELECTROWEAK RELATIONS  
THE FULL CALCULATIONS AND SO-CALLED EW-FIT  
ARE PERFORMED TO HIGHER ORDER  
(programs as TOPAZO and ZFITTER)

### ALL TOGETHER:

- MEASURING COUPLINGS  $\rightarrow$  SENSITIVE TO  $\sin^2\theta_W$
  - MEASURE  $m_W$
- $\Rightarrow$  THESE PARAMETERS ARE AT THE HEART OF THE ELECTROWEAK THEORY  
& SENSITIVE TO  $m_t$  &  $m_H$  VIA LOOP CORRECTIONS

THE ABOVE EQUATIONS PROVIDE THE BASIS  
FOR THE INTERPRETATIONS

... AT THE END OF LECTURE (2).

HENCE APART FROM OTHER FERMION MASSES AND MIXINGS, THE STANDARD MODEL HAS 3 FREE PARAMETERS:  $\alpha$ ,  $G_F$ ,  $m_Z$  (+  $m_H$ ,  $m_t$  VIA LOOPS)

(i) • MEASURE  $g$  COUPLINGS PRECISELY  $\Rightarrow \sin^2 \theta_W$

(ii) • MEASURE THE "FREE" PARAMETERS PRECISELY (PREDICT eg.  $\sin^2 \theta_W$  &  $m_W$ )

↳ POSSIBLE SINCE THE PRECISE  $m_Z$  MEASUREMENT

• TEST CONSISTENCY BETWEEN eg.  $\sin^2 \theta_W$  BETWEEN

(i) & (ii)

• CONSTRAIN UNMEASURED QUANTITY  $m_H$

• FIND THE HIGGS BOSON ...

$\alpha = 1 / 137.03599911 (46)$  (eg. magnetic moment  $e^\pm$ )  
(defined at very low energy scales)



run up to higher scales  $\alpha = \alpha(M_Z)$

$G_F = 1.16637 (1) \times 10^{-5} \text{ GeV}^{-2}$  (muon life time)

$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$  (lineshape)

⇒ GET EXPERIMENTS WHICH CAN MEASURE OBSERVABLES SENSITIVE TO  $\sin^2 \theta_W$  &  $m_W$  WHICH IS ON HIS TURN SENSITIVE TO  $m_H$  AND  $m_t$

⇒ LECTURE ① :  $\sin^2 \theta_W$  ⇒ LEP I & SLC

LECTURE ② :  $m_W$  &  $m_t$  ⇒ LEP II & Tevatron



## ②. THE EXPERIMENTS FOR $\sin^2 \theta_w$

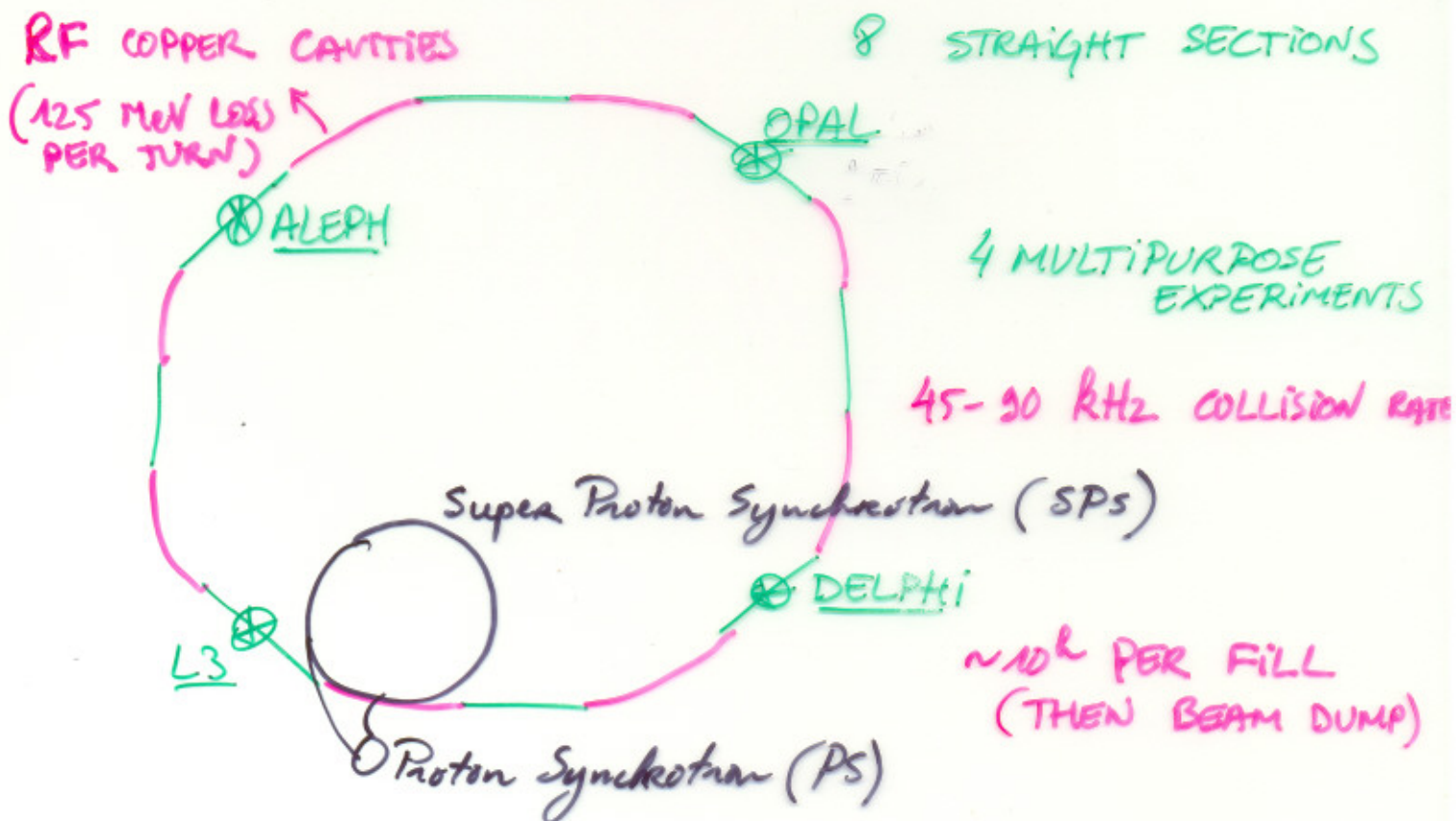
THE PROCESS UNDER STUDY IS  $e^+e^- \rightarrow f\bar{f}$



TWO ACCELERATORS DESIGNED DURING THE 1980s TO ESTIMATE THE Z-POLE PARAMETERS WITH HIGH PRECISION: **LEP** & **SLC**

### I. LEP: LARGE ELECTRON-POSITRON COLLIDER (1989-2000)

THE LARGEST PARTICLE ACCELERATOR IN THE WORLD WITH 27 km CIRCUMFERENCE.

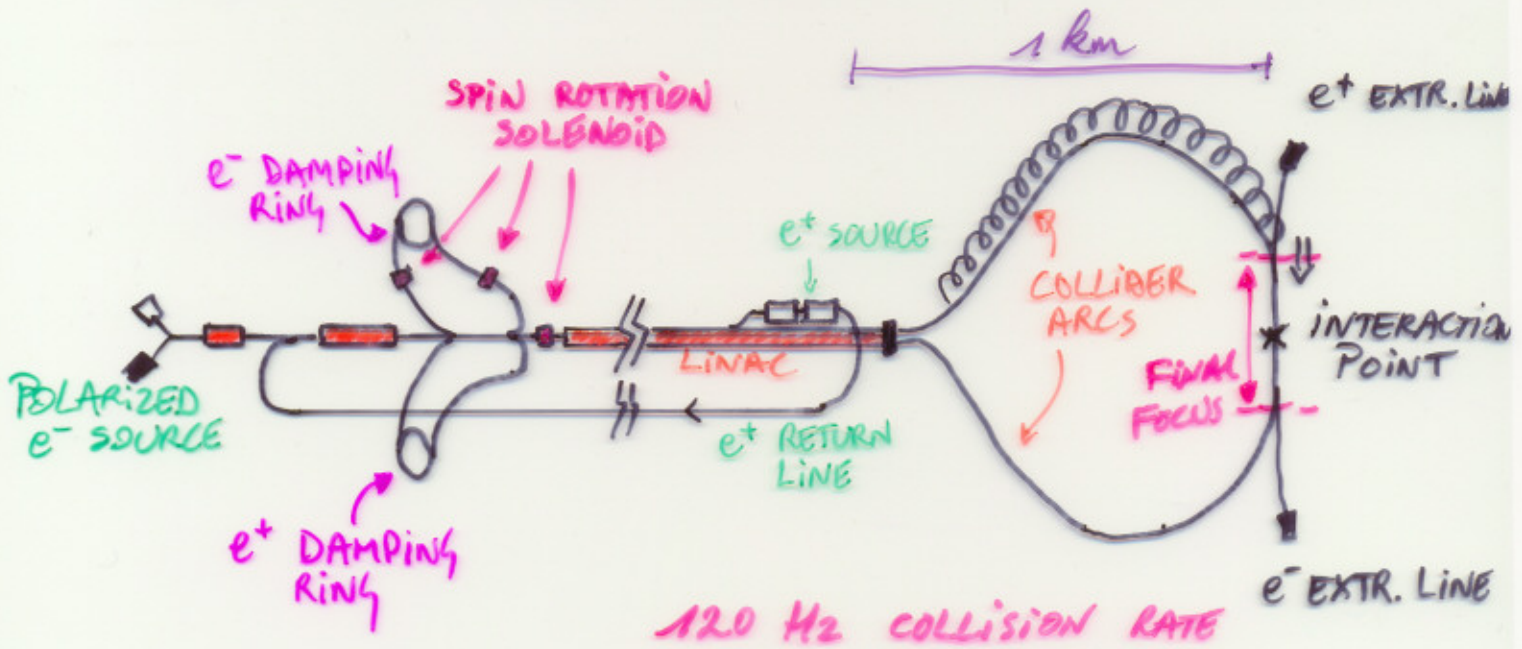


AT PEAK LUMINOSITY ( $2 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ ) EACH EXPERIMENT COLLECTED ABOUT 1000 Z BOSONS EVERY HOUR.



## II. STANFORD LINEAR COLLIDER @ SLAC

THE FIRST  $e^+e^-$  LINEAR COLLIDER OF 3.2 km LENGTH



DAMPING RINGS REDUCE THE SIZE & ENERGY SPREAD OF THE ELECTRON & POSITRON BUNCHES

- STARTS WITH 2 CLOSELY SPACED ELECTRON BUNCHES (1 LONGITUDINALLY POLARISED)
- STORED IN DAMPING RINGS AT 1.2 GeV
- BACK IN LINAC AT 30 GeV TO A TARGET FOR POSITRON CREATION (THEY GO BACK TO START AT 200 MeV AND GO TO DAMPING RING)
- SYNCHRONIZATION WITH SECOND 2-ELECTRON BUNCH START
- FINAL ENERGY OF 46.5 GeV (1 GeV LOST IN ARCS)
- ELECTRONS MANIPULATED TO GET LONGITUDINAL POLARISATION AT INTERACTION POINT

MAIN ADVANTAGE COMPARED TO LEP IS THE POLARIZATION (LOTS OF WORK TO KEEP POLARIZATION FROM SOURCE TO COLLISION)



CRUCIAL : MEASURE THE POLARIZATION  
 (PRECISION OF 0.5% ACHIEVED WHILE DESIGNED WAS 1%)  
 HEAD-ON COMPTON SCATTERING OF A POLARIZED  
 HIGH-POWER LASER BEAM WITH THE ELECTRON BEAM

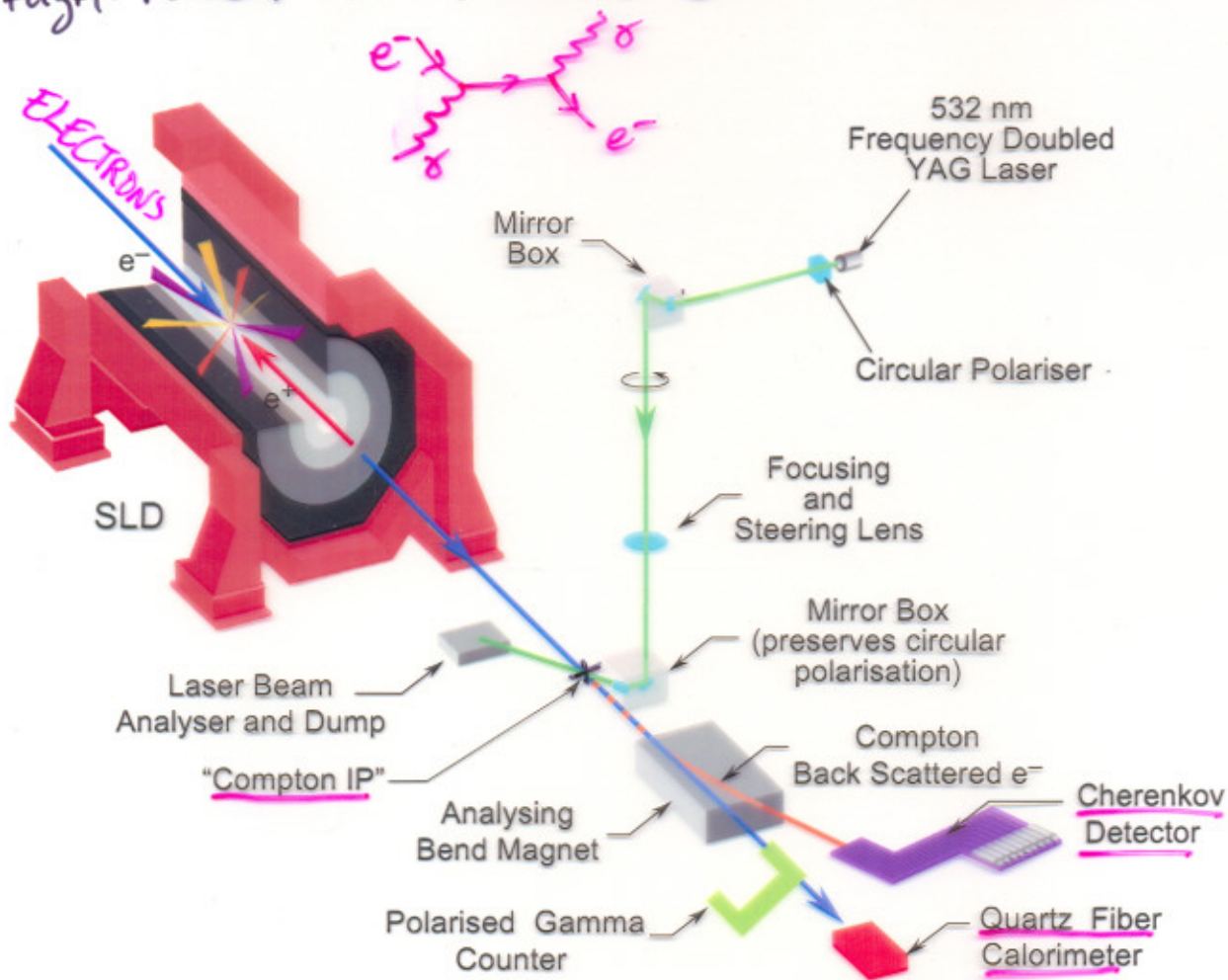


Figure 3.1: A conceptual diagram of the SLD Compton Polarimeter. The laser beam, consisting of 532 nm wavelength 8 ns pulses produced at 17 Hz and a peak power of typically 25 MW, were circularly polarised and transported into collision with the electron beam at a crossing angle of 10 mrad approximately 30 meters from the IP. Following the laser/electron-beam collision, the electrons and Compton-scattered photons, which are strongly boosted along the electron beam direction, continue downstream until analysing bend magnets deflect the Compton-scattered electrons into a transversely-segmented Cherenkov detector. The photons continue undeflected and are detected by a gamma counter (PGC) and a calorimeter (QFC) which are used to cross-check the polarimeter calibration.

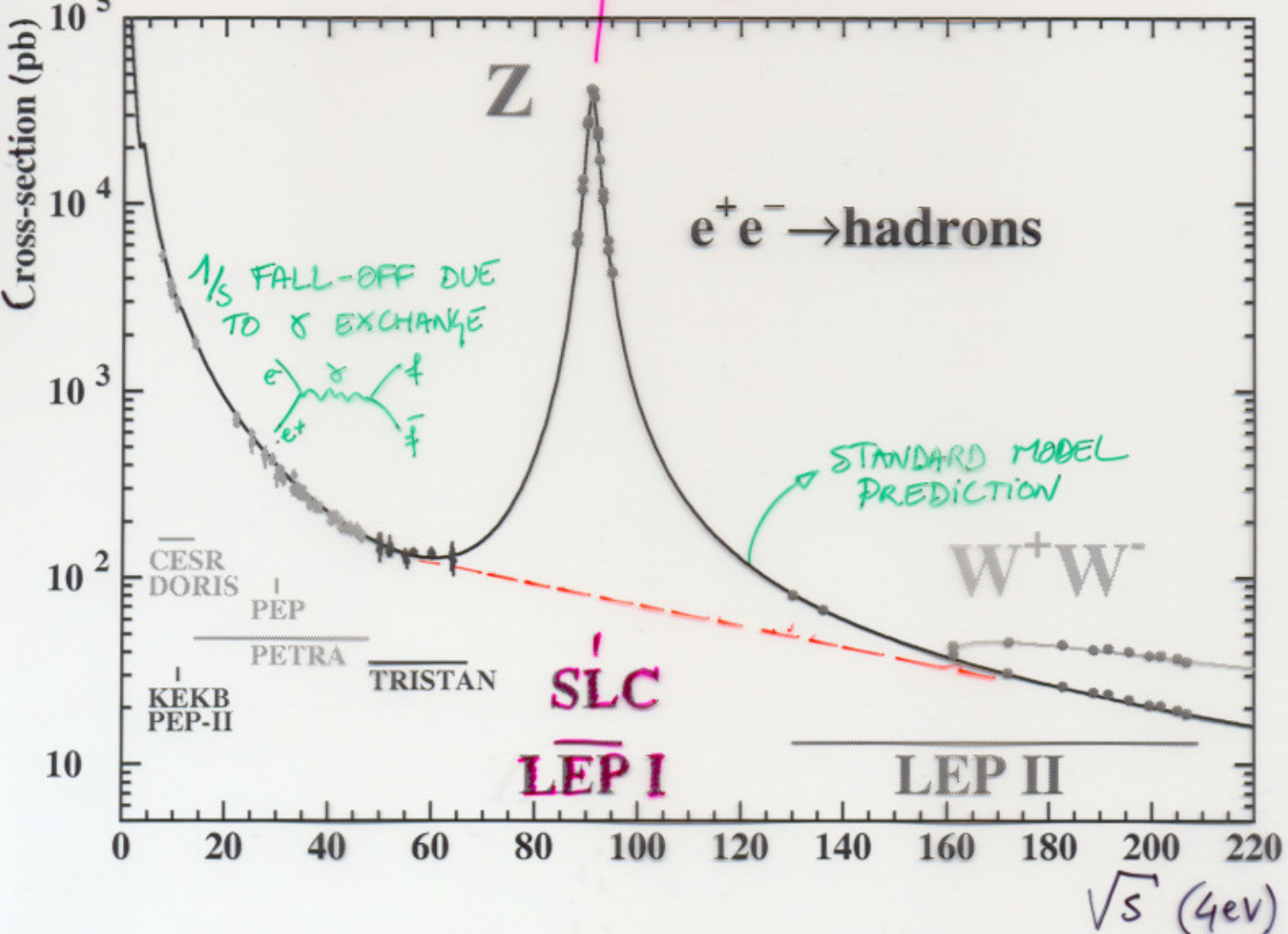
# HADRONIC CROSS SECTION VERSUS $\sqrt{s}$

} LEP : 1989  $\rightarrow$  2000  $\sqrt{s} = m_z \pm 3 \text{ GeV} \rightarrow 17 \text{ M } z^0$   
 $\rightarrow 0.6 \text{ M } z^0$   
 } SLC : 1989  $\rightarrow$  1998  
 (with longitudinal polarization)

OPTIMAL ENERGY REGION TO STUDY THE  $e^+e^- \rightarrow f\bar{f}$  PROCESSES

VERY HIGH CROSS SECTION  $\Rightarrow$  LOSS OF  $z$  BOSONS

LOG-SCALE!





# EXAMPLE OF THE ACHIEVEMENTS OF LEP & SLC

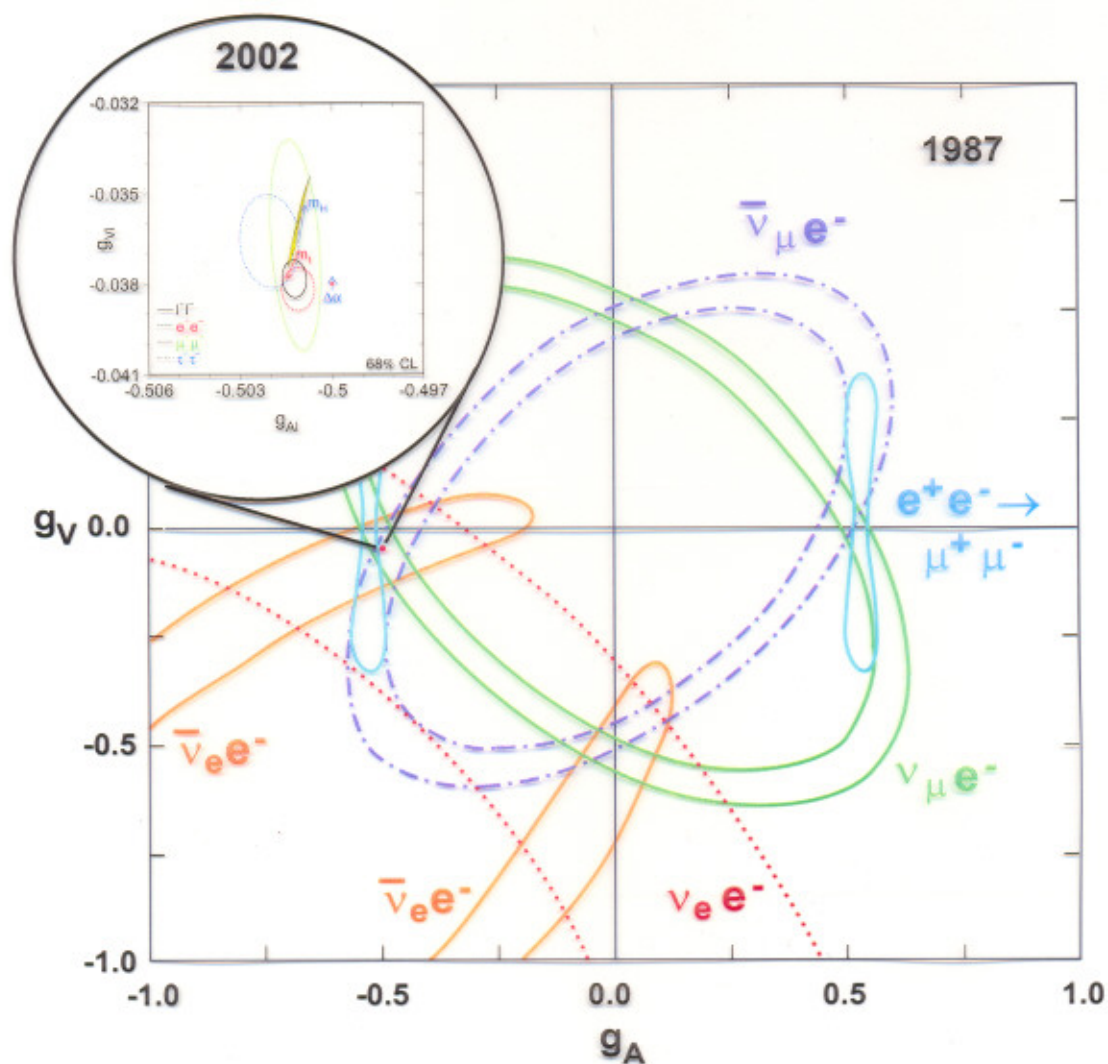


Figure 1.15: The neutrino scattering and  $e^+e^-$  annihilation data available in 1987 constrained the values of  $g_{V\ell}$  and  $g_{A\ell}$  to lie within broad bands, whose intersections helped establish the validity of the SM and were consistent with the hypothesis of lepton universality. The inset shows the results of the LEP/SLD measurements at a scale expanded by a factor of 65 (see Figure 7.3). The flavour-specific measurements demonstrate the universal nature of the lepton couplings unambiguously on a scale of approximately 0.001.

### ③. BASIC MEASUREMENTS

THE DETAILED DETECTORS AROUND THE COLLISION POINTS ARE ABLE TO MEASURE PRECISELY THE  $e^+e^- \rightarrow \gamma/Z \rightarrow f\bar{f}$  PROCESS. ALSO THE FLAVOURS OF LEPTONS AND SOME QUARKS CAN BE DISTINGUISHED.

(NOT THE MAIN TOPIC OF THIS LECTURE)

- TOTAL CROSS-SECTIONS

$$\sigma = \frac{N_{sel} - N_{back}}{E_{sel} \cdot L}$$

$N_{back}$  &  $E_{sel}$   
FROM SIMULATION

- CROSS-SECTION VERSUS  $\sqrt{s}$

NEEDED FOR  $Z$  BOSON MASS AND WIDTH

- RATIO OF CROSS-SECTIONS OF DIFFERENT DECAYS

FOR PARTIAL WIDTHS & RELATIVE STRENGTH OF  $Z$  COUPLINGS

- ASYMMETRIES OF ANGULAR DISTRIBUTIONS

MIXTURE OF VECTOR & AXIAL-VECTOR COUPLINGS

→ HERE THE POLARIZATION OF THE COLLIDING  $e^-$  AND  $e^+$  CAN HELP

$$\textcircled{\otimes} A_{FB} = \frac{N_F - N_B}{N_F + N_B} \quad \text{forward-backward asymmetry}$$

(NEEDS 4 $\pi$  ACCEPTANCE, HENCE  $A_{FB}$  USUALLY FROM FITS ON ANGULAR DISTRIBUTIONS)

$$\textcircled{\otimes} A_{LR} = \frac{N_L - N_R}{N_L + N_R} \cdot \frac{1}{\langle P_e \rangle} \quad \text{left-right asymmetry}$$

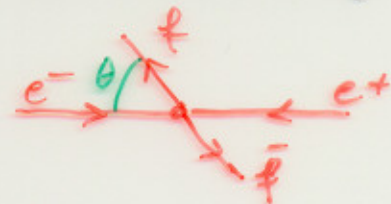
DOES NOT NEED ACCEPTANCE

( $N_L$ : #  $Z$  FOR LH  $e^-$  BUNCHES) ( $\langle P_e \rangle$ : MAGN. OF POLARISATION)



# ④. THE PROCESS $e^+e^- \rightarrow f\bar{f}$

DIFFERENTIAL CROSS-SECTION AROUND Z-POLE  
USING THE COMPLEX-VALUED EFFECTIVE COUPLING  
CONSTANTS



$$\frac{2s}{\pi} \frac{1}{N_c^f} \frac{d\sigma_{EW}}{d\cos\theta} (e^+e^- \rightarrow f\bar{f}) =$$

$$\sigma^0 \quad |\alpha(s) Q_f|^2 (1 + \cos^2\theta)$$

$$\sigma^{\delta-Z \text{ interference}} \quad - 8 \operatorname{Re} \left\{ \alpha^*(s) Q_f \chi(s) \left[ g_{Ve} g_{Vf} (1 + \cos^2\theta) + 2 g_{Ae} g_{Af} \cos\theta \right] \right\}$$

$$\sigma^Z \quad + 16 |\chi(s)|^2 \left[ (|g_{Ve}|^2 + |g_{Ae}|^2) \cdot (|g_{Vf}|^2 + |g_{Af}|^2) \cdot (1 + \cos^2\theta) \right. \\ \left. + 8 \operatorname{Re} \left\{ g_{Ve} g_{Ae}^* \right\} \operatorname{Re} \left\{ g_{Vf} g_{Af}^* \right\} \cos\theta \right]$$

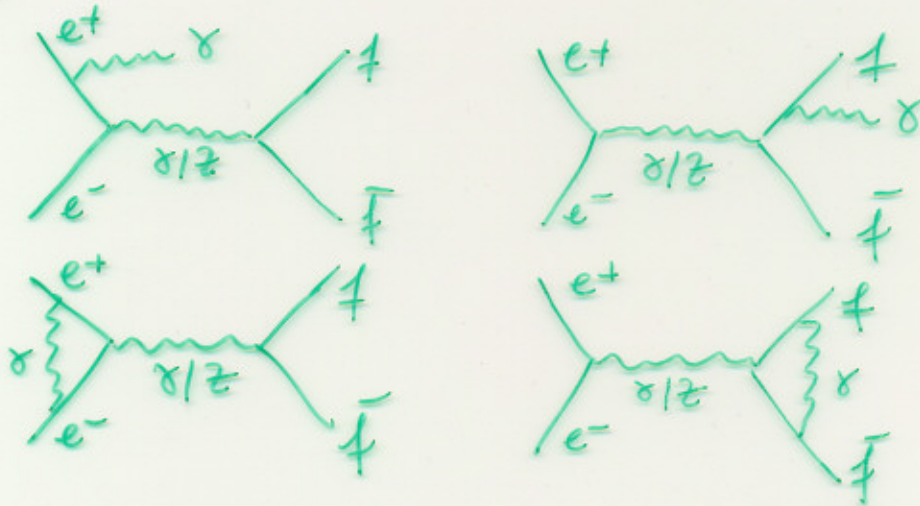
WITH  $\chi(s) = \frac{g_F m_Z^2}{8\pi\sqrt{2}} \cdot \frac{s}{s - m_Z^2 + i s \frac{\Gamma_Z}{m_Z}}$  propagator term

$N_c^f$ : one for leptons & three for quarks

RATHER MODEL-INDEPENDENT IF COUPLINGS ARE FREE  
ONLY ASSUMPTION IS THAT THE Z PROCESSES VECTOR  
& AXIAL-VECTOR COUPLINGS TO FERMIONS, HAS  
SPIN 1 AND INTERFERES WITH THE PHOTON.

[FOR ELECTRONS  $e^+e^- \rightarrow e^+e^-$  ALSO THE BHASKHA TERM]

PHOTON RADIATION FROM INITIAL & FINAL STATES LIKE



AND THEIR INTERFERENCE ARE TREATED BY CONVOLUTING THE EW KERNEL CROSS-SECTION  $\sigma_{ew}(s)$  WITH A QED RADIATOR

$$\sigma(s) = \int_0^1 \frac{4\pi\alpha^2}{s} dz H_{QED}^{tot}(z, s) \sigma_{ew}(zs)$$

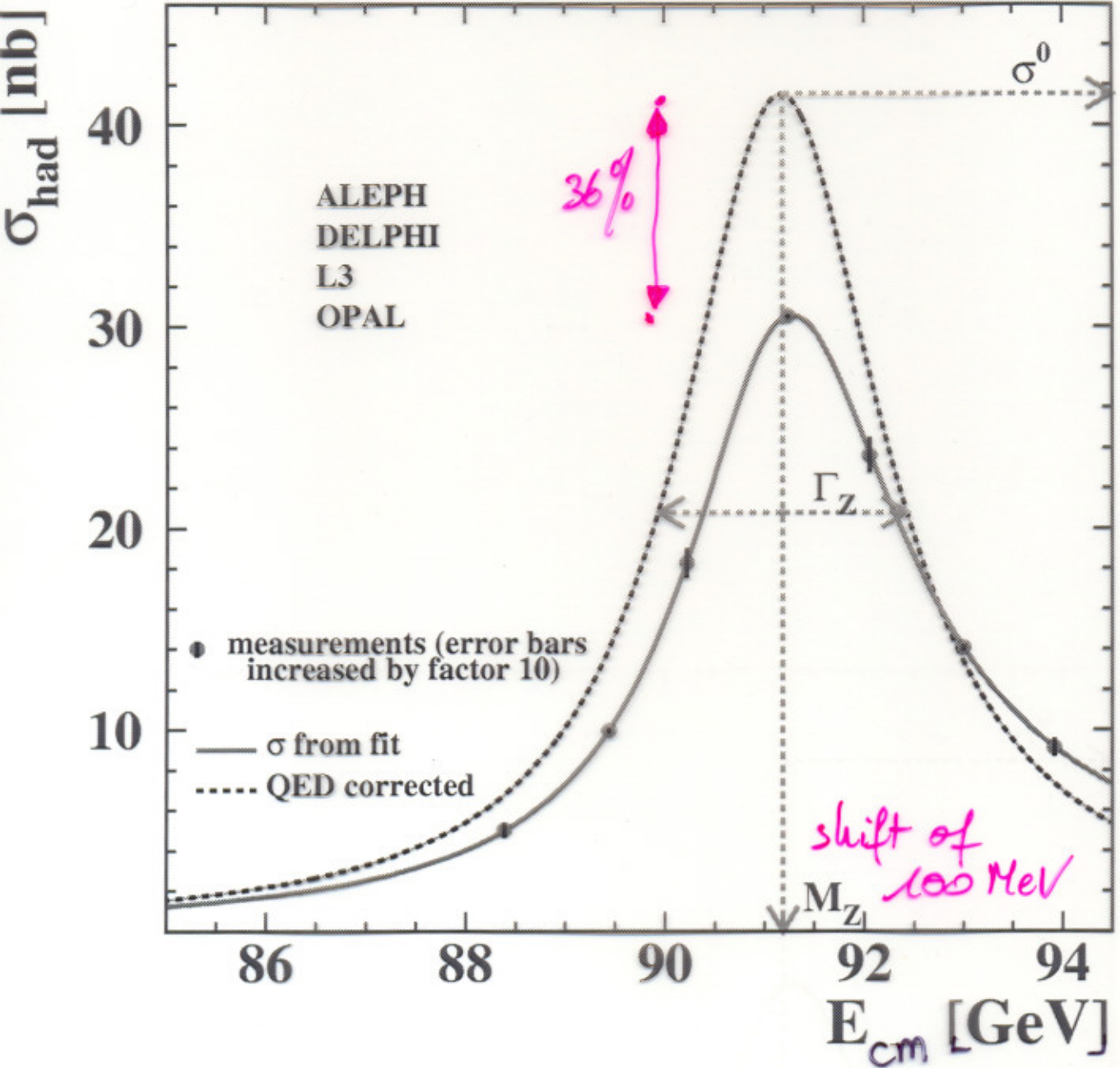
THE SAME PROCEDURE IS USED FOR THE FORWARD-BACKWARD ASYMMETRIES  $\sigma_F - \sigma_B$  WITH  $H_{QED}^{FB}$ .  
[calculated to 3<sup>rd</sup> ORDER]

THESE CORRECTIONS ARE IMPORTANT AND ESSENTIALLY INDEPENDENT OF THE EW CORRECTIONS DISCUSSED PREVIOUSLY.

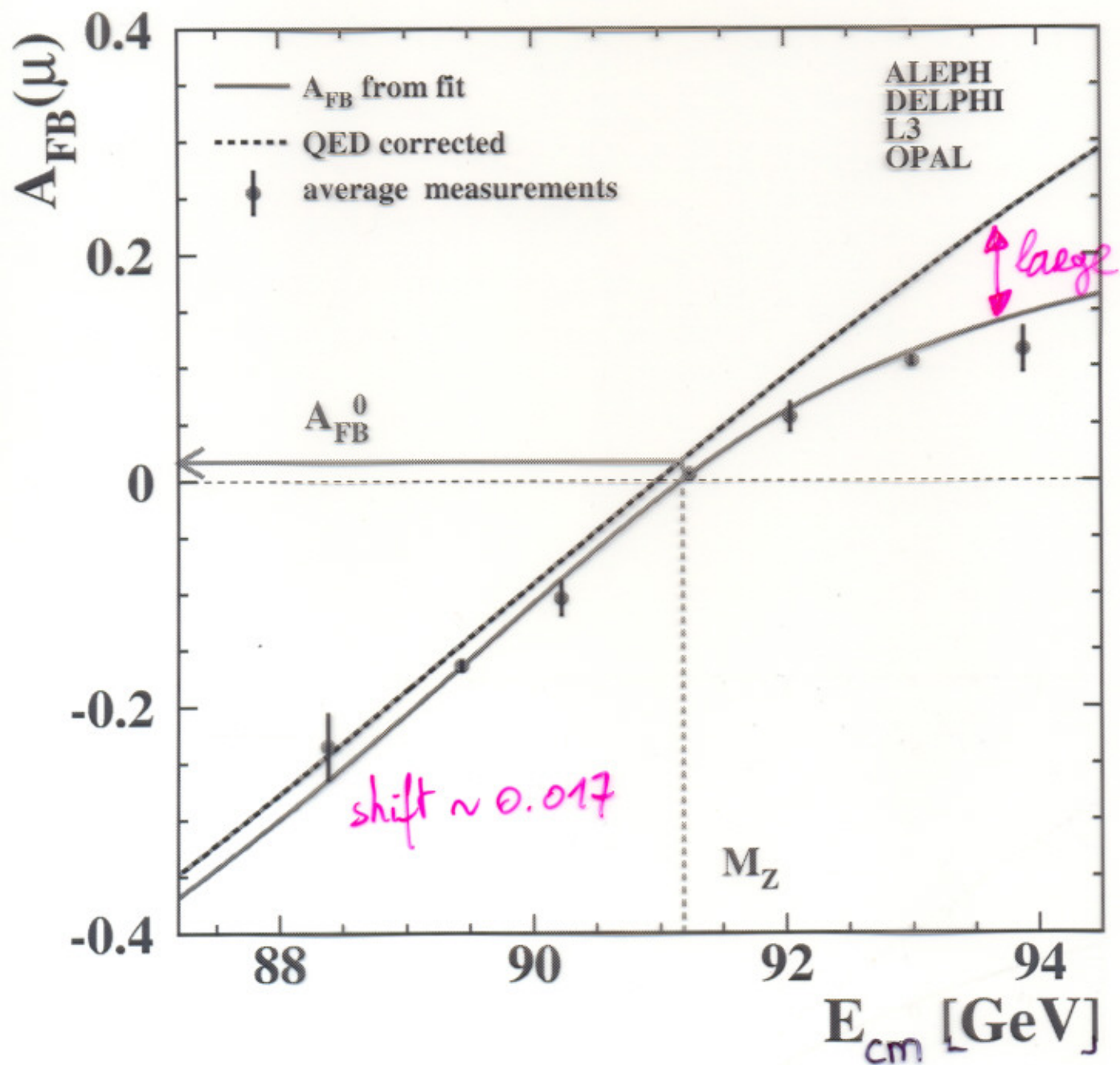
⇒ HENCE THE PARAMETERS IN EQUATION  $\frac{d\sigma_{ew}}{d\cos\theta}$  CAN BE EXTRACTED FROM DATA IN A MODEL-INDEPENDENT WAY



# EFFECT OF QED RADIATIVE CORRECTIONS ON THE LINESHAPE OF THE Z (hadronic cross section)



EFFECT OF QED RADIATIVE CORRECTIONS  
ON THE FORWARD-BACKWARD ASYMMETRIES.  
( $e^+e^- \rightarrow \mu^+\mu^-$ )





# \* CROSS SECTIONS & PARTIAL WIDTHS

THE CROSS SECTION FROM THE  $\cos\theta$ -SYMMETRIC  $Z$  PRODUCTION TERM CAN ALSO BE WRITTEN AS:

$$\sigma_{ff}^Z = \sigma_{ff}^{\text{peak}} \frac{s \Gamma_Z^2}{(s - m_Z^2)^2 + s^2 \frac{\Gamma_Z^2}{m_Z^2}}$$

removes QED corr. WITH

$$\sigma_{ff}^{\text{peak}} = \frac{1}{R_{\text{QED}}} \sigma_{ff}^0$$

$$\sigma_{ff}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{ff}}{\Gamma_Z^2}$$

WHERE YOU HAVE THE PARTIAL DECAY WIDTHS OF THE INITIAL ( $\Gamma_{ee}$ ) AND FINAL ( $\Gamma_{ff}$ ) STATES.

THE OVERALL HADRONIC WIDTH IS GIVEN AS

$$\Gamma_{\text{had}} = \sum_{q \neq t} \Gamma_{qq}$$

HENCE THE TOTAL WIDTH CAN BE WRITTEN AS

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv.}}$$

$$\Gamma_{\text{inv.}} = N_U \Gamma_{\nu\nu}$$

AS WE MEASURE CROSS-SECTIONS WHICH DEPEND ON SEVERAL PARTIAL WIDTHS, THESE MEASUREMENTS ARE CORRELATED. THE USE OF A SET OF 6 PARAMETERS (MOTIVATED EXPERIMENTALLY):

- $m_Z$

- $\Gamma_Z$

- $\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{\text{had}}}{\Gamma_Z^2}$  hadronic pole con-n.

- $R_{ee}^0 = \Gamma_{\text{had}} / \Gamma_{ee}$

- $R_{\mu\mu}^0 = \Gamma_{\text{had}} / \Gamma_{\mu\mu}$

- $R_{\tau\tau}^0 = \Gamma_{\text{had}} / \Gamma_{\tau\tau}$

} if universality is assumed this becomes 1 parameter

TRADITIONALLY THE BRANCHING RATIOS TO HEAVY QUARKS ARE TREATED INDEPENDENTLY

$$R_b^0 = \frac{\Gamma_{bb}}{\Gamma_{had}}$$
$$R_c^0 = \frac{\Gamma_{cc}}{\Gamma_{had}}$$

THIS IS POSSIBLE WITH THE PRECISE TRACKING DETECTORS IN THE ZEP & SLC DETECTORS.

SLD slightly better in heavy quark identification.

## \* INVISIBLE WIDTH & # NEUTRINOS

ASSUMING LEPTON UNIVERSALITY AND WE OBTAIN

$$R_{inv}^0 = \Gamma_{inv} / \Gamma_{\ell\ell}$$

$$R_{inv}^0 = \sqrt{\frac{12\pi R_e^0}{\sigma_{had}^0 m_z^2}} - R_e^0 - (3 + \delta_z)$$

↓ effect of  $z$  mass  
 $\delta_z \approx -0.23\%$

HENCE ASSUMING ONLY INVISIBLE DECAYS TO NEUTRINOS AND THE SM PREDICTION FOR  $\Gamma_{\nu\bar{\nu}} / \Gamma_{\ell\ell}$  WE CAN ESTIMATE THE NUMBER OF NEUTRINOS

$$R_{inv}^0 = N_\nu \left( \frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\ell\ell}} \right)_{SM}$$

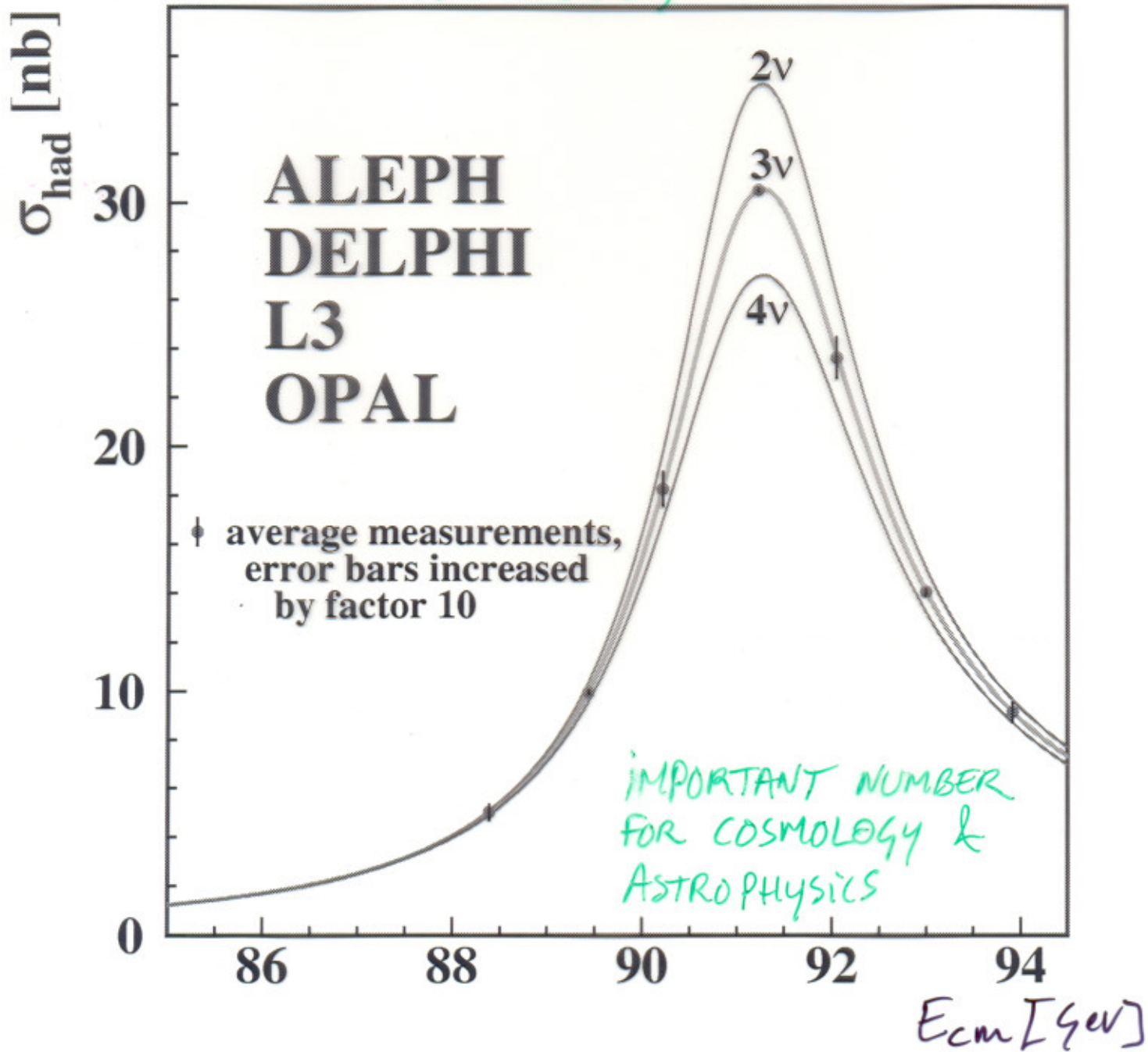
DEPENDS ON HADRONIC CROSS SECTION ( $\rightarrow$  cfr. plot)



# DEPENDENCY OF HADRONIC CROSS SECTION ON THE NUMBER OF NEUTRINO SPECIES.

$$N_\nu = 2.9840 \pm 0.0082$$

(25 below 3)



# \* ASYMMETRIES & POLARISATION

ADDITIONAL OBSERVABLES ARE INTRODUCED TO DESCRIBE THE  $\cos\theta$  DEPENDENCY in  $d\sigma_{eff}/d\cos\theta$ . THEY QUANTIFY THE AMOUNT OF PARITY VIOLATION OF THE NEUTRAL CURRENT, HENCE THE VECTOR & AXIAL-VECTOR COUPLINGS TO THE Z BOSON.

$\Rightarrow$  MEASURE OF  $\sin^2\theta_{eff}^+$

- (i). EVEN IF THE INITIAL ELECTRONS & POSITRONS ARE NOT POLARISED, THE Z BOSON CAN HAVE A LONGITUDINAL POLARIZATION IN ITS DECAY. THIS BECAUSE THE LEFT-RIGHT-HANDED COUPLING TO FERMIONS ARE UNEQUAL. HENCE THE ANGULAR DISTRIBUTION WILL BE FORWARD-BACKWARD ASYMMETRIC.

THE Z EXCHANGE CROSS SECTION CAN BE WRITTEN AS

$$\frac{d\sigma_{eff}}{d\cos\theta} = \frac{3}{8} \sigma_{ff}^{tot} \left[ (1 - P_e A_e) (1 + \cos^2\theta) + 2 (A_e - P_e) A_f \cos\theta \right]$$

electron beam polarisation  
(assuming no positron polarisation)

WITH

$$A_f = \frac{g_{L_f}^2 - g_{R_f}^2}{g_{L_f}^2 + g_{R_f}^2} = \frac{2 g_{V_f} g_{A_f}}{g_{V_f}^2 + g_{A_f}^2} = 2 \frac{g_{V_f}/g_{A_f}}{1 + (g_{V_f}/g_{A_f})^2}$$

WHERE THE LAST TERM CLEARLY SHOWS THE DEPENDENCY ON  $\sin^2\theta_w$



WHEN INTEGRATING THE CROSS SECTIONS OVER THE FORWARD OR BACKWARD HEMISPHERE WE OBTAIN

$\sigma_F$  : forward  
 $\sigma_B$  : backward

IDENTICAL FOR RIGHT & LEFT ELECTRON HELICITIES.  
THREE BASIC ASYMMETRIES CAN BE MEASURED

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

→ picks out the coefficient  $A_e A_f$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \cdot \frac{1}{\langle P_e \rangle}$$

→ picks out the coefficient  $A_e$

$$A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \cdot \frac{1}{\langle P_e \rangle}$$

→ picks out the coefficient  $A_f$

(ii) POLARISATION OF A FINAL-STATE FERMION IS THE DIFFERENCE BETWEEN THE CROSS SECTIONS FOR RIGHT- AND LEFT-HANDED FINAL STATE HELICITIES DIVIDED BY THEIR SUM

$$P_f = \frac{d(\sigma_R - \sigma_L)/d\cos\theta}{d(\sigma_R + \sigma_L)/d\cos\theta}$$

AGAIN WE CAN INTEGRATE OVER FORWARD AND BACKWARD HEMISPHERES :

$$\langle P_f \rangle = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \quad \rightarrow \text{picks out } A_f$$

$$A_{FB}^{pol} = \frac{(\sigma_R - \sigma_L)_F - (\sigma_R - \sigma_L)_B}{(\sigma_R + \sigma_L)_F + (\sigma_R + \sigma_L)_B} \quad \rightarrow \text{picks out } A_e$$

THESE VARIABLES CAN BE OBTAINED FROM A MEASUREMENT OF

$$P_f(\cos\theta) = - \frac{A_f(1 + \cos^2\theta) + 2A_e \cos\theta}{(1 + \cos^2\theta) + 2A_f A_e \cos\theta}$$

WHICH IS ONLY MEASURED FOR Z-LEPTONS IN THE FINAL STATE OF WHICH WE CAN OBTAIN THE POLARISATION

HENCE ALL TOGETHER WHEN WE MEASURE THE ASYMMETRIES (FORWARD-BACKWARD AND/OR LEFT-RIGHT) WE CAN RELATE THEM TO THE PARAMETERS  $A_f$ :

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

$$A_{LR}^0 = A_e$$

$$A_{LRFB}^0 = \frac{3}{4} A_f$$

$$\langle P_z^0 \rangle = -A_e$$

$$A_{FB}^{pol,0} = -\frac{3}{4} A_e$$

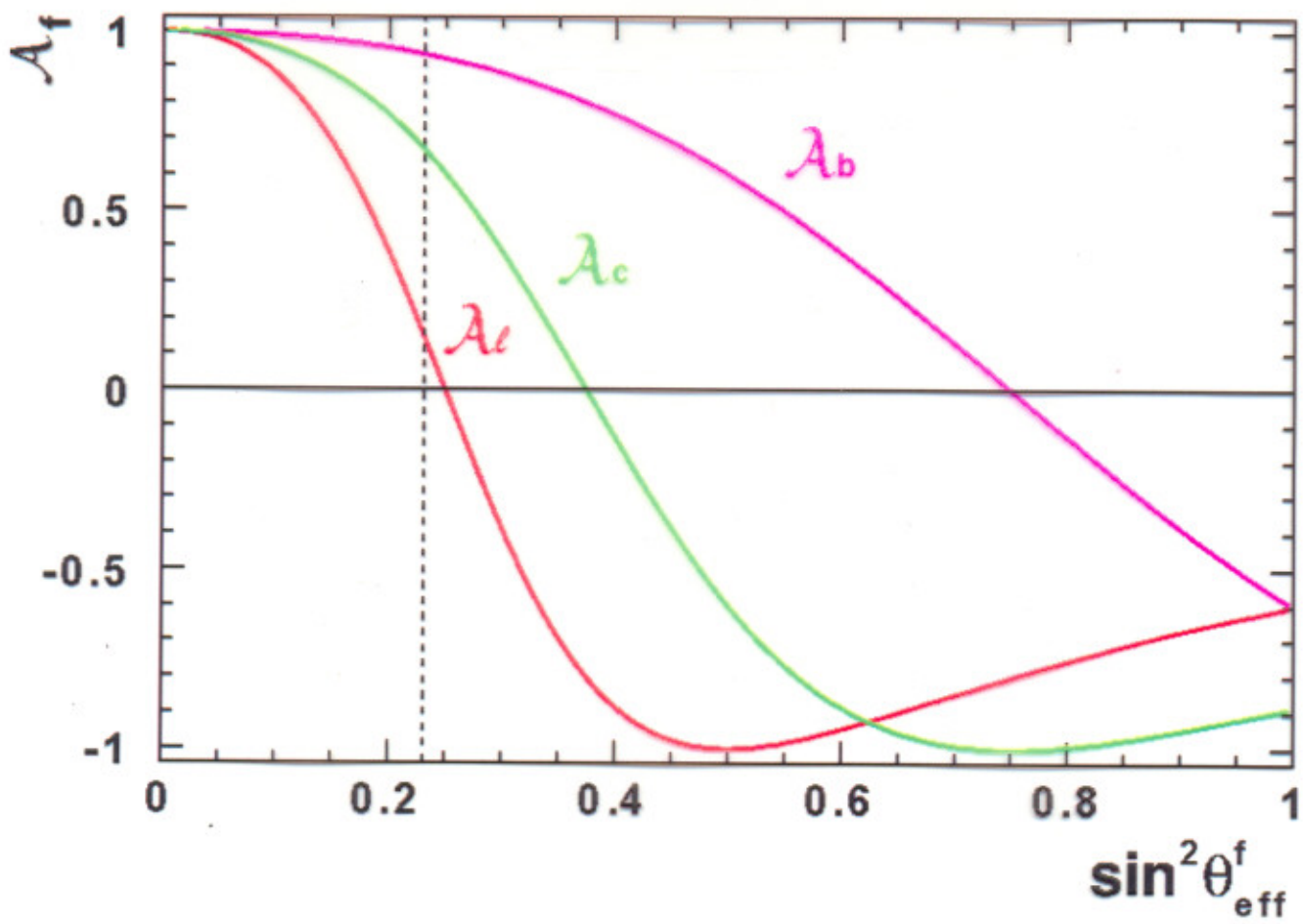
→ using this LEP can also measure  $A_f$

LEP:  $A_{FB}^0$  for all final states &  $P_z$

SLD:  $A_{LR}^0$ ,  $A_{LRFB}^0$  for all final states



SENSITIVITY OF  $A_f$  TO  $\sin^2 \theta_{eff}^f$



# FROM THEORY TO EXPERIMENT

THE ABOVE PARAMETERS ARE NOT "REALISTIC OBS" BUT WHICH HAVE SIGNIFICANT THEORY CORRECTIONS

→ PSEUDO-OBSERVABLES

(denoted by superscript 0)

eg.  $\left\{ \begin{array}{l} \sigma_{\text{had}} \text{ is the measured hadronic cross section} \\ \sigma_{\text{had}}^0 \text{ is the pole cross-section derived from } \sigma_{\text{had}} \\ R_b \text{ is the measurement of b-quark cross section} \\ \text{divided by the hadronic one } \sigma_{b\bar{b}}/\sigma_{\text{had}} \\ R_b^0 \text{ is } \Gamma_{b\bar{b}}/\Gamma_{\text{had}} \text{ derived from this} \end{array} \right.$

THE EXPERIMENTAL CROSS SECTIONS & ASYMMETRIES ARE MEASURED IN THE ACCEPTANCE OF THE DETECTOR

→ CORRECT THEM BY EXTRAPOLATING TO PERFECT (= FULL) ACCEPTANCE

NINE PSEUDO-OBSERVABLES DESCRIBE THE Z RESONANCE IN A MODEL INDEPENDENT WAY.

("THEORY" & "EXPERIMENT" REMAIN DISTINCT)

$m_Z, \Gamma_Z, \sigma_{\text{had}}^0, R_b^0, A_{\text{FB}}^{0,f}$

$A_{\text{LR}}^0, A_{\text{LRFB}}^0, \langle P_Z^0 \rangle, A_{\text{FB}}^{\text{pole},0}$

need a fit to take into account the correlations between them, only then an interpretation is possible



# LEP RESULTS IN THE LEPTON SECTOR

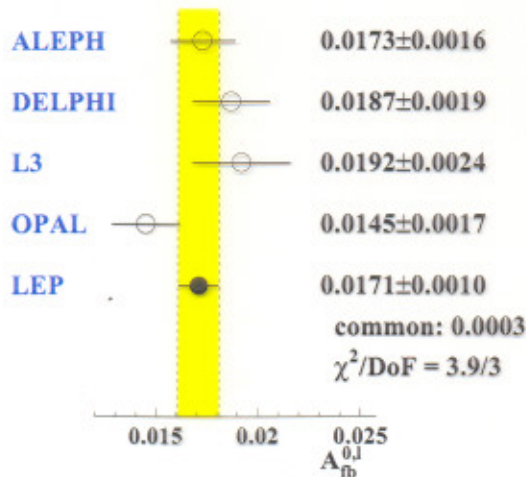
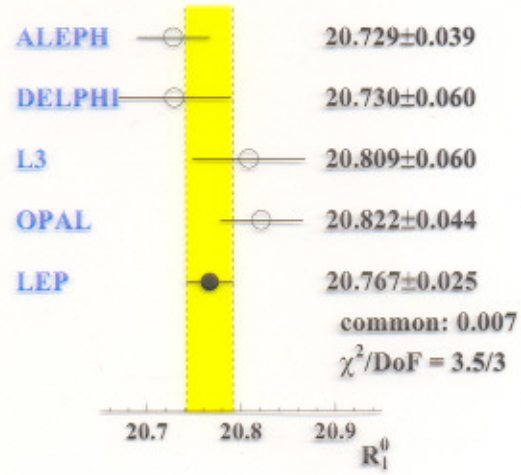
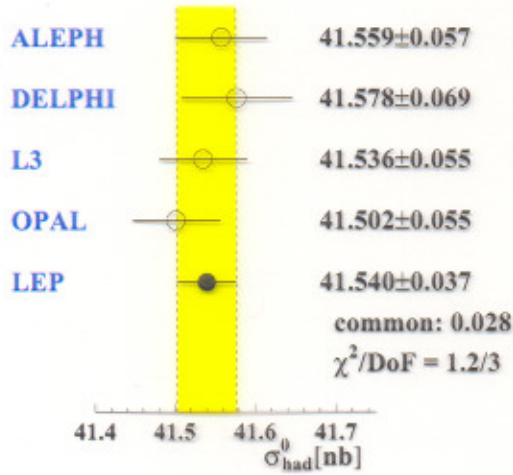
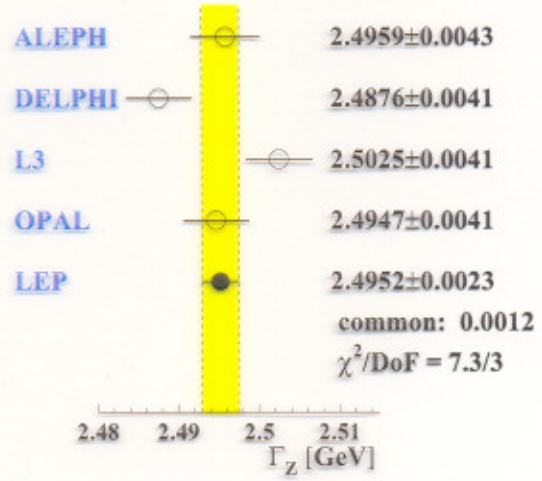
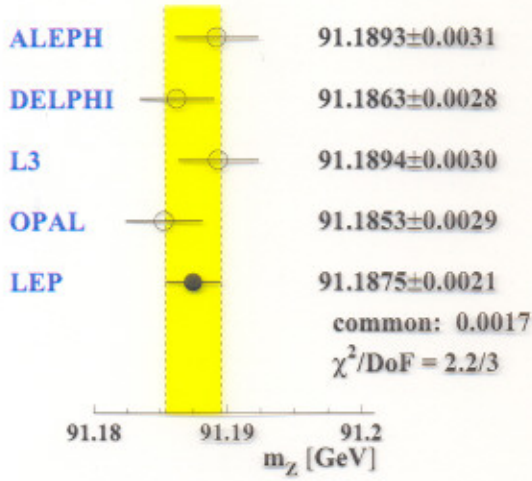


Figure 2.9: Measurements of  $m_Z$ ,  $\Gamma_Z$ ,  $\sigma_{\text{had}}^0$ ,  $R_\ell^0$  and  $A_{\text{FB}}^{0,\ell}$ . The averages indicated were obtained using the common errors and combination method discussed in the text. The values of  $\chi^2$  per degree of freedom were calculated considering error correlations between measurements of the same parameter, but not error correlations between different parameters.

# GOOD COMPARISON BETWEEN LEPTON FLAVOURS

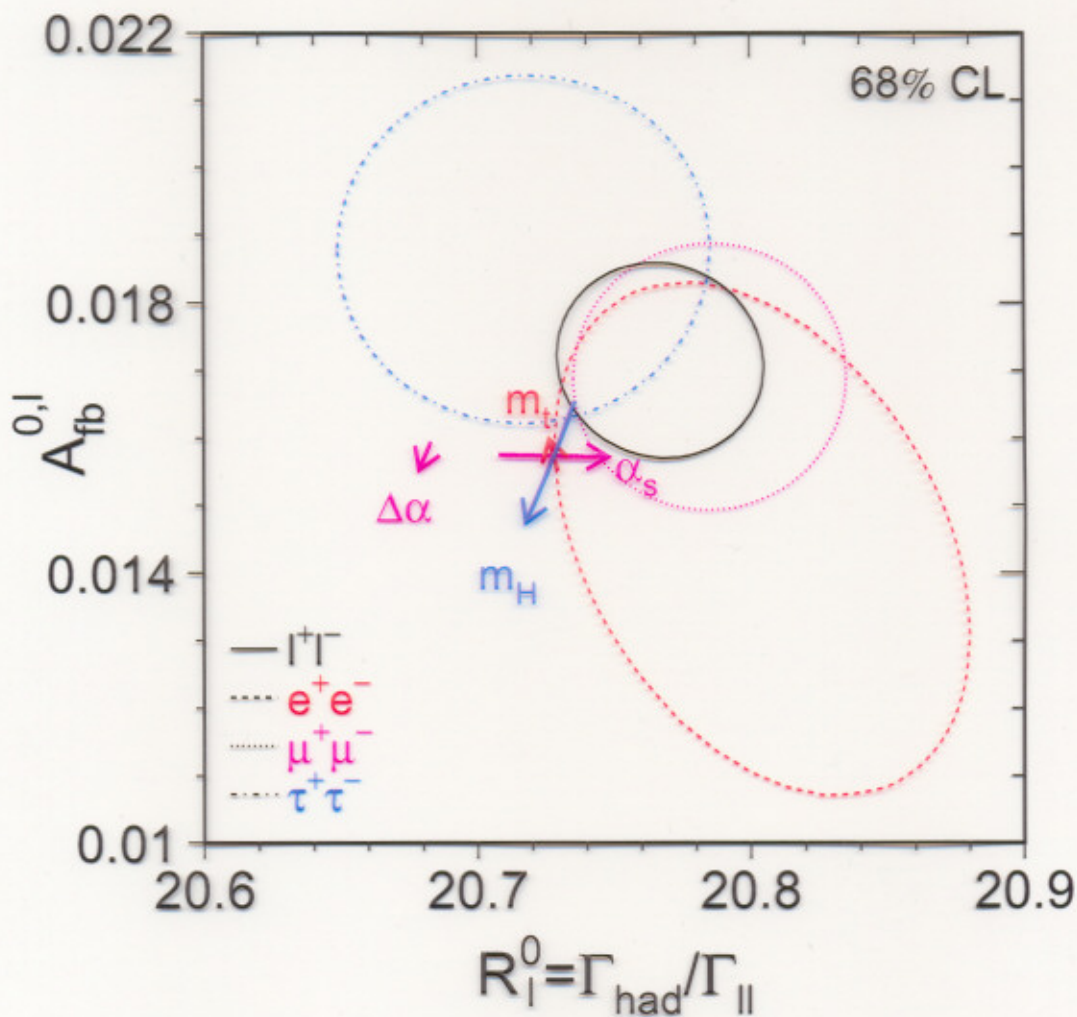


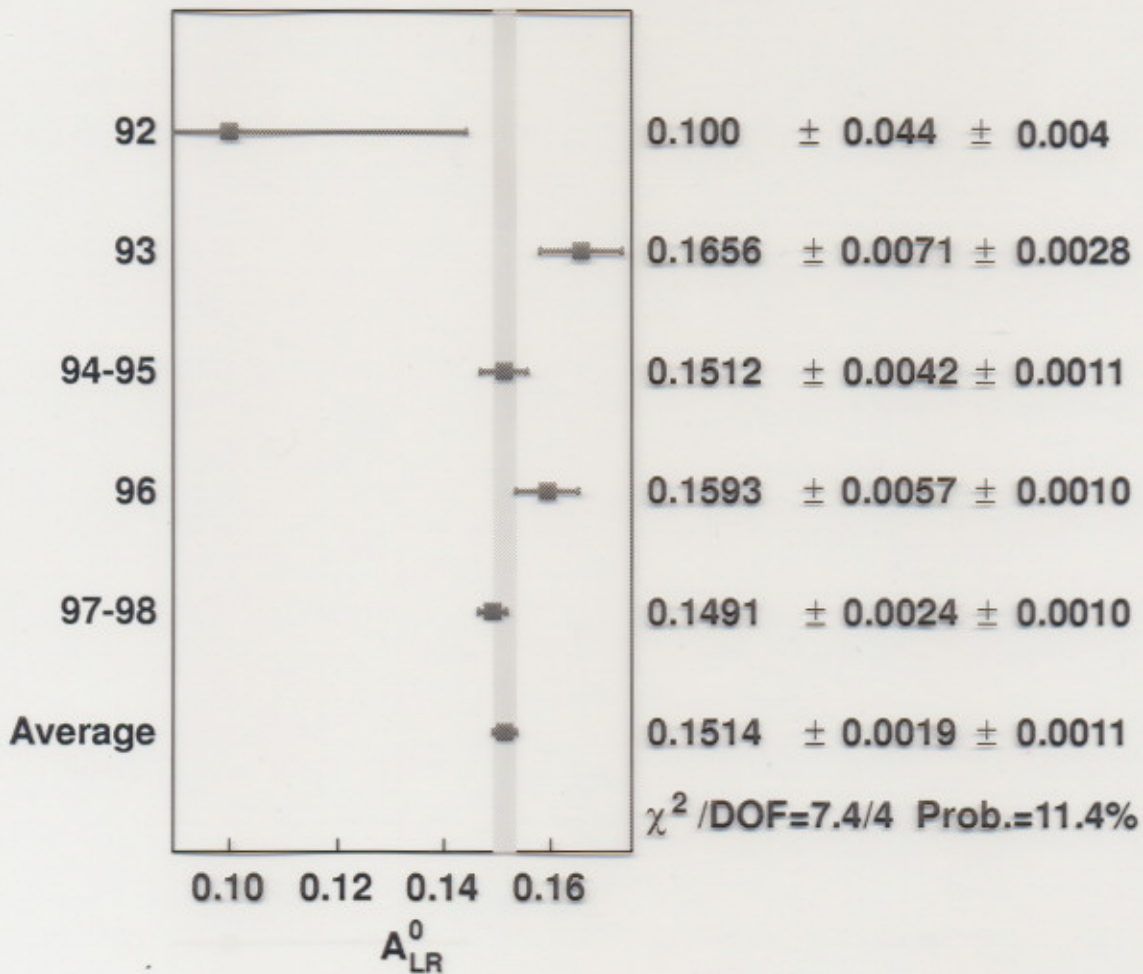
Figure 2.11: Contour lines (68% CL) in the  $R_l^0 - A_{FB}^{0,\ell}$  plane for  $e^+e^-$ ,  $\mu^+\mu^-$  and  $\tau^+\tau^-$  final states and for all leptons combined. For better comparison the results for the  $\tau$  lepton are corrected to correspond to the massless case. The SM prediction for  $m_Z = 91.1875$  GeV,  $m_t = 178.0$  GeV,  $m_H = 300$  GeV, and  $\alpha_S(m_Z^2) = 0.118$  is also shown as the intersection of the lines with arrows, which correspond to the variation of the SM prediction when  $m_t$ ,  $m_H$  and  $\alpha_S(m_Z^2)$  are varied in the intervals  $m_t = 178.0 \pm 4.3$  GeV,  $m_H = 300_{-186}^{+700}$  GeV, and  $\alpha_S(m_Z^2) = 0.118 \pm 0.003$ , respectively. The arrow showing the small dependence on the hadronic vacuum polarisation  $\Delta\alpha_{had}^{(5)}(m_Z^2) = 0.02758 \pm 0.00035$  is displaced for clarity. The arrows point in the direction of increasing values of these parameters.



# MEASUREMENT OF THE LEFT-RIGHT ASYMMETRY BY SLC

NEEDED FOR A PRECISE DETERMINATION OF  $A_e$   
 COUNT THE NUMBER OF Z BOSONS PRODUCED  
 BY LEFT AND RIGHT LONGITUDINALLY POLARISED  
 ELECTRONS

$$A_{LR} = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\langle P_e \rangle}$$



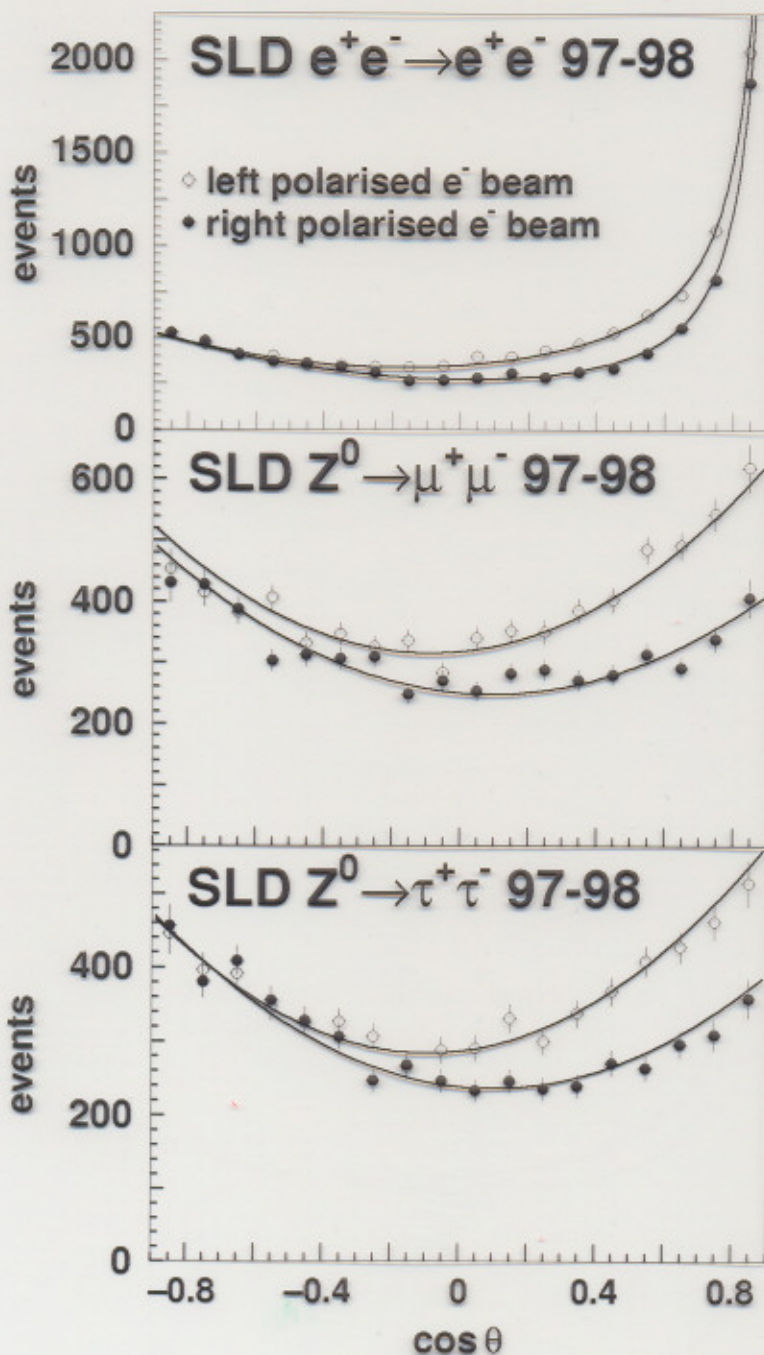
$$A_{LR}^0 = \frac{2(1 - 4 \sin^2 \theta_{\text{eff}}^{\text{lept}})}{1 + (1 - 4 \sin^2 \theta_{\text{eff}}^{\text{lept}})^2}$$

$$\Rightarrow \sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23097 \pm 0.00027$$

# LEPTON ASYMMETRY MEASUREMENTS

VIA MEASUREMENTS OF  $A_{LRFB}^{0,l} = \frac{3}{4} |P_e| A_e$

↓ OBTAINED FROM A FIT ON  $d\sigma/d\cos\theta$



$$A_e = 0.1513 \pm 0.001$$

$$\Rightarrow \sin^2 \theta_{eff}^{lept} = 0.23098 \pm 0.00026$$



# THE $Z$ POLARISATION MEASUREMENTS

DEPENDS ON THE DEPENDENCE OF KINEMATIC DISTRIBUTIONS OF THE OBSERVED  $Z$  DECAY ON THE HELICITY OF THE PARENT  $Z$  LEPTON.

$\Rightarrow$  EXTRACT  $P_Z$  AS A FUNCTION OF  $\cos\theta_Z$

ALEPH  $0.1451 \pm 0.0060$

DELPHI  $0.1359 \pm 0.0096$

L3  $0.1476 \pm 0.0108$

OPAL  $0.1456 \pm 0.0095$

$A_\tau$  (LEP)  $0.1439 \pm 0.0043$

$$P_Z(\cos\theta_Z) = - \frac{A_Z(1 + \cos^2\theta_Z) + 2A_e \cos\theta_Z}{(1 + \cos^2\theta_e) + \frac{8}{3}A_{FB}^Z \cos\theta_e}$$

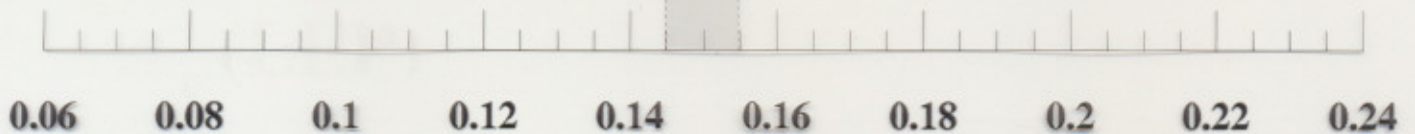
ALEPH  $0.1504 \pm 0.0068$

DELPHI  $0.1382 \pm 0.0116$

L3  $0.1678 \pm 0.0130$

OPAL  $0.1454 \pm 0.0114$

$A_e$  (LEP)  $0.1498 \pm 0.0049$



$A_1$  (LEP)  $= 0.1465 \pm 0.0033$

$\chi^2/\text{DoF} = 4.7/7$

$$\Rightarrow \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23159 \pm 0.00041$$

# HEAVY FLAVOUR PARTIAL WIDTH (using b- and c-tagging)

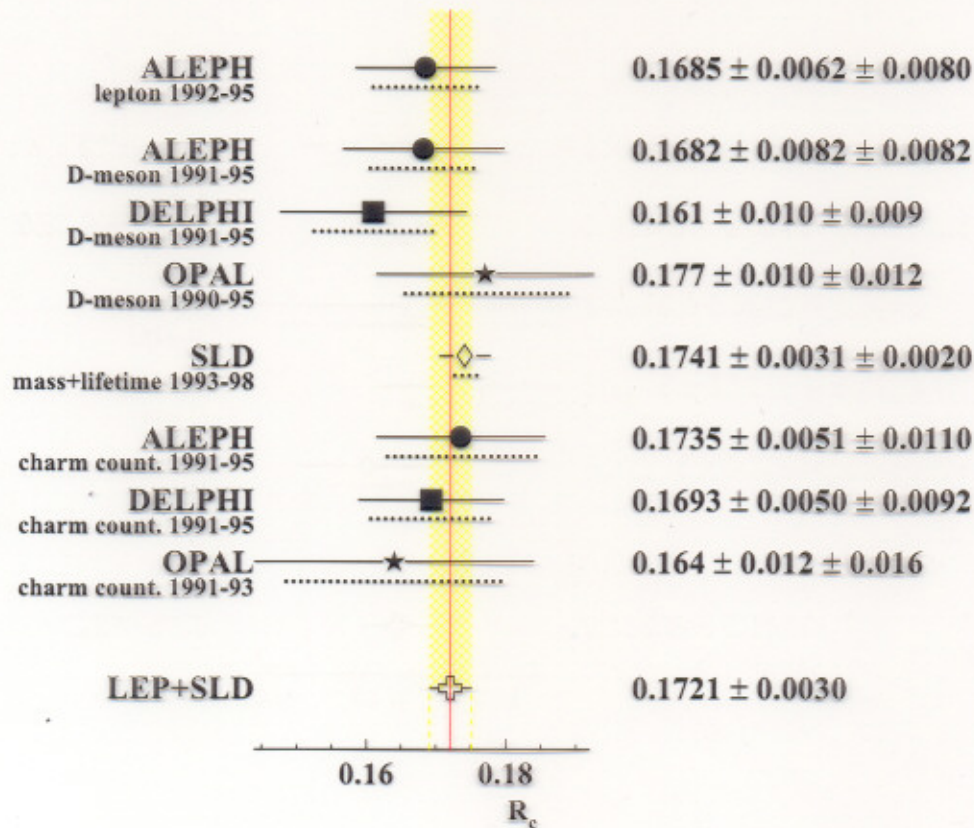
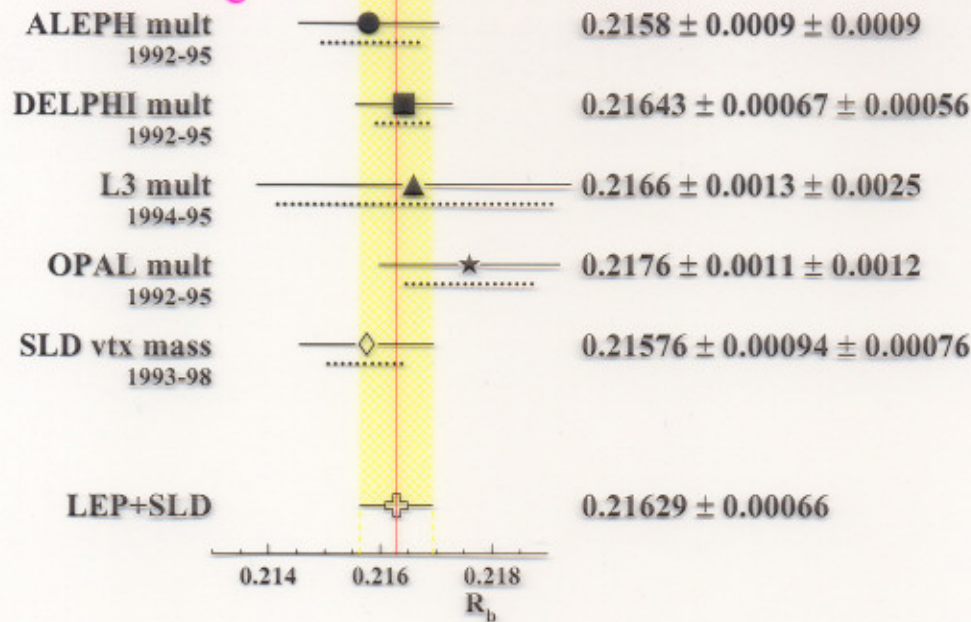


Figure 5.13:  $R_b^0$  and  $R_c^0$  measurements used in the heavy flavour combination, corrected for their dependence on parameters evaluated in the multi-parameter fit described in the text. The dotted lines indicate the size of the systematic error.



# HEAVY FLAVOUR ASYMMETRIES

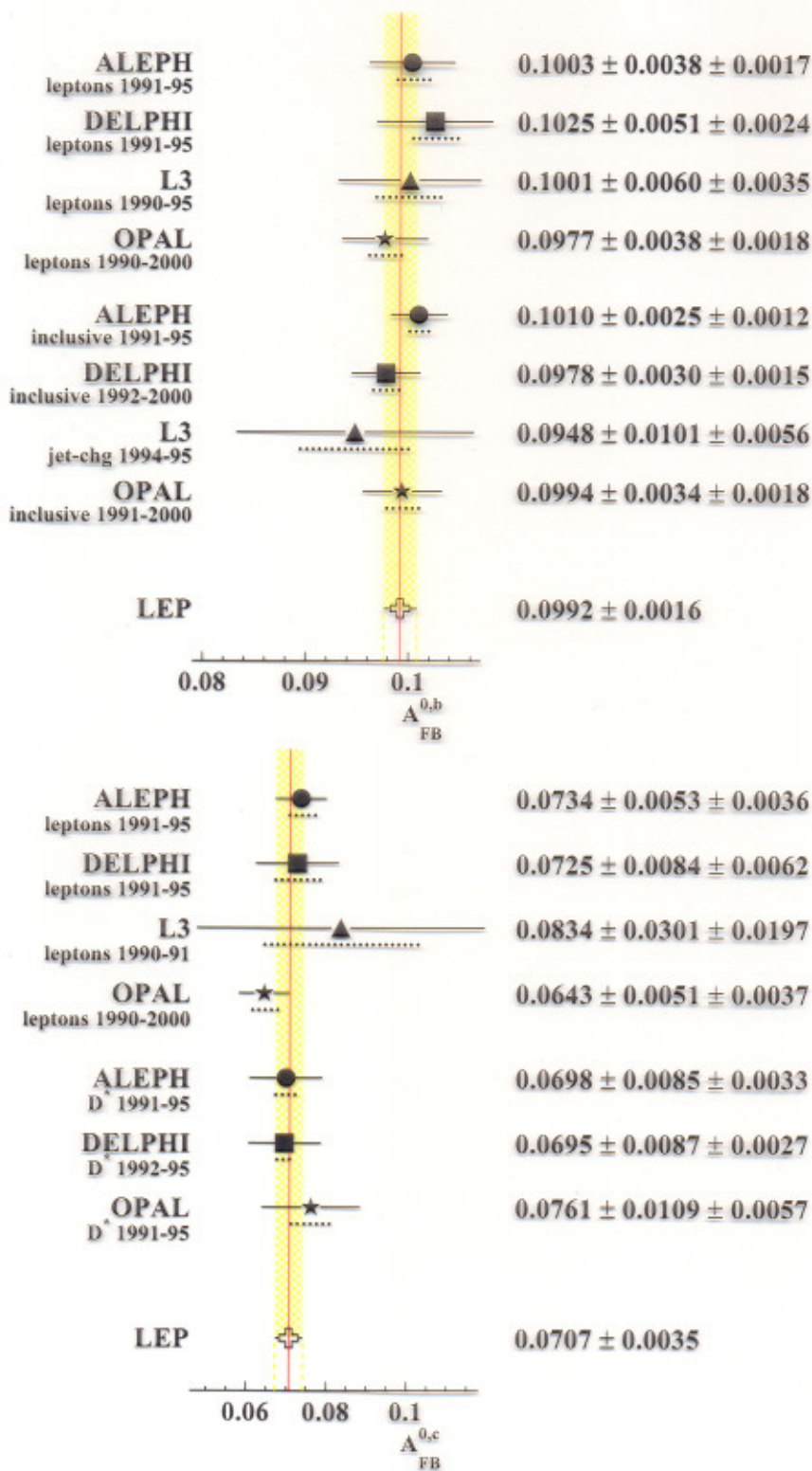


Figure 5.14:  $A_{FB}^{0,b}$  and  $A_{FB}^{0,c}$  measurements used in the heavy flavour combination, corrected for their dependence on parameters evaluated in the multi-parameter fit described in the text. The  $A_{FB}^{0,b}$  measurements with D-mesons do not contribute significantly to the average and are not shown in the plots. The experimental results are derived from the ones shown in Tables C.3 to C.8 combining the different centre of mass energies. The dotted lines indicate the size of the systematic error.

# ENERGY DEPENDENCE

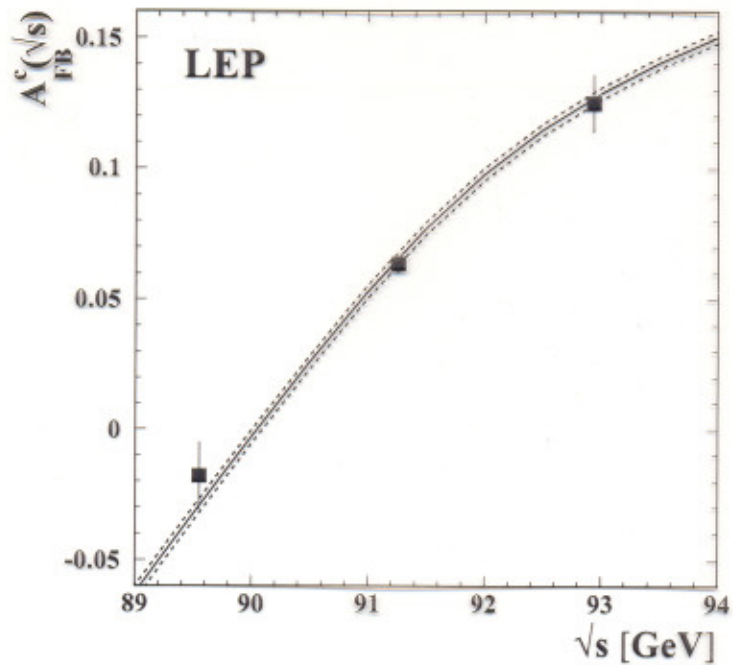
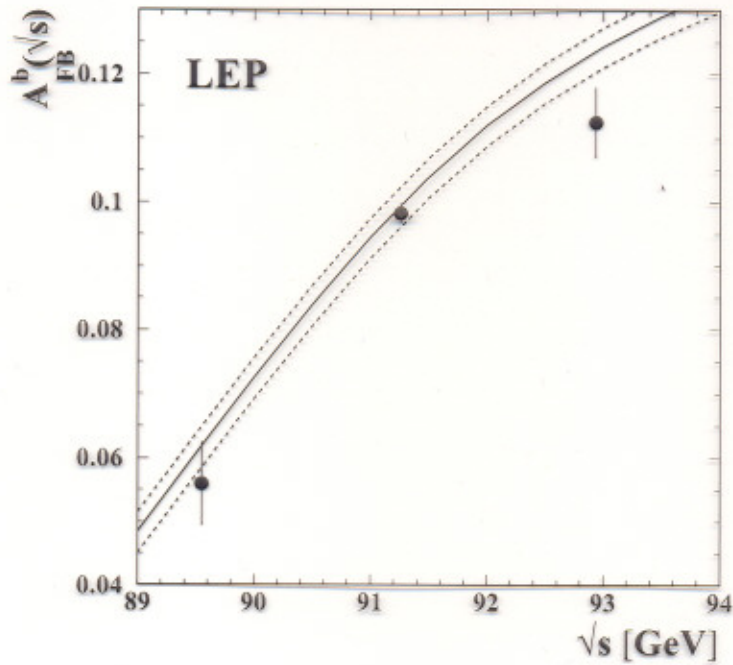


Figure 5.16: Energy dependence of  $A_{FB}^b$  and  $A_{FB}^c$ . The solid line represents the SM prediction for  $m_t = 178$  GeV,  $m_H = 300$  GeV, the upper (lower) dashed line is the prediction for  $m_H = 100$  (1000) GeV.



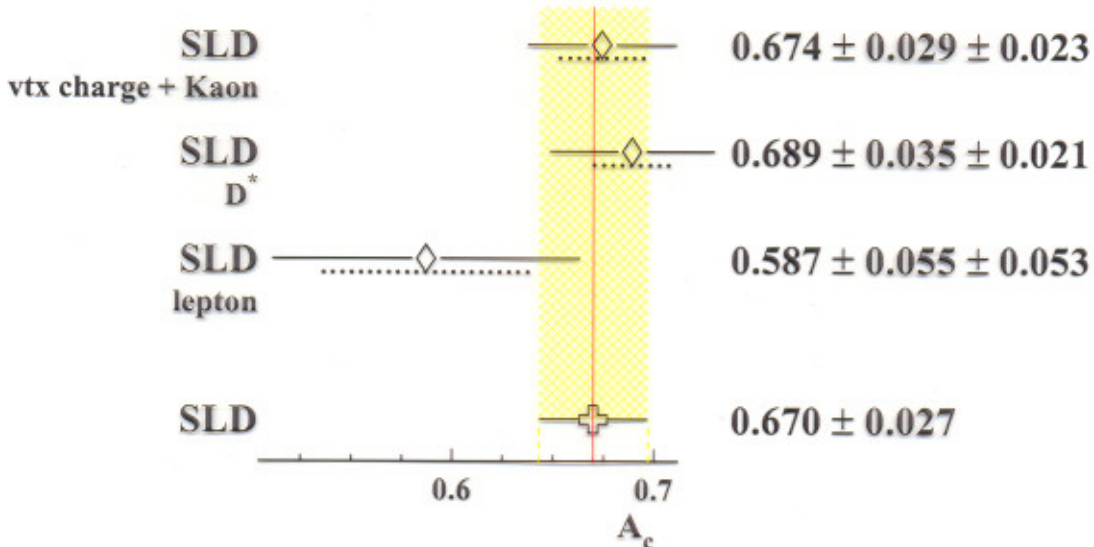
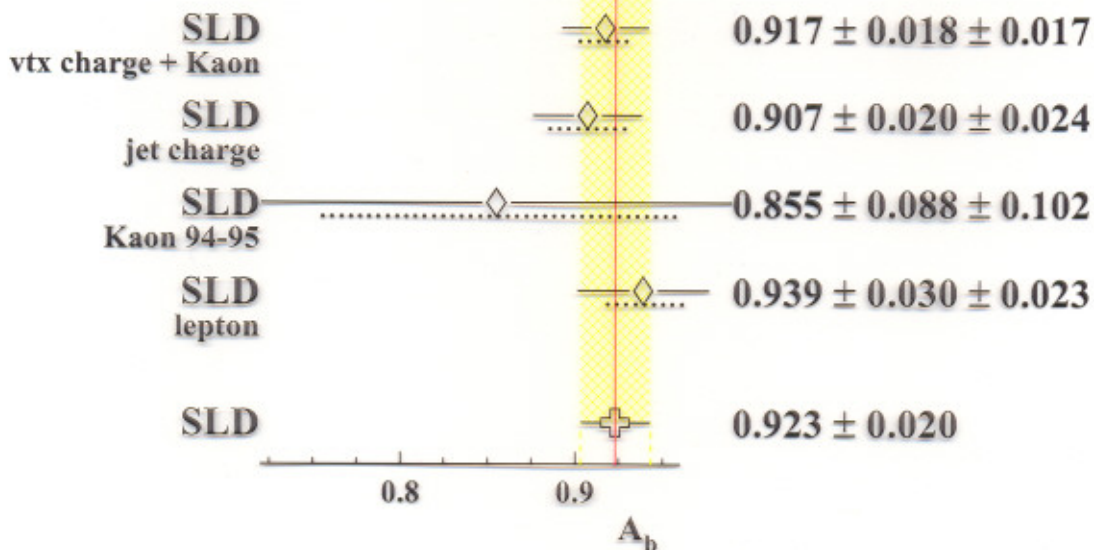
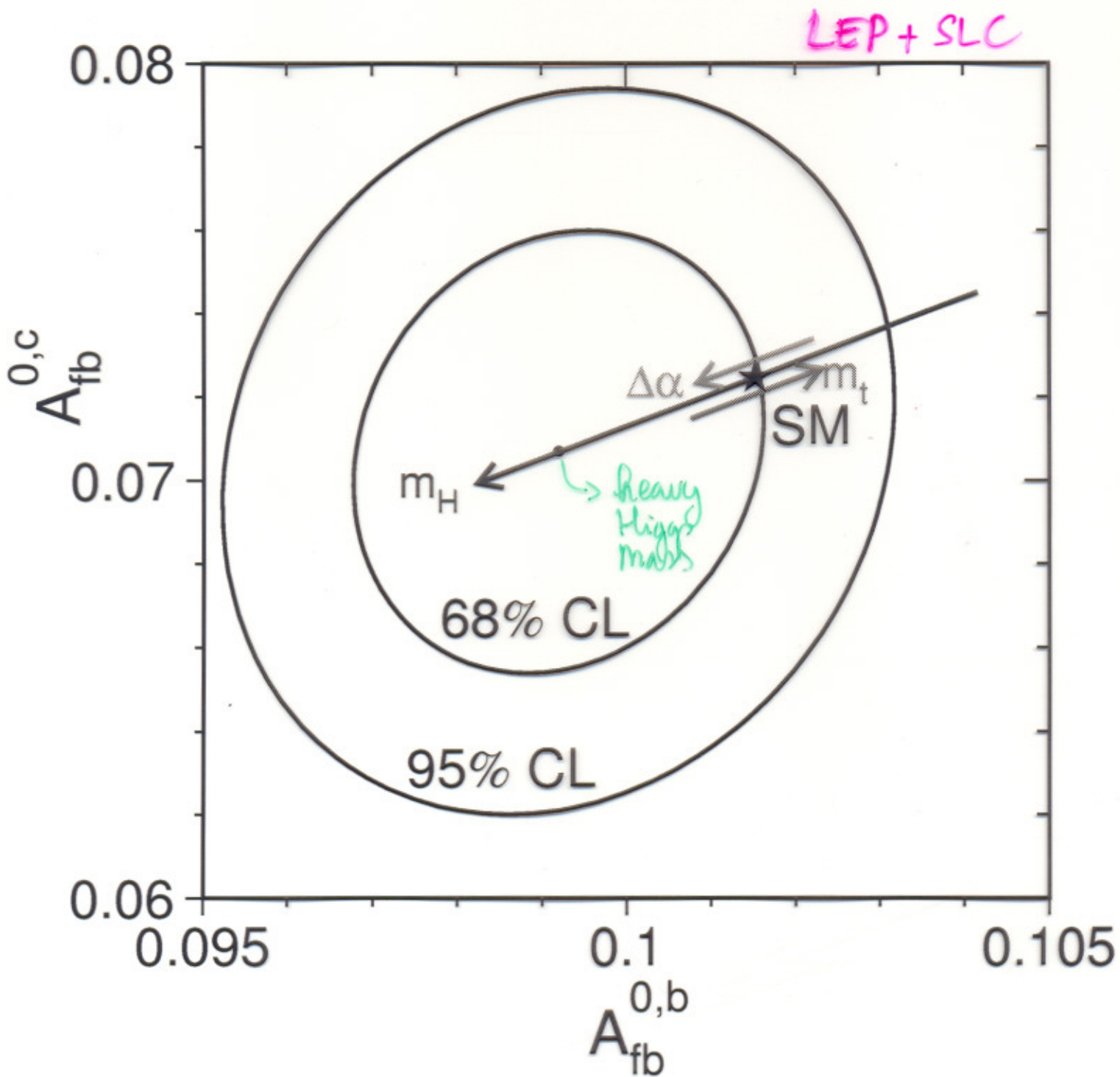


Figure 5.15:  $\mathcal{A}_b$  and  $\mathcal{A}_c$  measurements used in the heavy flavour combination, corrected for their dependence on parameters evaluated in the multi-parameter fit described in the text. The dotted lines indicate the size of the systematic error.

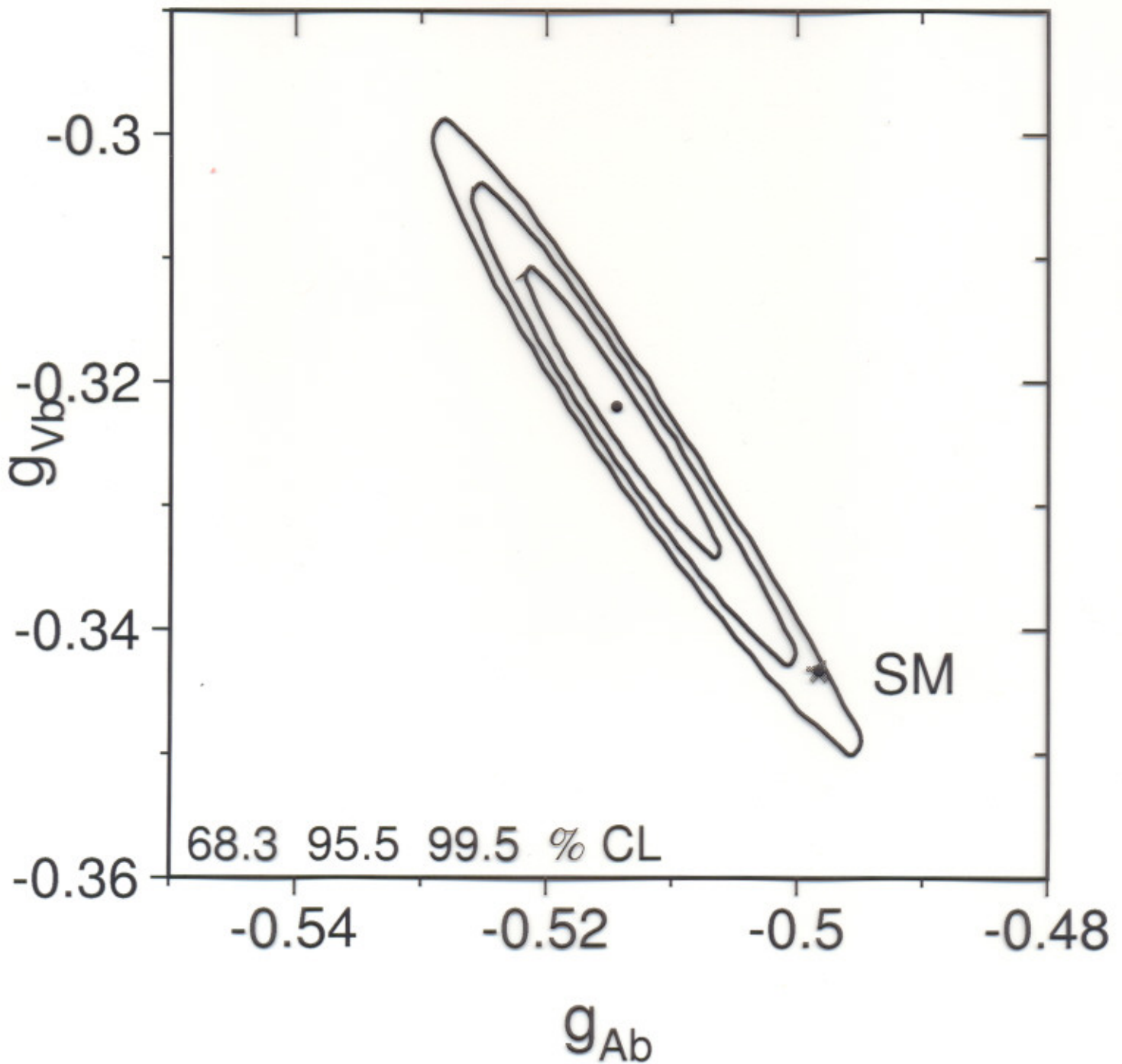
FINAL VALUES FOR  $R_b^0, R_c^0, A_{FB}^{0,b}, A_{FB}^{0,c}, A_b, A_c$   
 ARE OBTAIN FROM A MULTIPARAMETER FIT  
 → GOOD AGREEMENT ILLUSTRATED BELOW

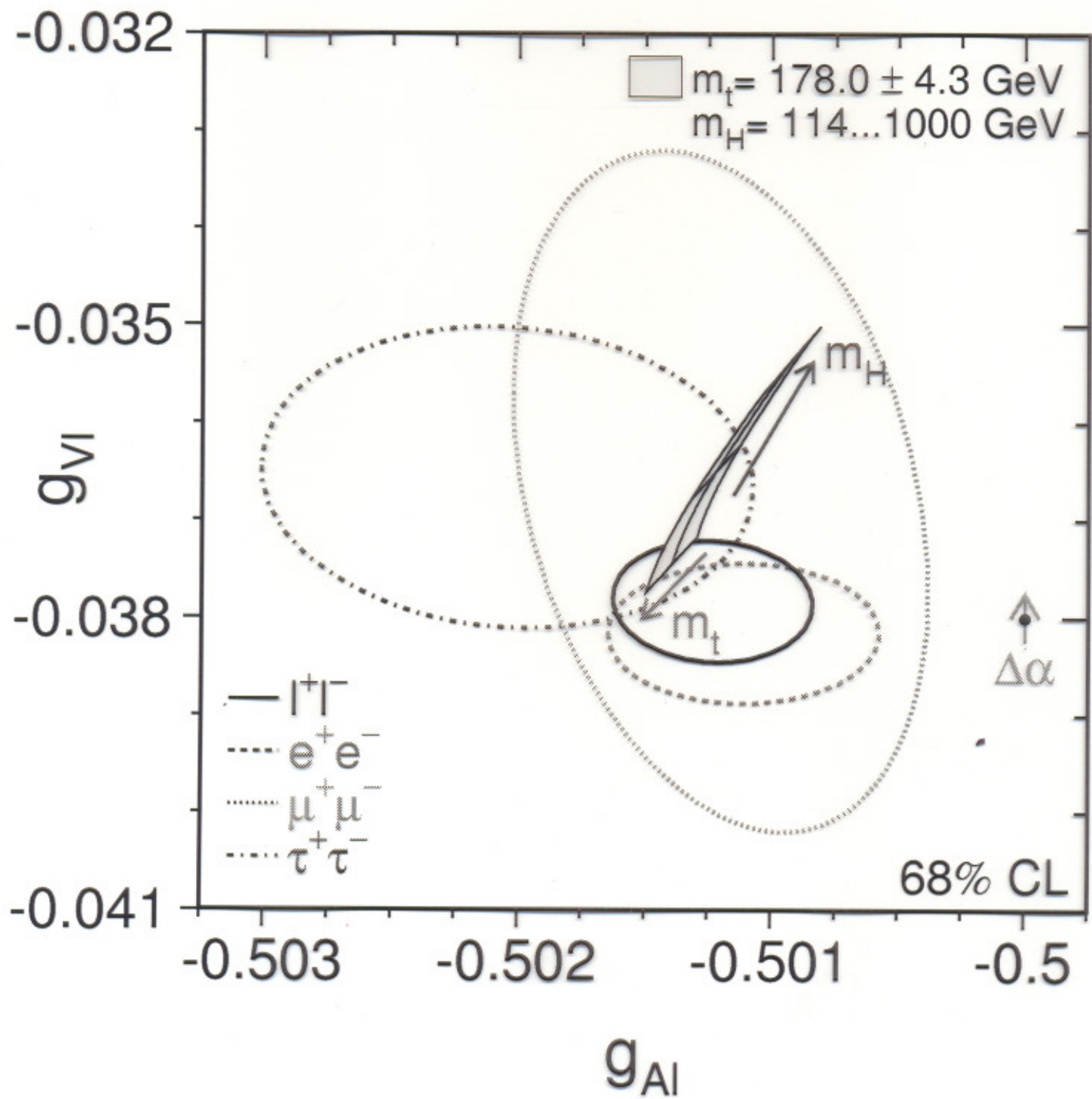




WITH THE SM EQUATIONS WE HAVE SEEN  
 WE CAN TRANSFORM THE PSEUDO-OBSERVABLES  
 INTO EFFECTIVE COUPLINGS OF THE NEUTRAL  
 WEAK CURRENT :

$A_f, (g_{Vf}^+, g_{Af}^+), (g_{Lf}^+, g_{Rf}^+), P_f, \sin^2 \theta_{eff}^f$   
 $\downarrow$   $\downarrow$  EXAMPLES

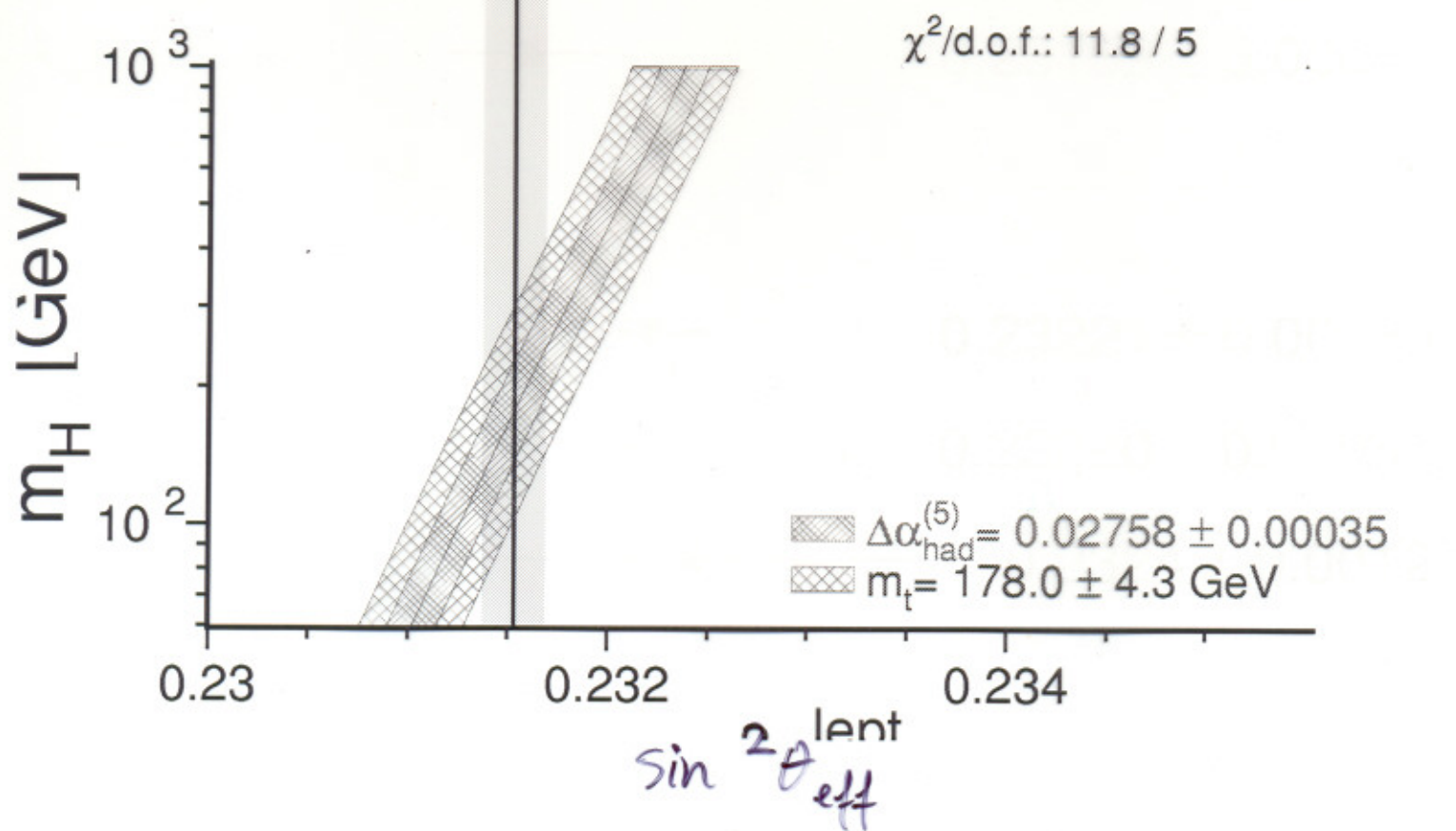
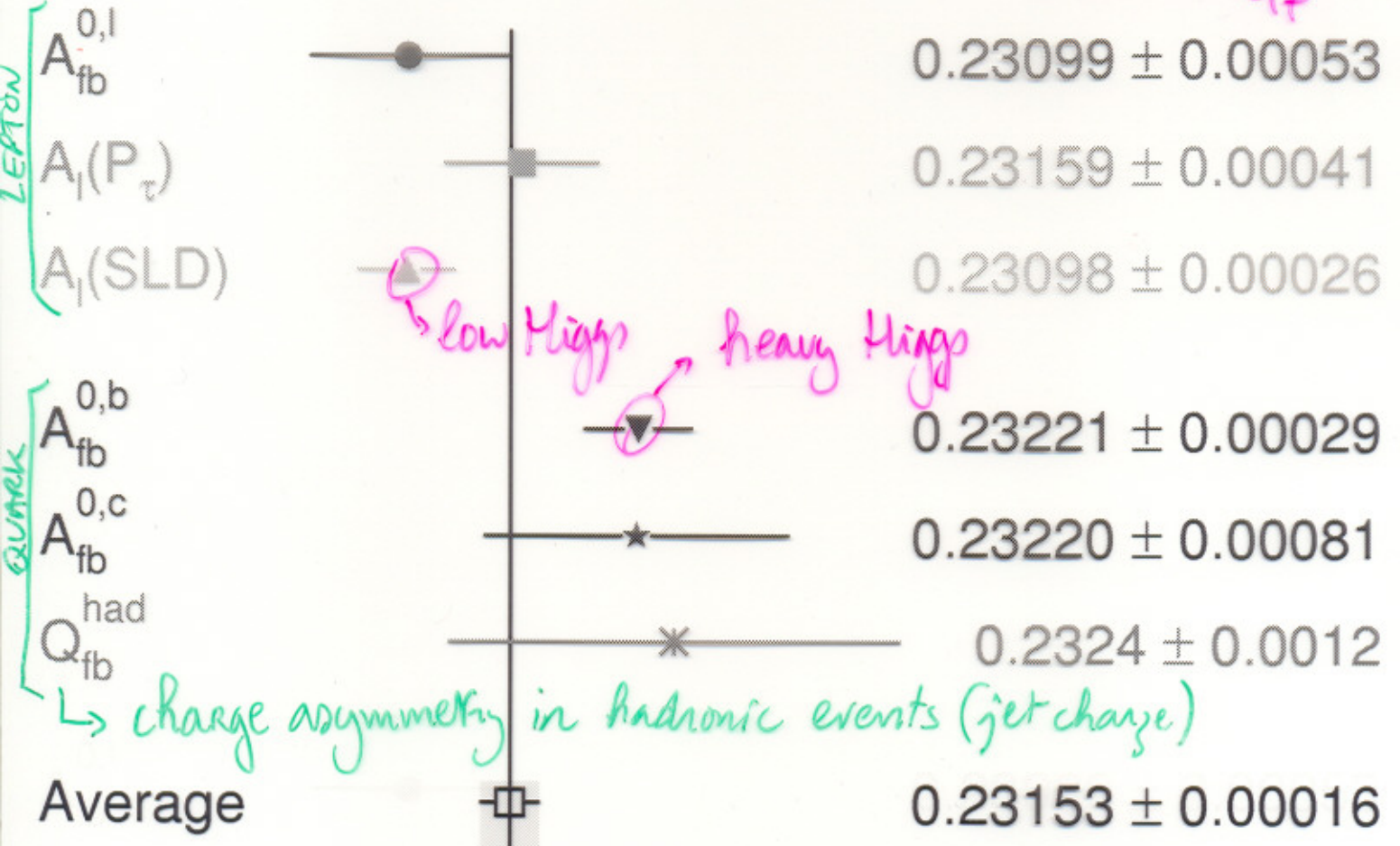






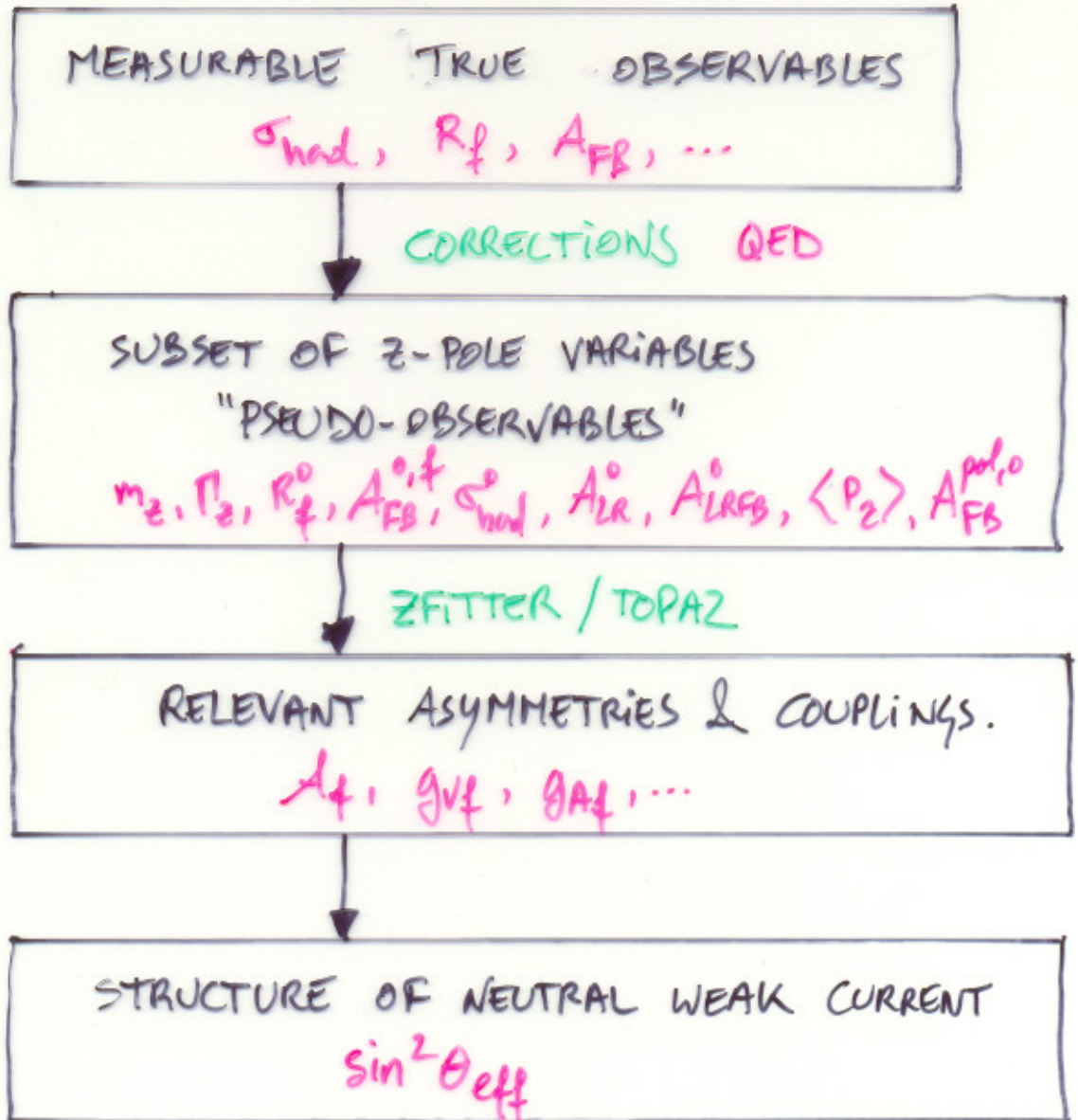
# PUTTING THE ASYMMETRIES TOGETHER

$\sin^2 \theta_{\text{eff}}^{\text{lept}}$



# LECTURE 1: THE Z POLE

TWO KEY EXPERIMENTS: LEP-I (17M<sup>20</sup>) & SLC (0.6M<sup>20</sup>)



MEASUREMENTS OF  
 $m_W$  &  $m_t$

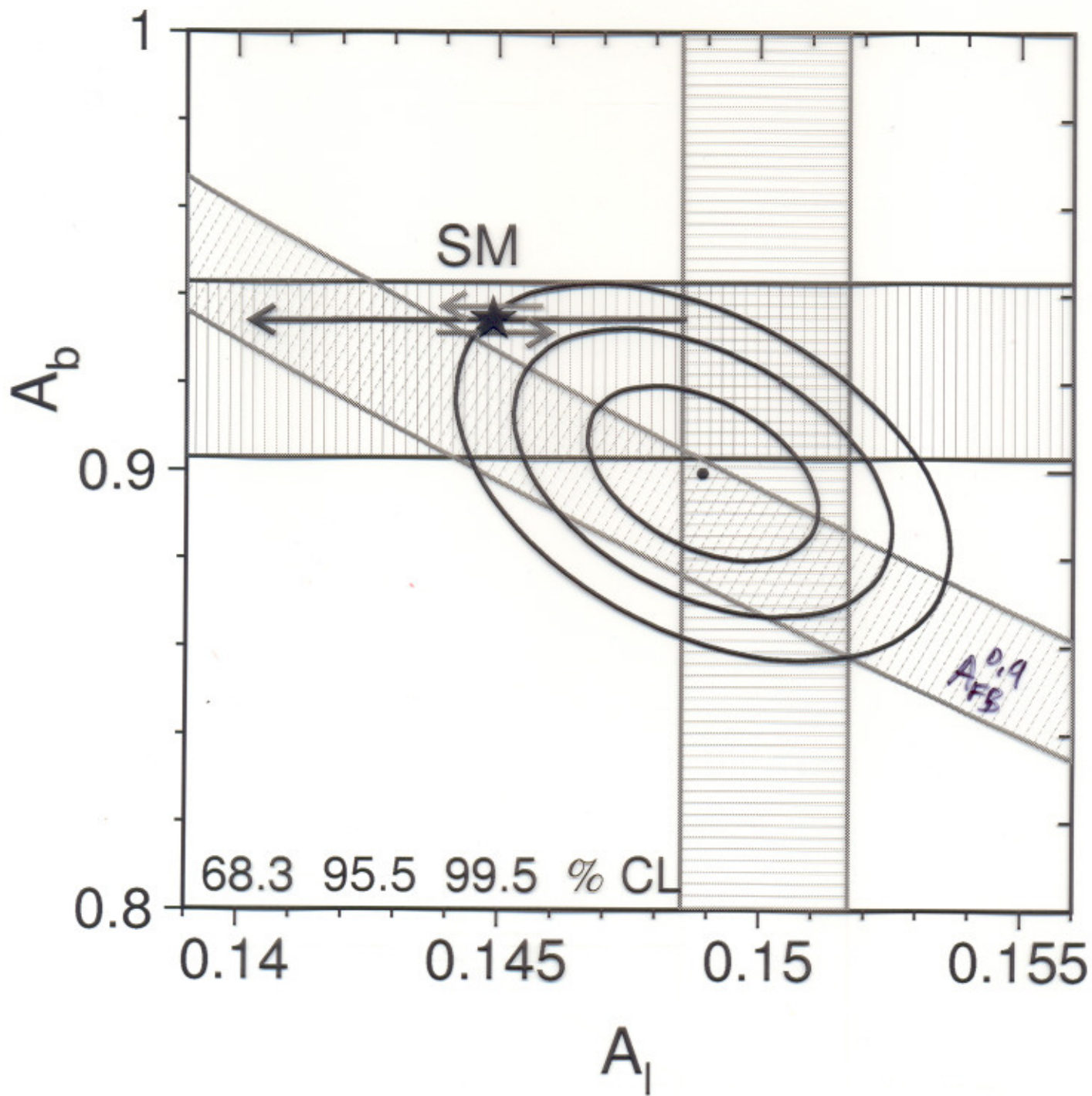
LECTURE 2

ELECTROWEAK FIT USING  $\alpha, G_F, m_Z, m_t, m_H$   
(RADIATIVE CORRECTIONS EW)

LECTURE 2

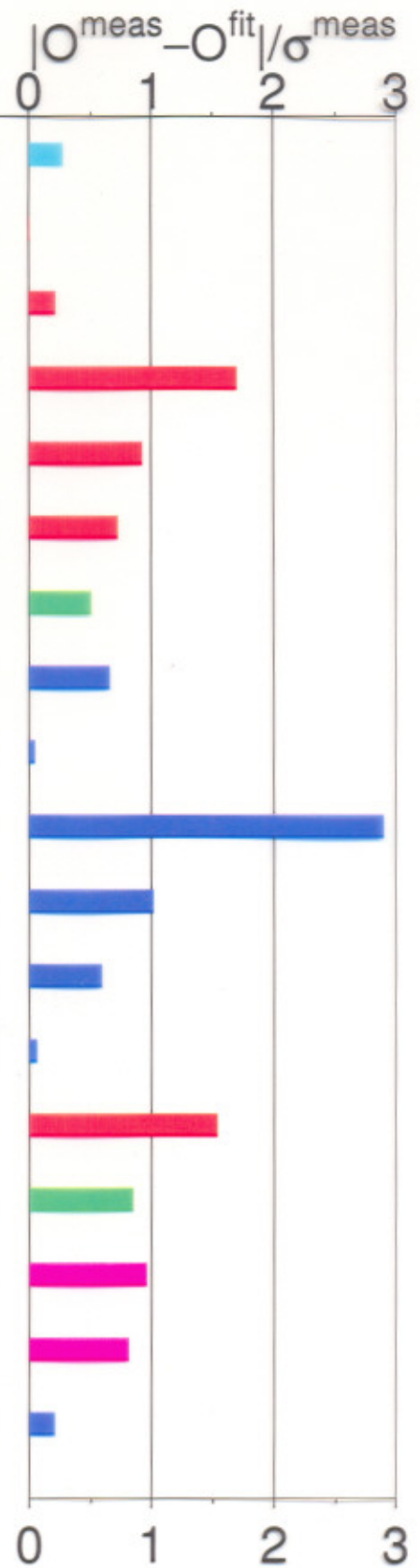
! HIGGS BOSON BLUEBAND PLOT !





Measurement

Fit

 $\Delta\alpha_{\text{had}}^{(5)}(m_Z)$  $0.02758 \pm 0.00035$  0.02768 $m_Z$  [GeV] $91.1875 \pm 0.0021$  91.1875 $\Gamma_Z$  [GeV] $2.4952 \pm 0.0023$  2.4957 $\sigma_{\text{had}}^0$  [nb] $41.540 \pm 0.037$  41.477 $R_l$  $20.767 \pm 0.025$  20.744 $A_{\text{fb}}^{0,l}$  $0.01714 \pm 0.00095$  0.01645 $A_l(P_\tau)$  $0.1465 \pm 0.0032$  0.1481 $R_b$  $0.21629 \pm 0.00066$  0.21586 $R_c$  $0.1721 \pm 0.0030$  0.1722 $A_{\text{fb}}^{0,b}$  $0.0992 \pm 0.0016$  0.1038 $A_{\text{fb}}^{0,c}$  $0.0707 \pm 0.0035$  0.0742 $A_b$  $0.923 \pm 0.020$  0.935 $A_c$  $0.670 \pm 0.027$  0.668 $A_l(\text{SLD})$  $0.1513 \pm 0.0021$  0.1481 $\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{fb}})$  $0.2324 \pm 0.0012$  0.2314 $m_W$  [GeV] $80.398 \pm 0.025$  80.374 $\Gamma_W$  [GeV] $2.140 \pm 0.060$  2.091 $m_t$  [GeV] $170.9 \pm 1.8$  171.3

LECTURE -1 @ THE Z



# DISCUSSION

THE  $Z$ -POLE OBSERVABLES HAVE BEEN MEASURED WITH HIGH PRECISION. THIS GIVES A COMPLETE PICTURE OF THE NEUTRAL WEAK CURRENT AND THE  $Z$  BOSON COUPLINGS.

→ SOME DEVIATIONS BETWEEN  $A_e(\text{SLC})$  &  $A_{\text{FB}}^{0,1,b}$  THOROUGHLY CHECKED BY EXPERIMENTS AND ASSUMED TO BE A FLUCTUATION

ONLY WITH THE LINEAR COLLIDER WE WILL IMPROVE THIS KNOWLEDGE SIGNIFICANTLY.

→ ALLOWS THE USE OF  $m_Z$  AS A PARAMETER FOR INPUT TO SM FITS.

$(\alpha, G_F, m_Z, m_t, m_H)$

$\downarrow$   $\rightarrow$  to be found.  
next lecture

NEXT LECTURE:

- $m_W$  measurement
- $m_{\text{top}}$  measurement
- global Standard Model fit  $\Rightarrow m_H$
- interpretation