Introduction to

Low-energy Supersymmetry and its Experimental Aspects

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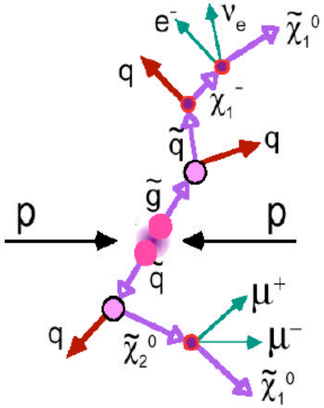
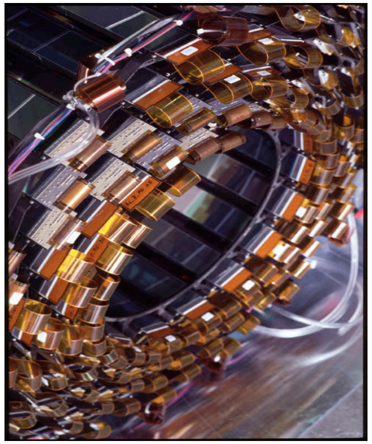




Programme

- Introduction to SUSY and its phenomenology
  - open questions in the Standard Model
  - motivations for low-energy supersymmetry
  - Minimal Supersymmetric Standard Model (MSSM)
  - MSSM particle content
  - MSSM Higgs sector

- Searching for low-energy supersymmetry
  - Accelerators and detectors
  - Inclusive searches for supersymmetry
  - Exclusive searches and determination of properties
  - MSSM Higgs searches

- Searching for dark matter
  - SUSY candidates for dark matter
  - direct and indirect searches

Literature

Useful references:

- S. Martin, “A Supersymmetry Primer” , hep-ph/9709356
- L. Pape and D. Treille, “Supersymmtry facing experiment”, Rept.Prog.Phys.69:2843-3067, 2006

Useful books:

- H. Baer and X. Tata, “Weak-scale Supersymmetry” , 2006
- Drees, Godbole, Roy,

“Theory and phenomenology of sparticles”, 2005

- I. Aitchinson, “Supersymmetry in Particle Physics” , 2007

This lecture

Today:

- why go Beyond the Standard Model?
- are there theoretical models for BSM physics?
- are there experiments that may reveal BSM physics?

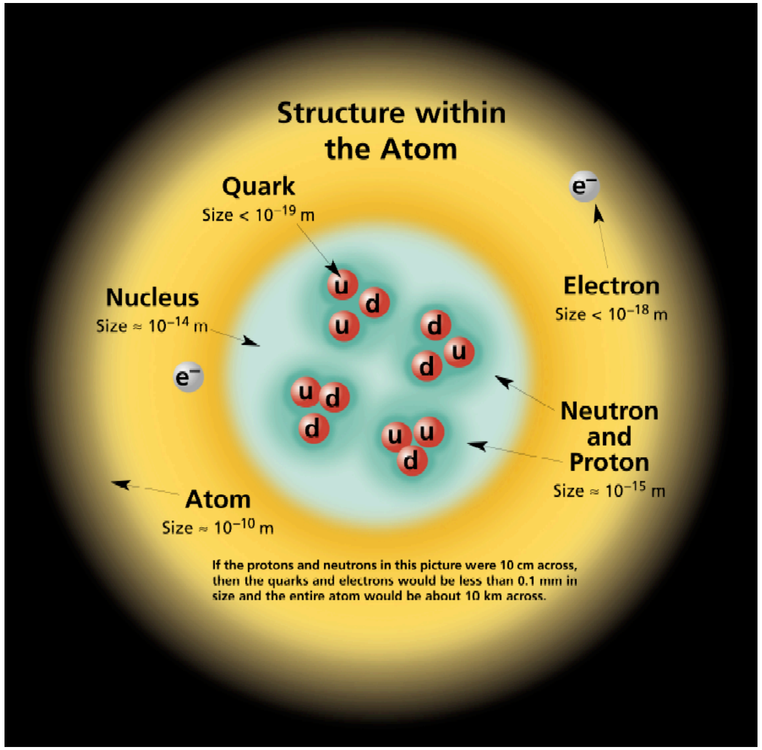
Particle Physics

There are two basic questions in Particle Physics:

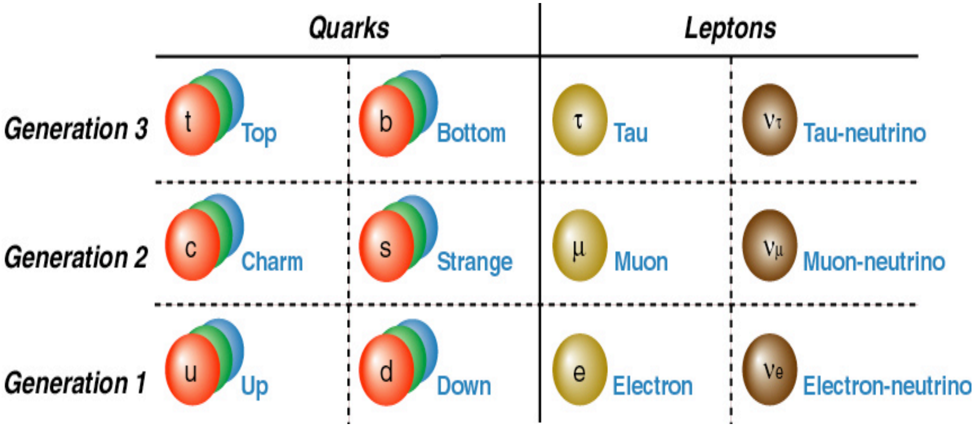
- What are the fundamental constituents of matter?
- What are the fundamental interactions between them?

Elementary Particles

1) Leptons and quarks are the fundamental constituents of matter



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

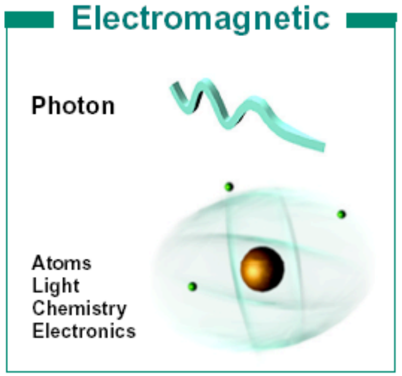


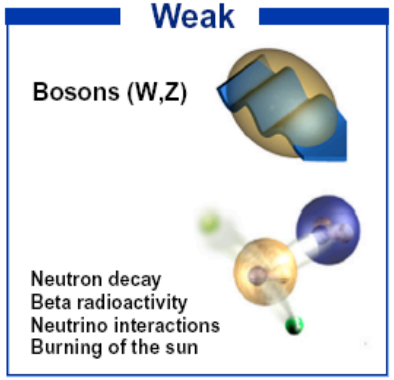
	Quarks		Leptons	
Generation 3	t Top	b Bottom	τ Tau	ν_τ Tau-neutrino
Generation 2	c Charm	s Strange	μ Muon	ν_μ Muon-neutrino
Generation 1	u Up	d Down	e Electron	ν_e Electron-neutrino

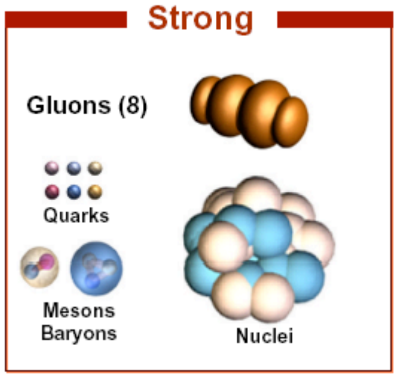
Leptons and quarks together are called fermions

Basic Forces

2) There exist four fundamental interactions between the matter particles



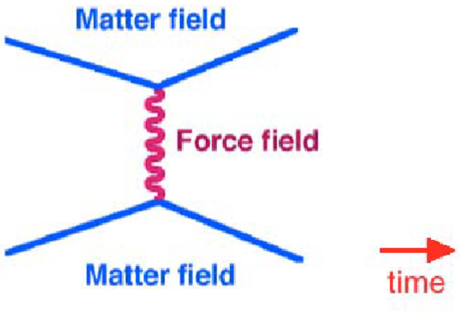




The forces are communicated to the matter particles by means of messenger particles

⇩

bosons



Messenger particles

Interaction	Messenger particle	Range (m)	Relative Strength	Example
Strong	gluon	10^{-15}	1	proton (quarks)
Electro-magnetic	photon	infinite	$< 10^{-2}$	atoms, chemistry
Weak	$W^{\pm}$, Z boson	$< 10^{-17}$	10^{-5}	radioactivity, sun
Gravity	graviton?	infinite	10^{-38}	solar system, galaxies

The Standard ~~Model~~ Theory

The Standard Model (SM) describes the fundamental laws of the Universe in terms of only a few input parameters

- Such a theory must include the two pillars of modern physics: (special) relativity and quantum mechanics

→ relativistic quantum field theory

- (Local) gauge symmetries dictate the form of the fundamental interactions:

U(1) x SU(2) x SU(3)

(works for 3 of the 4 forces, gravity is missing)

Has been tested to very high precision

Why we're still not happy with it

Still many open questions:

- What about particle masses? Does the Higgs boson exist and what is its mass? And why is $\mu^2 < 0$?
- Can gravity be included in the theory?
- Can the forces be unified to one fundamental force?
- Why is $M_{W,Z} << M_{Planck}$? (hierarchy problem)
- What is dark matter?
- What about neutrino masses?
- Are there 3+1 = 4 dimensions or more? …

Where are the masses?

Problem with the Standard Model: all particles are massless.
Introduction of mass terms ruins the gauge invariance. Oops.

Solution proposed by Brout-Englert-Higgs:

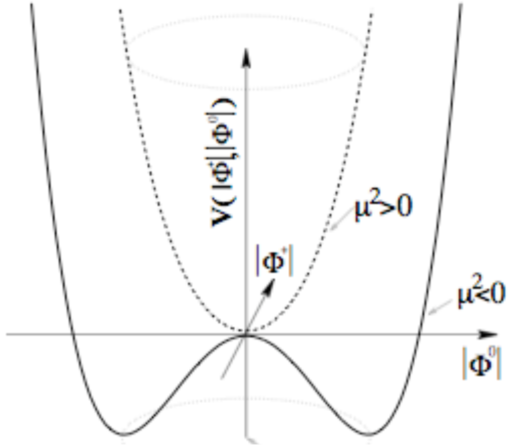
- assume the existence of a scalar field that pervades the universe
- particles interacting with this field acquire mass – the stronger the interaction, the larger the mass
- the particle associated with the Higgs field is the Higgs boson

EWSB mechanism

More technically: scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential: $V(\phi) = \mu^2 \left|\Phi^\dagger\Phi\right| + \lambda \left|\Phi^\dagger\Phi\right|^2, \quad \lambda > 0$

$\mu^2 < 0$: spontaneous symmetry breaking



minimum of the potential at

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$

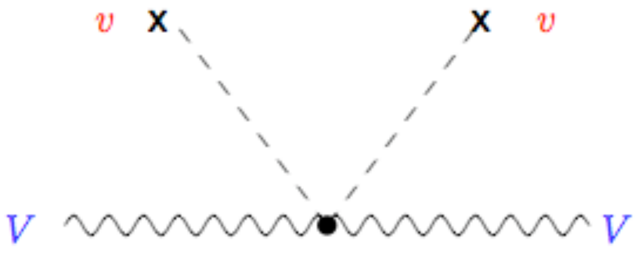
Why is $\mu^2 < 0$? "Because it is."

Either sign of μ^2 is possible in principle; there is no preference.

EWSB(2)

$\mathcal{L}_{\rm Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi);$ unitary gauge: $\Phi = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$

$VV\Phi\Phi$ coupling:



$\Rightarrow$ VV mass terms: $\frac{1}{2} g_2^2 v^2 \equiv M_{\rm W}^2, \quad \frac{1}{2}(g_1^2 + g_2^2) v^2 \equiv M_{\rm Z}^2$

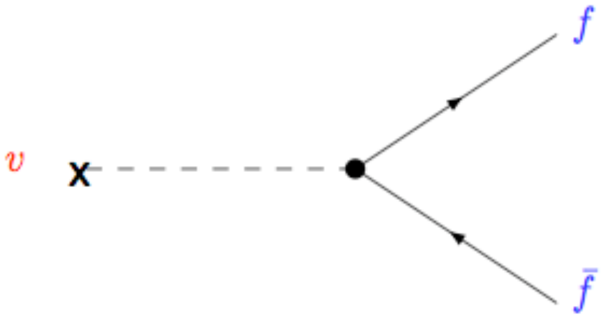
3 components of Higgs doublet

$\longrightarrow$ longitudinal components of $W^\pm$, Z

H elementary scalar field, Higgs boson

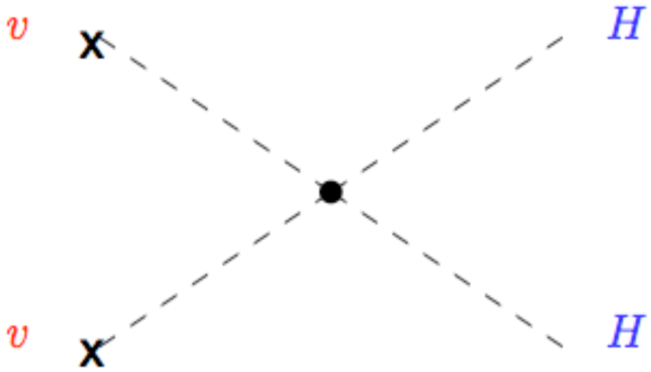
EWSB(3)

Fermion mass terms: Yukawa couplings



$m_f = v\, g_f$ free parameters

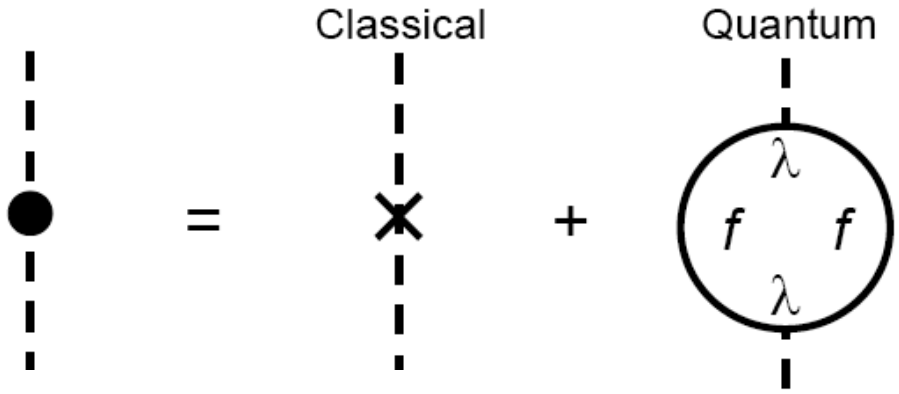
Mass of the Higgs boson: self-interaction



$M_{\mathrm{H}} = v\sqrt{\lambda}$ free parameter

⇒ Higgs couplings proportional to masses of the particles

Hierarchy problem



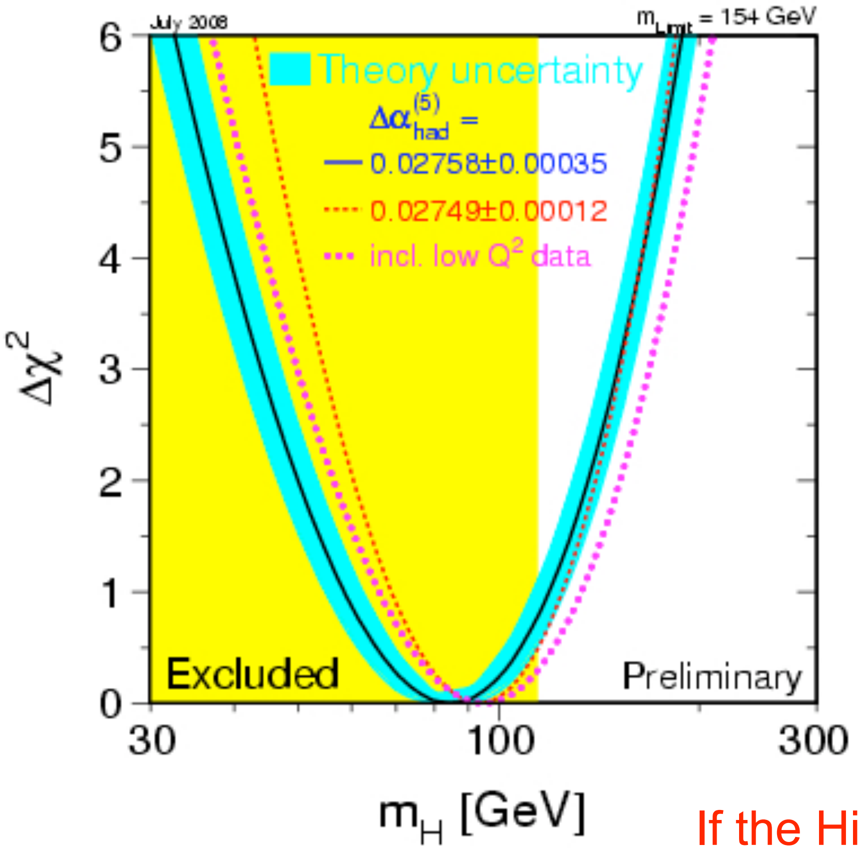
$$\mu^2 = \mu^2_{\mathrm{bare}} - \frac{1}{16\pi^2}\lambda^2\Lambda^2$$

In the SM, m_h is naturally ~ Λ, the largest energy scale

m_h ~ 100 GeV, Λ ~ 10^{19} GeV → cancellation of 1 part in 10^{34}

Higgs mass limits

From LEP and Tevatron experiments:



Direct limit M_H > 114.4 GeV

Indirect constraints < 154 GeV (95%CL)

Preferred value:

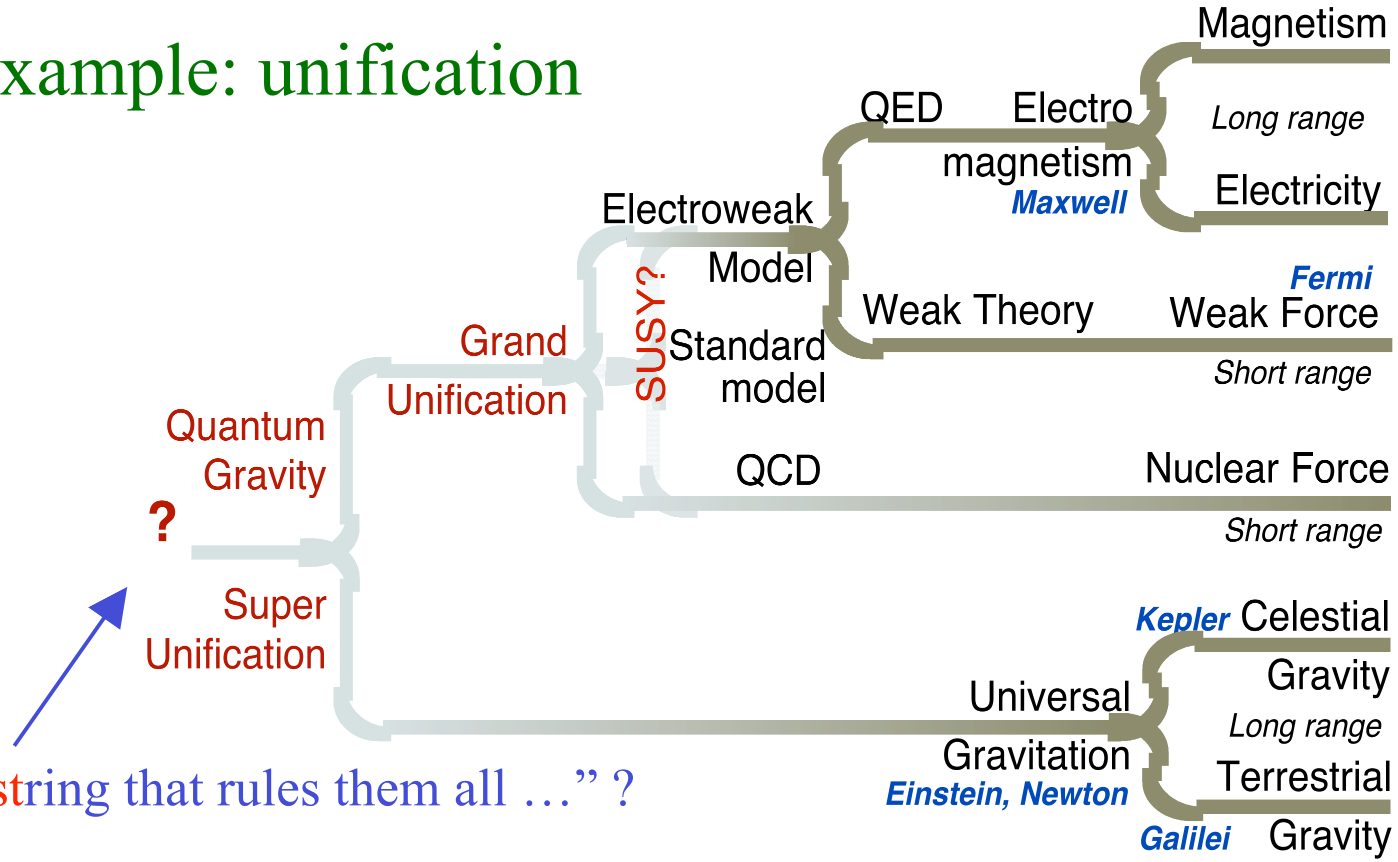
$M_H=84^{+34}_{-26}$ GeV

Significant tension!

If the Higgs boson exists, we will find it with the LHC

Unification

Example: unification



Magnetism

Long range

Electricity

QED

Electro magnetism

Maxwell

Electroweak Model

Fermi

Weak Theory

Weak Force

Short range

SUSY?

Standard model

Grand Unification

QCD

Nuclear Force

Short range

Quantum Gravity

?

Super Unification

Kepler Celestial Gravity

Long range

Universal Gravitation

Einstein, Newton

Terrestrial

Galilei Gravity

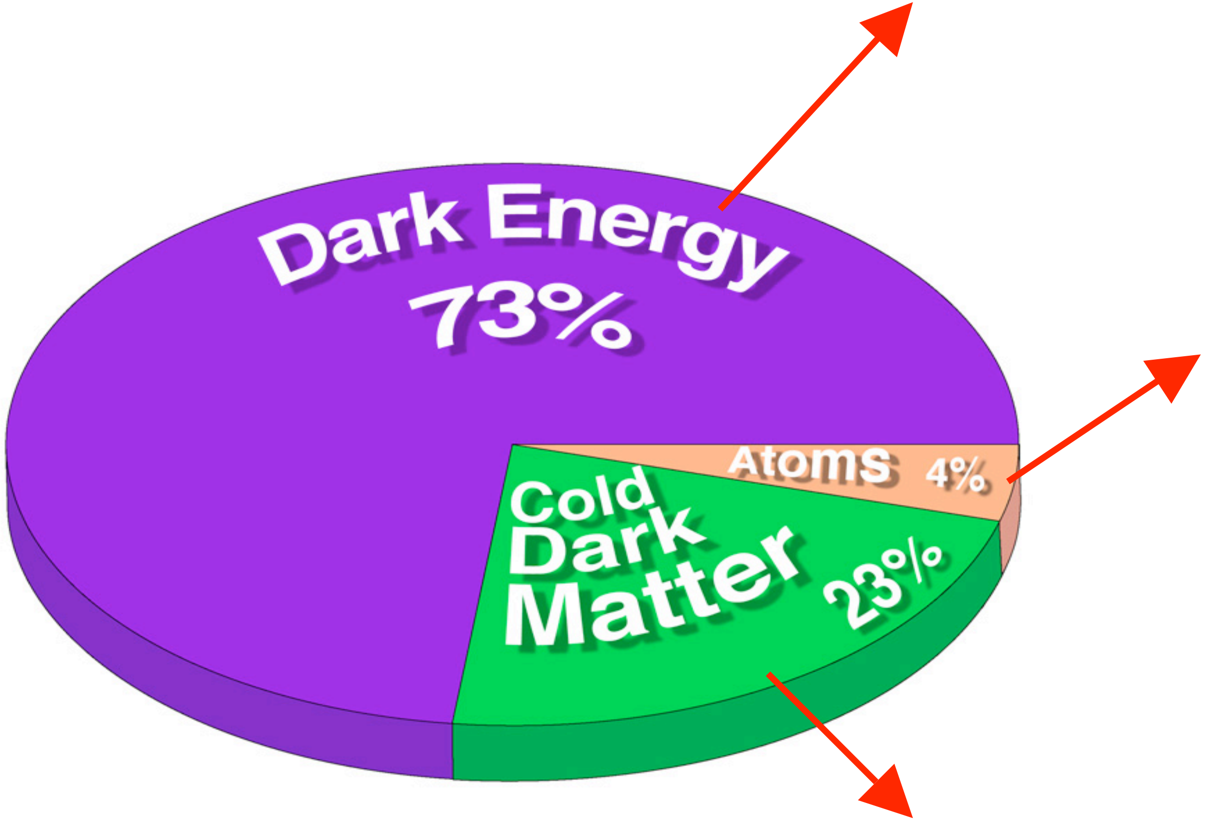
“one string that rules them all …” ?

Theories:

STRINGS? **RELATIVISTIC/QUANTUM** **CLASSICAL**

Dark matter and energy

Content of the Universe:



Main evidence: CMB radiation

Main evidence:
telescopic observations

Main evidence: rotation curves
Also “direct” observation

How to go beyond the SM?

A good BSM model should:

- incorporate the SM and extend it
- solve one or several of the SM defects
- describe all (present and future) experimental results

Supersymmetry

A possible extension of the SM :

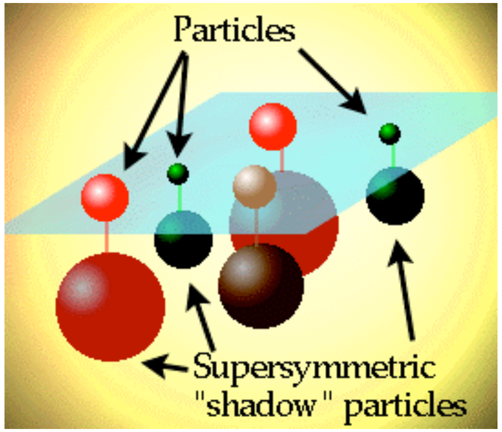
a symmetry between fermions and bosons
= supersymmetry

Solves several problems at once:

- dark matter candidate (= lightest supersymmetric particle)
- opening towards a theory of gravity
- unification of gauge couplings
- hierarchy problem
- allows to explain why the Higgs mechanism works

New sparticles

Need to introduce new particles :



leptons (f)	sleptons (b)	
quarks (f)	squarks (b)	
gauge bosons (b)	gauginos (f)	neutralinos
Higgs bosons (b)	higgsinos (f)	charginos

(f = fermion, b = boson)

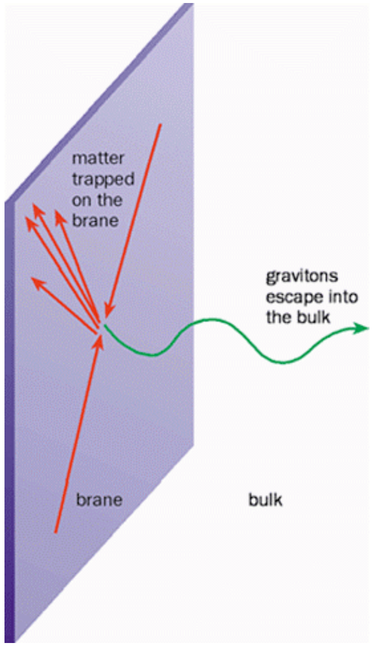
Extra dimensions?

What if the Planck scale is actually IS around the EW scale …
but it just looks much further because there are additional
(curved) extra dimensions?

→ New phenomenology of KK states and/or particles that can travel in the extra dimension …

$$M_{planck} = M_*^{n-2} \quad R^n$$

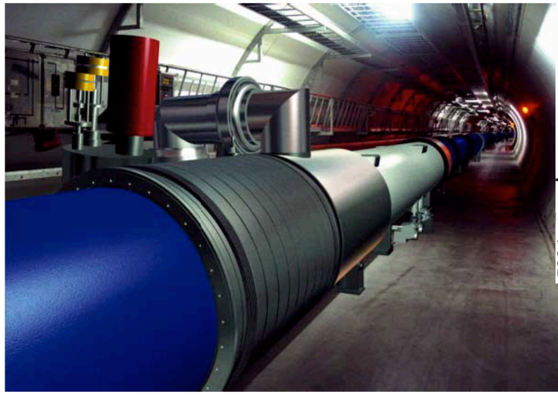
with n the number of the extra dimension
and R their radius

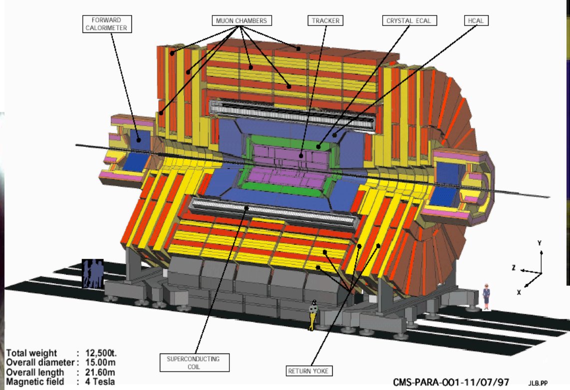


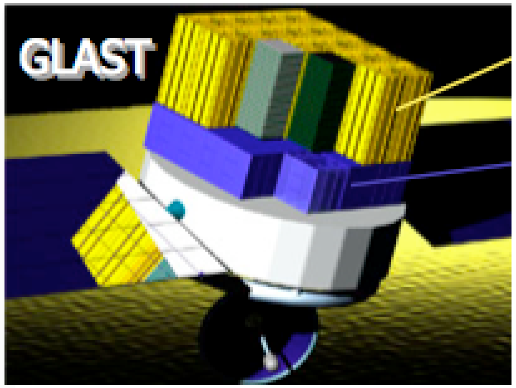
Experiment

… and how do we know any of the above is right?

→ Experimental search for new physics





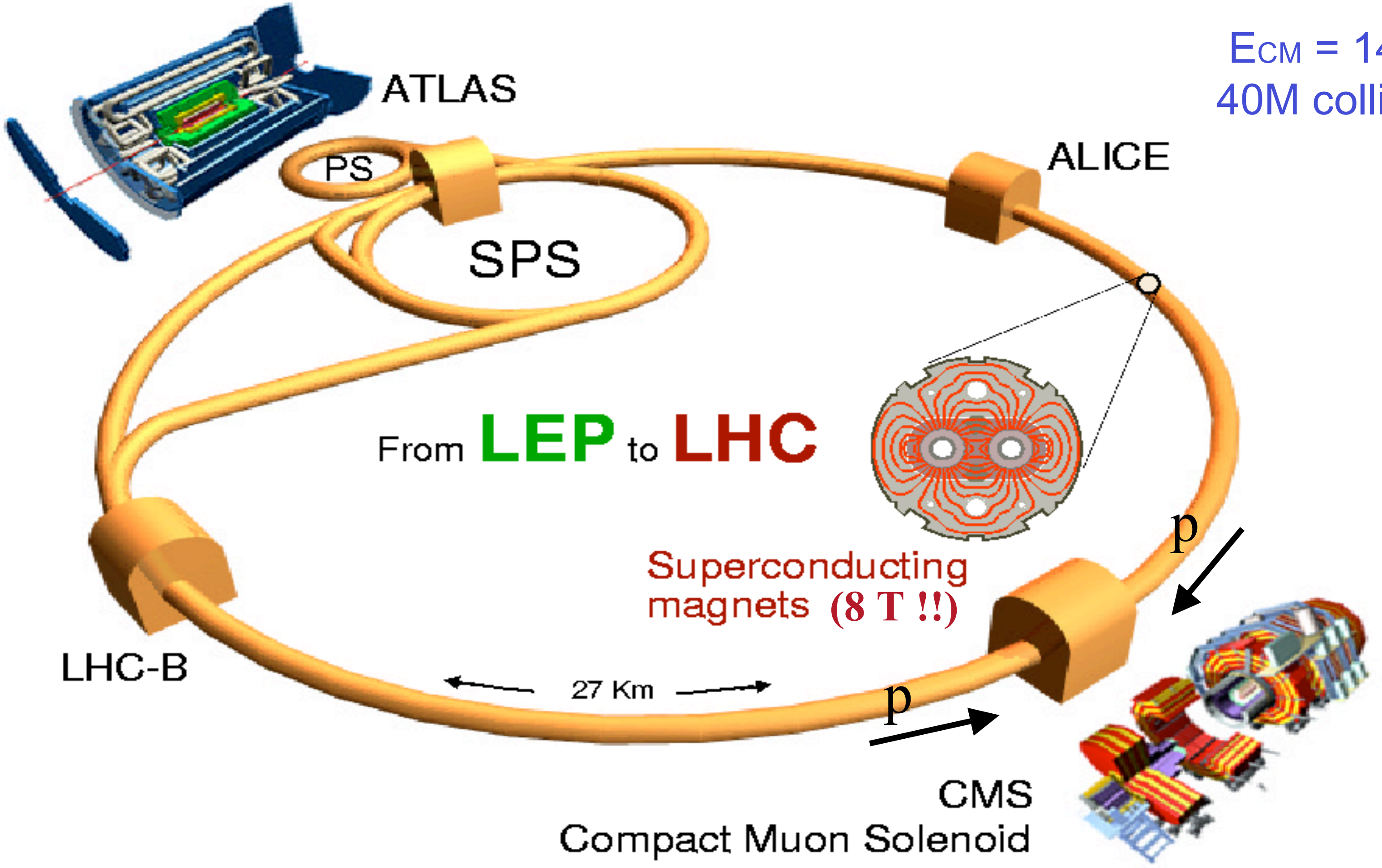


Large Hadron Collider

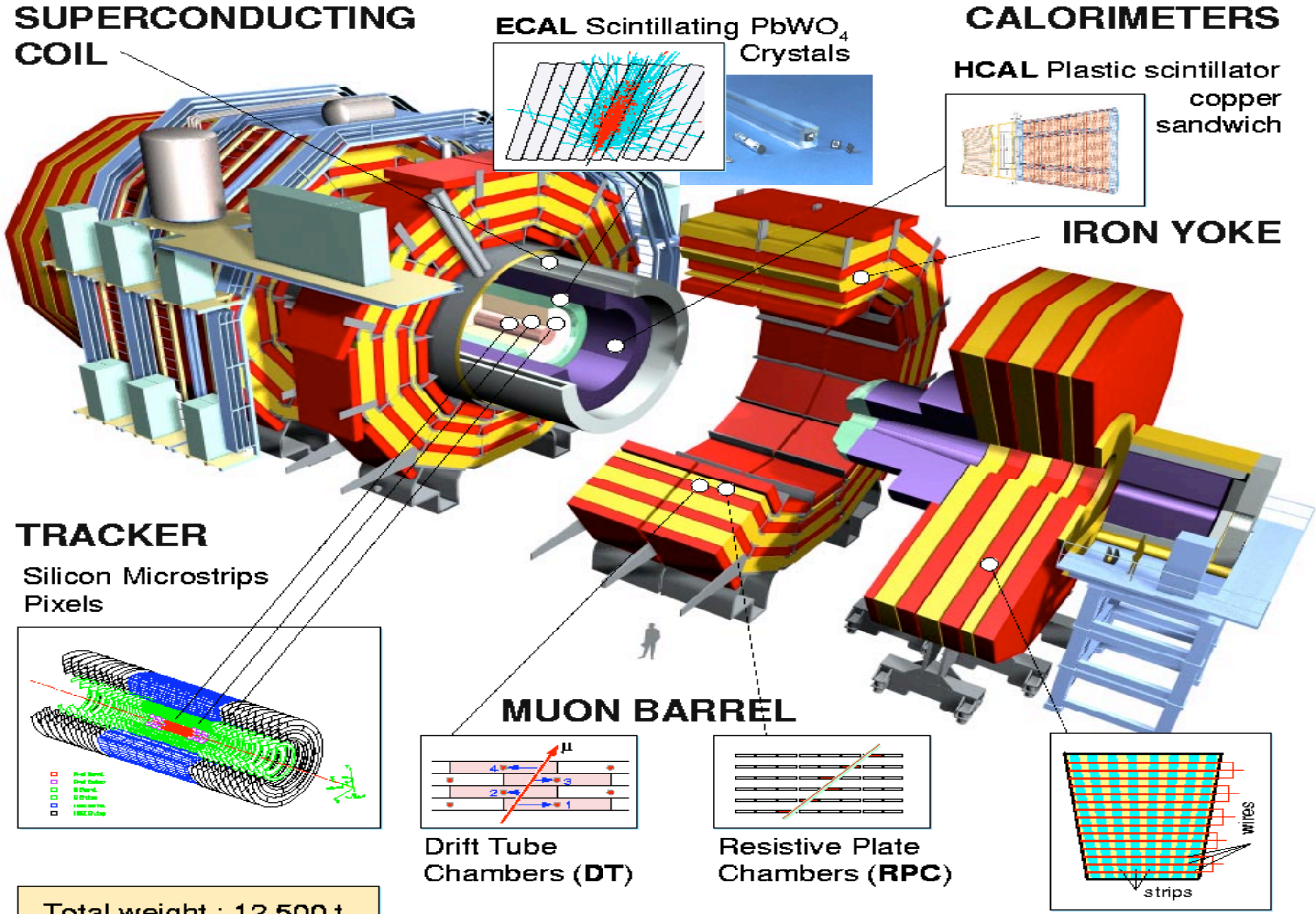
LHC ring:
27 km circumference

The LHC Collider

E_{CM} = 14 TeV
40M collisions/s



The Compact Muon Solenoid



SUPERCONDUCTING COIL

ECAL Scintillating $PbWO_4$ Crystals

CALORIMETERS

HCAL Plastic scintillator copper sandwich

IRON YOKE

TRACKER

Silicon Microstrips
Pixels

MUON BARREL

Drift Tube Chambers (**DT**)

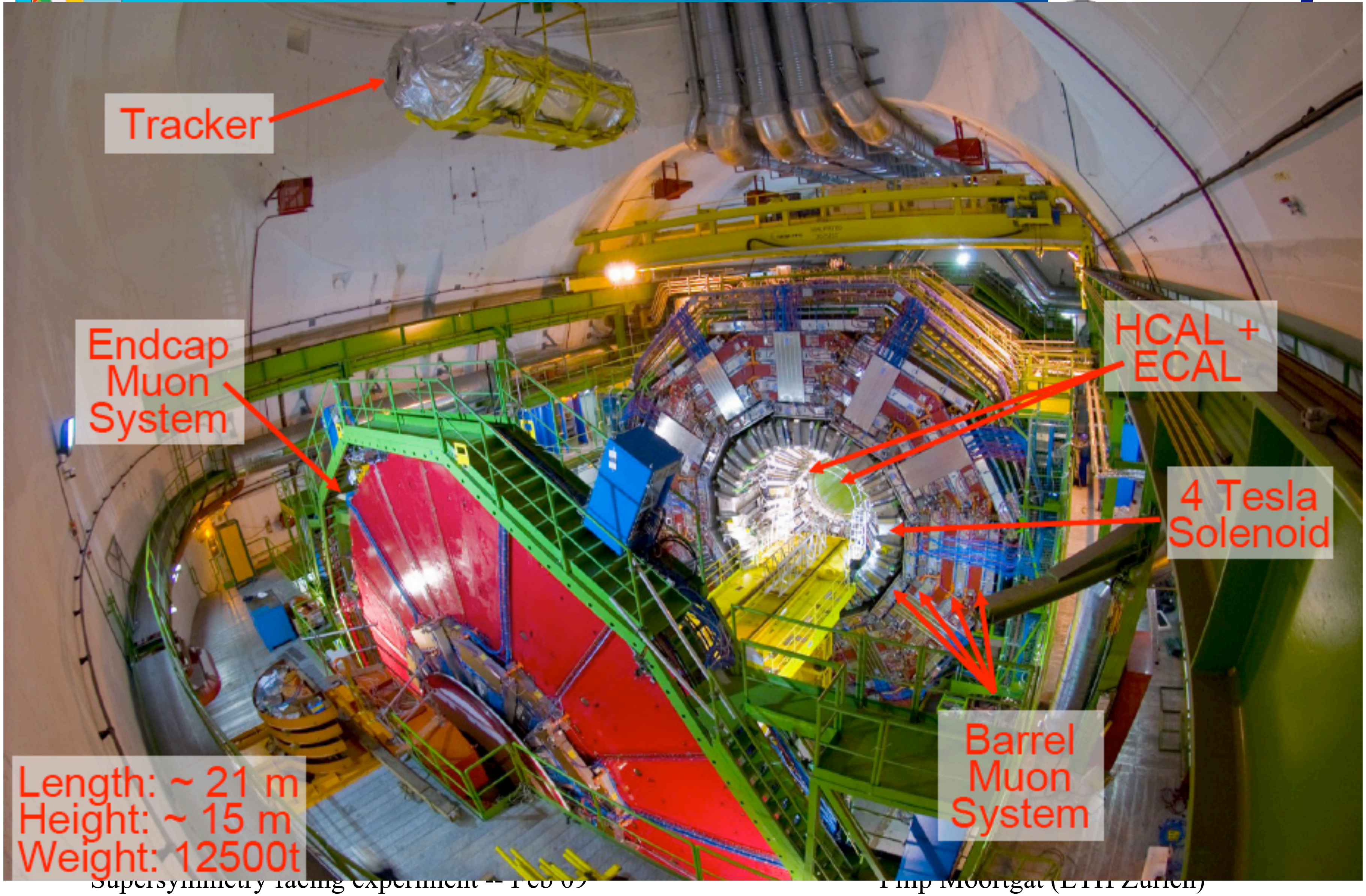
Resistive Plate Chambers (**RPC**)

Total weight : 12,500 t
Overall diameter : 15 m
Overall length : 21.6 m
Magnetic field : 4 Tesla

MUON ENDCAPS

Cathode Strip Chambers (**CSC**)
Resistive Plate Chambers (**RPC**)

The CMS detector



Event selection at the LHC

Per year, the LHC will provide about 10^{16} *pp* collisions.

An observation of ~ 10 events could be a discovery of new physics.

→ One has to find these 10 events among 10^{16} non-interesting ones!!

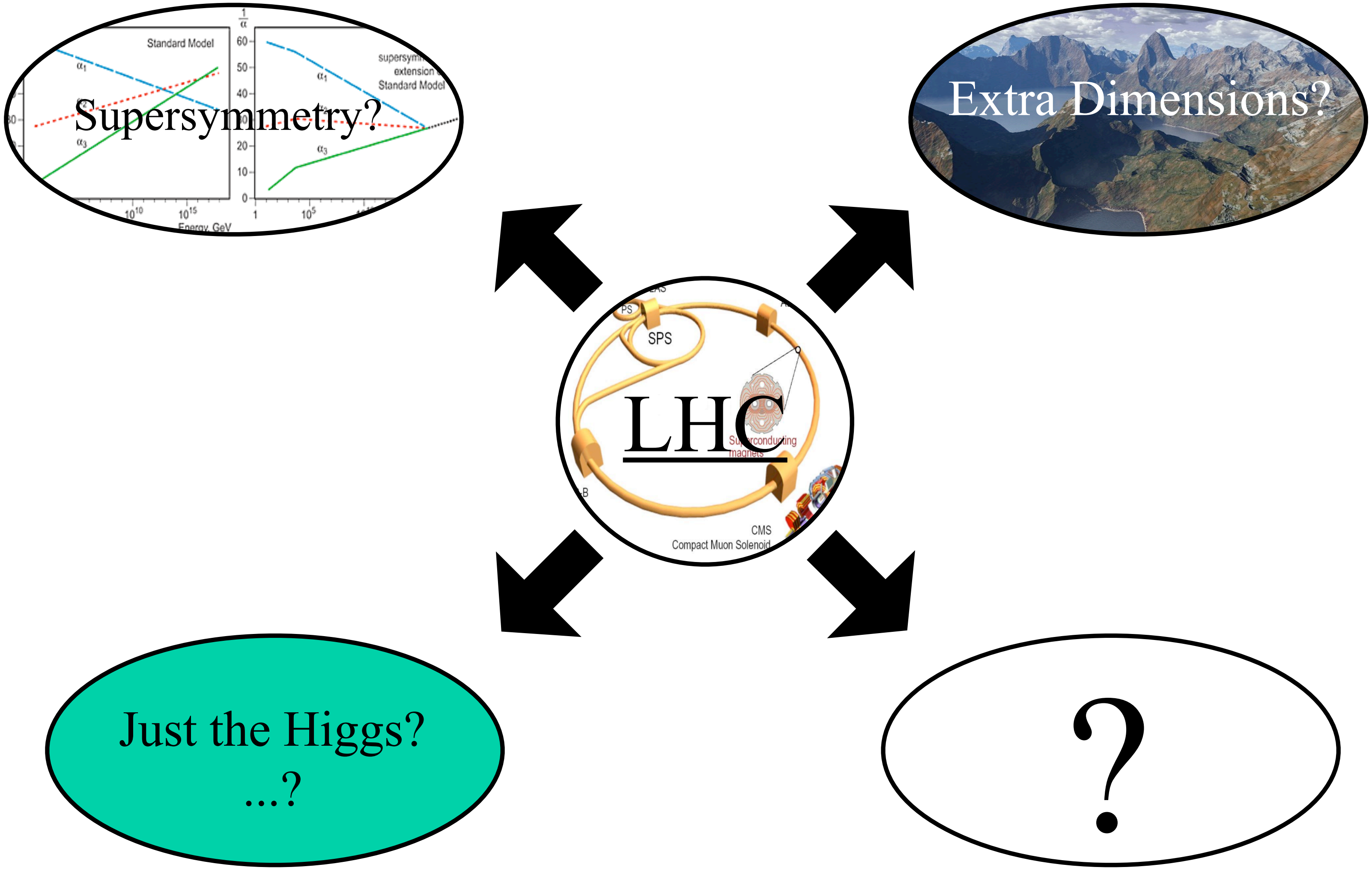
Searching for a needle in a hay stack?

- typical needle: 5 mm^3
- typical haystack: 50 m^3

→ needle : haystack = 1 : 10^{10}

→ Looking for new physics at the LHC is like looking for a needle in 100000 haystacks …

What will the LHC bring?



Supersymmetry?

Extra Dimensions?

LHC

Just the Higgs?
...?

?

Supersymmetry algebra

Supersymmetry

Algebra:

$$\{ Q_\alpha , \bar{Q}_\beta \}_+ = 2 \gamma^m_{\alpha\beta} P_m$$

$$\{ Q_\alpha , Q_\beta \}_+ = [P_m , Q_\alpha] = [P_m , P_n] = 0$$

Q_α : spinorial $\quad \bar{Q} = Q^\dagger \gamma^0$

$m = 0, 1, 2, 3$

J. Wess

Consequences

Q_α changes spin of particle by $\frac{1}{2}$

$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle$, $Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle$

Consider fermionic state $|f\rangle$ with mass m:

$\Rightarrow$ bosonic state $|b\rangle = Q_\alpha|f\rangle$

$P^2|f\rangle = m^2|f\rangle$

$\Rightarrow P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$

$\Rightarrow$ For each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ States are paired bosonic $\leftrightarrow$ fermionic

Supermultiplets

Number of degrees of freedom is constrained:

$$\boxed{n_F = n_B}$$

SM Fermion
(real spinor)

SUSY Scalar
(complex field)

$$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

SM fermions ⇒ sFermions
SM Higgs (complex scalar) ⇒ higgsino, L/R with $I = 1/2$

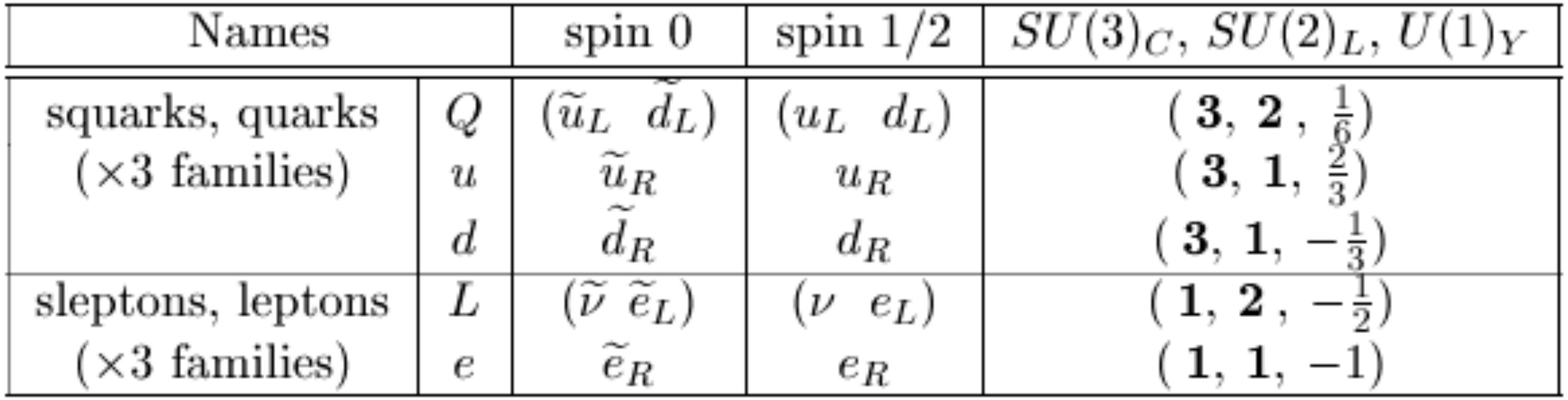
SM Gauge boson
$(\text{vector},\ \text{m} = 0)$

SUSY Fermion
(gaugino)

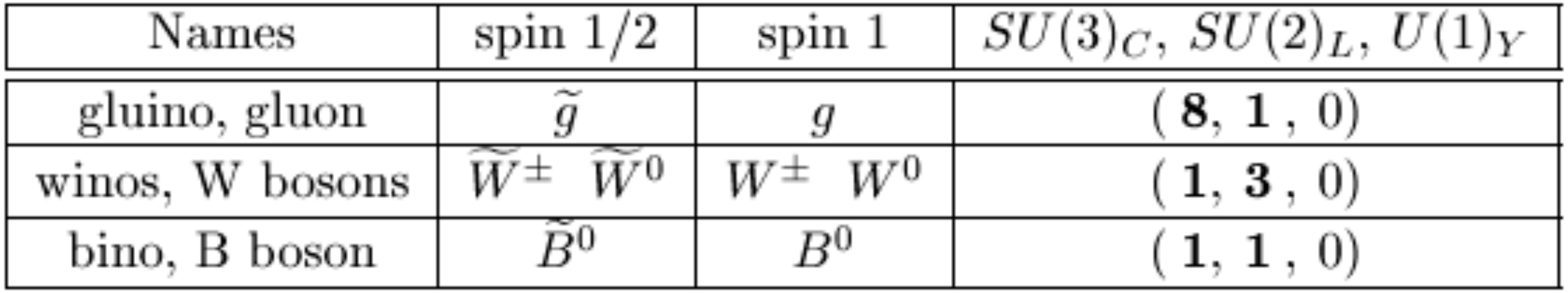
$$\begin{pmatrix} +1 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$$

Supermultiplet components have same gauge transformation properties: gauge bosons and gauginos are self-conjugate

Quantum numbers



Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks (×3 families)	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
	u	$\widetilde{u}_R$	u_R	$(\ \mathbf{3},\ \mathbf{1},\ \frac{2}{3})$
	d	$\widetilde{d}_R$	d_R	$(\ \mathbf{3},\ \mathbf{1},\ -\frac{1}{3})$
sleptons, leptons (×3 families)	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
	e	$\widetilde{e}_R$	e_R	$(\ \mathbf{1},\ \mathbf{1},\ -1)$



Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\ \ \widetilde{W}^0$	$W^{\pm}\ \ W^0$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

$$Q = I_3 + Y/2$$

Higgs doubling

- SUSY requires 2 Higgs doublets to cancel anomalies and to give mass to both up- and down-type particles

- E.g., anomaly cancelation requires $\Sigma Y^3 = 0$, where Y is hypercharge and the sum is over fermions. This holds in the SM

- SUSY adds an extra fermion with Y = –1:

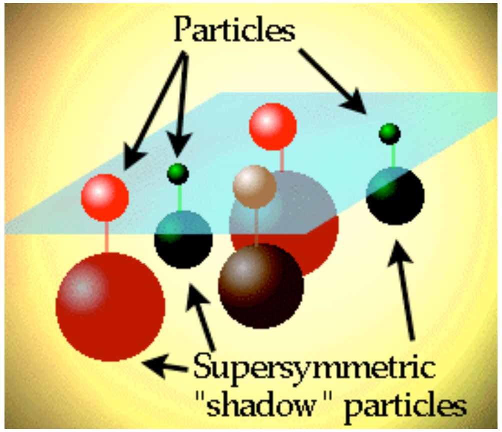
$$\begin{pmatrix} h^0 \\ h^- \end{pmatrix} \equiv \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$$

- To cancel the anomaly we add another Higgs doublet with Y = +1:

$$\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$$

New sparticles

Need to introduce new particles :



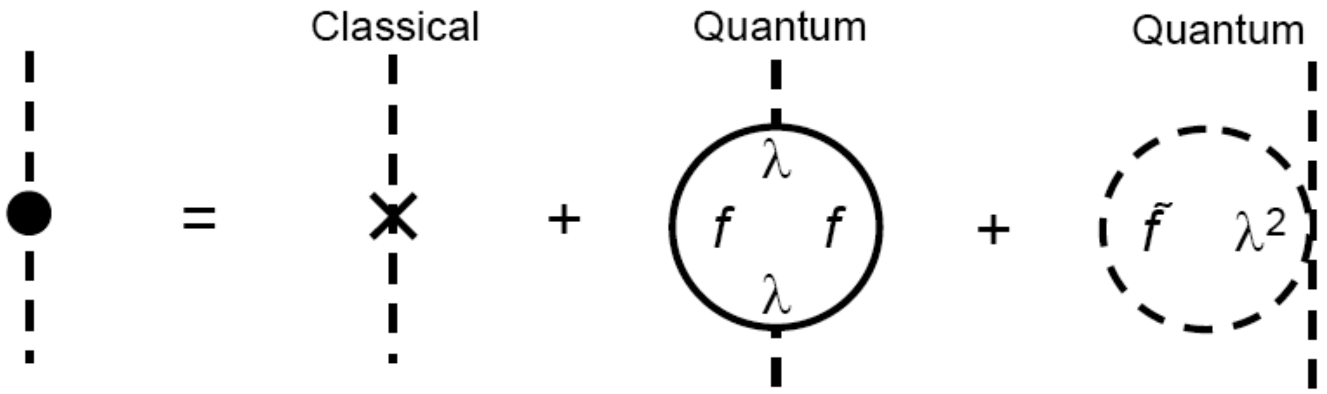
leptons (f)	→	sleptons (b)	$(\tilde{l}, \tilde{q})$
quarks (f)		squarks (b)	
gauge bosons (b)		gauginos (f)	} neutralinos
Higgs bosons (b)		higgsinos (f)	} charginos

$$(\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0)$$

$$(\chi_1^\pm, \chi_2^\pm)$$

(f = fermion, b = boson)

Hierarchy solution



$$\mu^2 = \mu^2_{\text{bare}} \underbrace{- \frac{1}{16\pi^2}\lambda^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2}$$

$$+ \frac{1}{16\pi^2}\lambda^2(m_{\tilde{f}}^2 - m_f^2)\ \ln(\Lambda/m_h)$$

Dependence on Λ is softened to a logarithm

SUSY solves the hierarchy problem, even if broken, provided that superpartner masses are < O(1 TeV)

Gauge coupling evolution

$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right), \quad \beta = \text{const.}$$

$g(\mu)$: "running coupling", $m(\mu)$: "running mass"

$\Rightarrow$ effective coupling $\alpha(Q)$ varies with scale Q

$$\alpha_i = g_i^2 / 4\pi$$

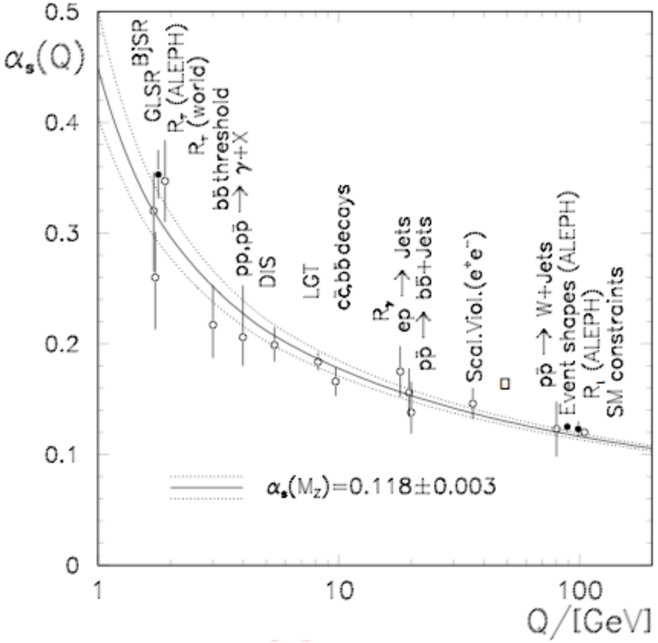
E.g. QED: $$\alpha(Q) = \frac{\alpha(Q_0)}{1 + 4\pi\beta\,\alpha(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta < 0$ in QED: $\beta \equiv b/(16\pi^2)$, $b = -4/3$ (1-loop)

$\Rightarrow$ $\alpha(Q)$ increases for increasing Q ($Q > Q_0$),
$\alpha(m_e) \approx 1/137$, $\alpha(M_Z) \approx 1/128$

Gauge coupling evolution(2)

E.g. QCD:



$$b^{\mathrm{QCD}} = \frac{1}{3}(11N_C - 2n_f) > 0 \text{ for } n_f = 6$$

$\Rightarrow \alpha_s(Q) \to 0$ for $Q \to \infty$: "asymptotic freedom"

For large Q: perturbation theory applicable, description in terms of quarks and gluons useful

$\alpha_s(Q)$ becomes strong for small Q:

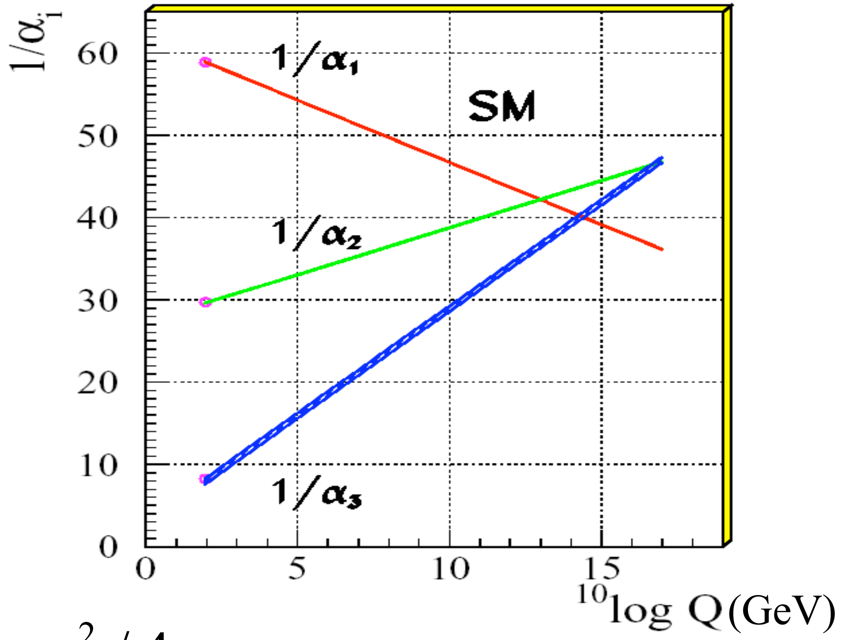
quarks and gluons bound together to form colourless hadrons

$\Rightarrow$ "confinement"

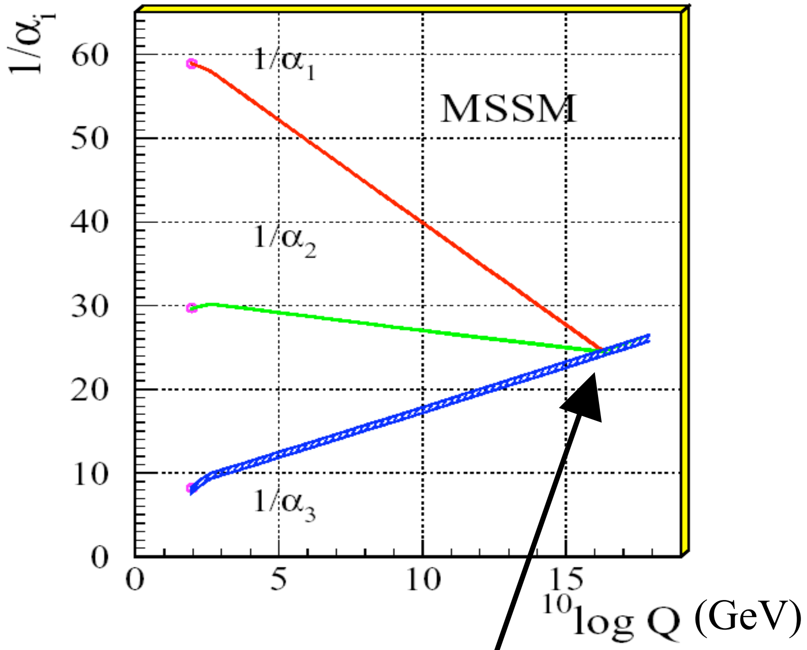
$$\alpha_s(Q) = \frac{12\pi}{(11N_C - 2n_f)\ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

Gauge coupling unification

SM



MSSM



$$\alpha_i = g_i^2 / 4\pi$$

(1 = electromagnetic, 2 = weak, 3 = strong)

- meet in 1 point
- energy scale ok with proton decay

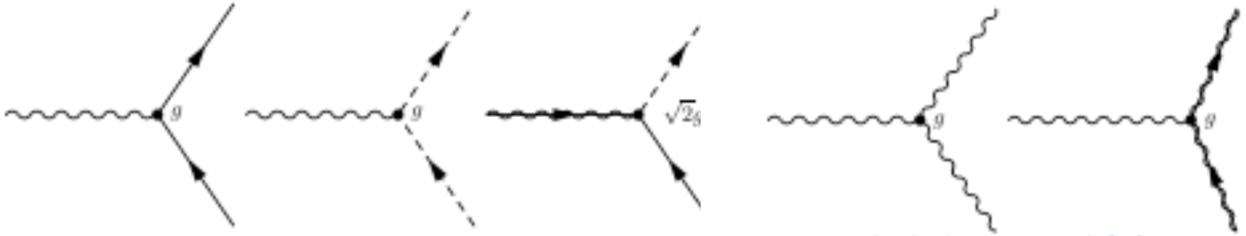
MSSM gauge couplings

SM multiplets ⟶ MSSM supermultiplets
by including superpartners differing by 1/2 unit in spin
Supermultiplets: Chiral = (ψ, ϕ) ; Gauge = (A, λ)

Same supermultiplet ⇒ same couplings in interactions
But: amplitudes must be scalar in spin space

Replace pair of SM particles by their superpartners

Gauge Interactions : (shorthand notation)
Trilinear interactions, as they determine production and decay



SM $(A\psi\psi)$ $(A\phi\phi)$ $(\lambda\phi\psi)$ SM (AAA) $(A\lambda\lambda)$

couplings:
$W^{\pm}(\tilde{W}^{\pm}) \propto g_2 I^3$
$\gamma(\tilde{\gamma}) \propto eQ = g_2 sin\theta_W Q$
$Z^0(\tilde{Z}^0) \propto g_2(I^3 - Q sin^2\theta_W)$

More formally: obtained from covariant derivatives

Supersymmetry breaking

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- Unbroken MSSM
    - Unbroken SUSY introduces new interactions but no new parameters
    - All particles have the same mass
    - Superpartners must be heavier than SM particles → SUSY broken
- Soft SUSY breaking (soft = no quadratic divergences)

$$\delta V = \Sigma_{\tilde{q},\tilde{l},H_{d,u}} m^2{}_{0,i} \,|\, \Phi_i \,|^2 + m_{1/2,a}\lambda_a\lambda_a + h.c.$$

$$+ A_{0,e}\widetilde{L}_L^i h_L \widetilde{E}_L^c H_d^j + A_{0,d}\widetilde{Q}_L^i h_D \widetilde{D}_L^c H_d^j + A_{0,u}\widetilde{Q}_L^i h_U \widetilde{U}_L^c H_u^j + B_0 \mu H_d H_u + h.c.$$

    - **m$_{0,i}$** are the scalar masses (matrix in generation space),
    - **m$_{1/2,a}$** are the gaugino masses,
    - **A$_{0,i}$** are the trilinear couplings,
    - μ is the Higgsino mass parameter
- Parametrization of our ignorance of the SUSY breaking mechanism

  → Effective Lagrangian to derive phenomenology

R-parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{V} = \mathcal{V}_{\mathrm{MSSM}} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^{i} L_i H_u}_{\text{violate lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijk} U_i D_j D_k}_{\text{violates baryon number}}$$

If both lepton and baryon number are violated

⇒ rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles ⇒ additional symmetry: “R parity”

$$R_p = (-1)^{3(B-L)+2S}$$

R-parity(2)

- SUSY particles can only be pair-produced, e.g. $gg \to \tilde{g}\tilde{g}$
- Decay of SUSY particles into SM particles + odd number of SUSY particles ⇒ Lightest SUSY particle (LSP) is stable
- All SUSY particles will eventually decay into LSP

  ⇒ Decay cascades of heavy SUSY particles
- LSP stable ⇒ some LSP must have survived from Big Bang

  ⇒ Weakly interacting massive particle ("WIMP")
  (otherwise ruled out from bounds on exotic isotopes etc.)

  ⇒ Candidate for cold dark matter in the Universe
- LSP neutral, uncoloured ⇒ leaves no traces in collider detectors ⇒ Typical SUSY signatures: "missing energy"

Simplest assumption: the Constrained MSSM

Assume universality at high energy scale ($M_{\mathrm{GUT}}, M_{\mathrm{Pl}}, \ldots$)

- Universal scalar masses: $\tilde{m}^2 = m_0^2$
- Universal gaugino masses: $M_i = m_{1/2}$ ("GUT relation")
- Universality of soft-breaking trilinear terms:

$$\mathcal{L}_{\mathrm{tri}} = A_0(H_U Q y_u \bar{u} + H_D Q y_d \bar{d} + H_D L y_l \bar{e})$$

$y_u, y_d, \ldots$ are the same matrices that appear in Yukawa couplings ("proportionality")

CMSSM(2)

Universality ansatz results in five parameters, if possible phases are ignored:

$$m_0^2,\ m_{1/2},\ A_0,\ b,\ \mu$$

Require correct value of M_Z

$\Rightarrow$ $|\mu|$, b given in terms of $\tan\beta \equiv v_u/v_d$, sign μ

$$\begin{aligned} \sin(2\beta) &= \frac{2b}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}, \\ m_Z^2 &= \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2. \end{aligned}$$

$\Rightarrow$ CMSSM characterised by

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\,\mu$$

SUSY breaking scenarios

“Hidden sector”: ⟶ Visible sector:

SUSY breaking MSSM

“Gravity-mediated”: SUGRA
“Gauge-mediated”: GMSB
“Anomaly-mediated”: AMSB
“Gaugino-mediated”

. . .

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

SUGRA

SUGRA: mediating interactions are gravitational

$\Leftrightarrow$ Connection of gravity and electroweak physics

Flavour off-diagonal and $\mathcal{CP}$-violating effects?

SUGRA with universality assumptions $\Rightarrow$ CMSSM, $\tilde{\chi}_1^0$ LSP

Other possibility:

Gravitino LSP, compatible with CDM, $\tilde{\tau}$ NLSP

$\tilde{\tau}$ could be long-lived charged particle, $\tilde{\tau} \rightarrow \tau\tilde{G}$, interesting phenomenology

CMSSW spectrum

SUGRA with universality assumptions ⇒ CMSSM

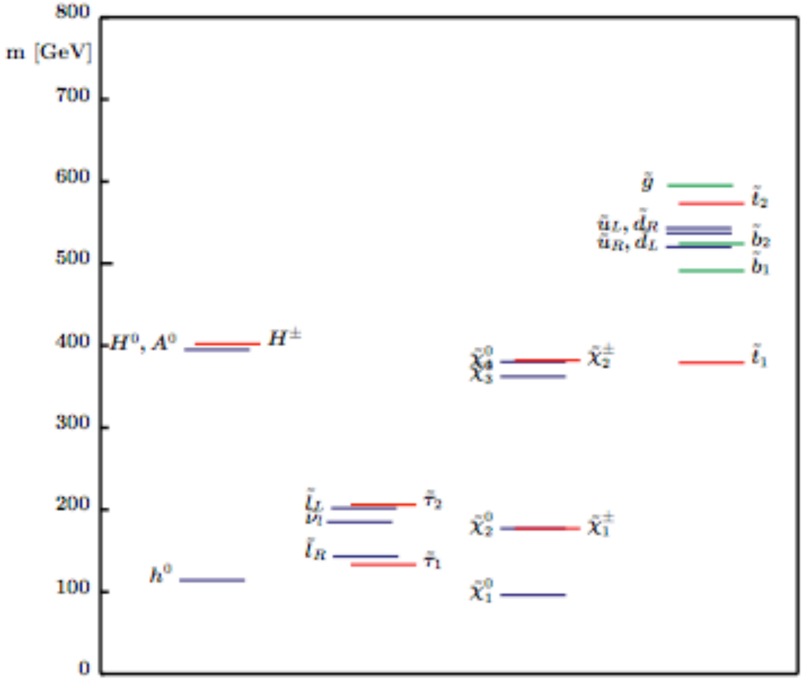
m_0, $m_{1/2}$, A_0: GUT scale parameters

⇒ Spectra from renormalisation group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

Gaugino masses run in same way as gauge couplings
⇒ gluino heavier than charginos, neutralinos

“Typical” CMSSM scenario (SPS 1a benchmark scen.):



Squark and slepton masses(1)

- First two families: start from m_0 at GUT scale
  - Yukawas are negligible
  - Running dominated by $m_{1/2}$ and α_is
  - Splitting by D-term (sfermion)^2(higgs)^2 after SU(2)xU(1) breaking
  - At weak scale (approx. formulae)

$$
\begin{aligned}
m^2(\tilde{u}_L) &= m_0^2 + 5.9 m_{1/2}^2 + 0.35 \cos 2\beta M_Z^2 \\
m^2(d_L) &= m_0^2 + 5.9 m_{1/2}^2 - 0.42 \cos 2\beta M_Z^2 \\
m^2(\tilde{u}_R) &= m_0^2 + 5.5 m_{1/2}^2 + 0.15 \cos 2\beta M_Z^2 \\
m^2(d_R) &= m_0^2 + 5.4 m_{1/2}^2 - 0.07 \cos 2\beta M_Z^2 \\
m^2(\tilde{e}_L) &= m_0^2 + .49 m_{1/2}^2 - 0.27 \cos 2\beta M_Z^2 \\
m^2(\tilde{\nu}_L) &= m_0^2 + .49 m_{1/2}^2 + 0.50 \cos 2\beta M_Z^2 \\
m^2(\tilde{e}_R) &= m_0^2 + .15 m_{1/2}^2 - 0.23 \cos 2\beta M_Z^2
\end{aligned}
$$

D-term sum rule: $m_{\tilde{e}_L}^2 - m_{\tilde{\nu}_L}^2 = m_{\tilde{d}_L}^2 - m_{\tilde{u}_L}^2 = -M_W^2 \cos 2\beta$

note that m_{gluino} is at most ~1.2 m_{squark} (for m_0=0)

Squark and slepton masses(2)

- Third family: Yukawa couplings cannot be neglected
  - At weak scale (tanβ = 10)

$$\begin{aligned}
m^2(\tilde{t}_L) &= m_t^2 + .69m_0^2 + 5.0m_{1/2}^2 + 0.35\cos 2\beta M_Z^2 \\
m^2(\tilde{b}_L) &= m_b^2 + .69m_0^2 + 5.0m_{1/2}^2 - 0.42\cos 2\beta M_Z^2 \\
m^2(\tilde{t}_R) &= m_t^2 + .33m_0^2 + 3.7m_{1/2}^2 + 0.15\cos 2\beta M_Z^2 \\
m^2(\tilde{b}_R) &= m_b^2 + m^2(\tilde{d}_R)
\end{aligned}$$

  - → Yukawa couplings decrease mass
  - Also L-R mixing: SUSY breaking and F-term

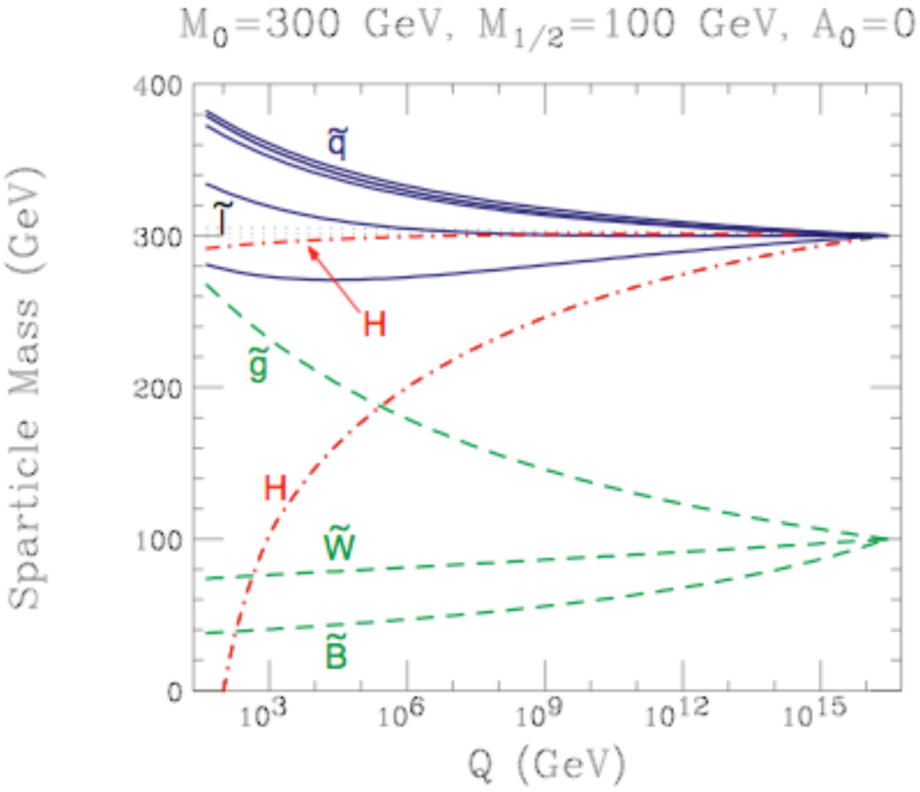
$$\begin{pmatrix} m^2(\tilde{t}_L) & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m^2(\tilde{t}_R) \end{pmatrix}$$

  - → $m^2_{\tilde{t}_{1,2}} = \frac{1}{2}\left[m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} \mp \sqrt{(m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R})^2 + 4m_t^2(A_t - \mu\cot\beta)^2}\right]$
  - Similar for sbottom/stau replacing cotβ by tanβ

Lightest squark: $\tilde{t}_1(\textit{low}\tan\beta), \tilde{b}_1(\textit{high}\tan\beta)$

Radiative EWSB

Universal boundary conditions at GUT scale, renormalisation group running down to weak scale



large corrections from top-quark Yukawa coupling

$\Rightarrow m^2_{H_u}$ driven to negative values

$\Rightarrow$ ew symmetry breaking

emerges naturally at scale $\sim 10^2$ GeV for $100\ \mathrm{GeV} \lesssim m_t \lesssim 200\ \mathrm{GeV}$

Gaugino mass RGEs

- If universal gaugino masses at GUT scale

→ Renormalization Group Equations (RGE):

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}$$

- Weak scale values:

$$\boxed{M_3 = M_{\tilde{g}} \cong 2.7 m_{1/2}}$$

$$\boxed{M_2 \cong 0.8 m_{1/2}} \qquad \boxed{M_1 \cong 0.4 m_{1/2} \cong 0.5 M_2}$$

- SU(3) unbroken: M_3=physical gluino mass, up to QCD corrections
- After SU(2)xU(1) breaking, Wino and Bino masses are mixed

Chargino/neutralino masses

- Gauginos mix with higgsinos
  - Off diagonal coupling $(\lambda, \varphi, \psi) \Rightarrow (\widetilde{W}^{+} H_d^0 \widetilde{H}_d^-)$
- Mass matrices:
  - Charginos (2x2) matrix: M_2, μ, $\tan\beta$
  - Neutralinos (4x4) matrix: M_1, M_2, μ, $\tan\beta$
  - In limit where neglect terms in $\tan\beta$, simplify to

$$M(\chi_1^{\pm}) \approx M_2, M(\chi_2^{\pm}) \approx \mu$$

$$M(\chi_1^0) \approx M_1, M(\chi_2^0) \approx M_2, M(\chi_3^0) \approx M(\chi_4^0) \approx \mu$$

  - → two extreme cases:

$\mu >> M_2$ ⟶ Lightest χ are "gaugino-like"

$M_2 >> \mu$ ⟶ Lightest χ are "higgsino-like"

  - In MSUGRA (+GMSB): usually gaugino-like, χ_1^0=Bino, χ_2^0,$\chi_1^{\pm}$=Winos

GMSB

Gaugino and scalar masses arise from loop contributions involving messenger fields

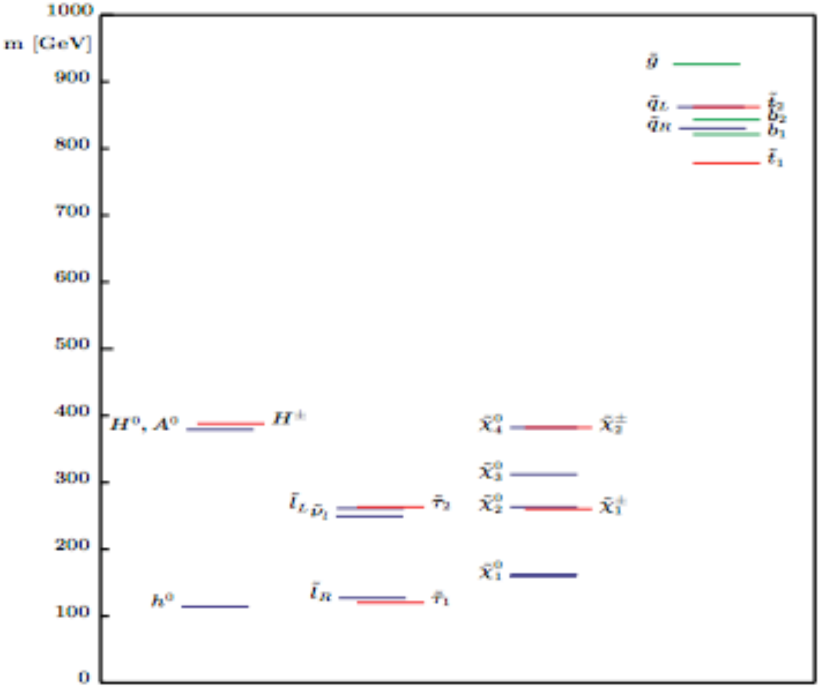
⇒ Typical mass hierarchy in GMSB scenario between strongly interacting and weakly interacting particles $\sim \alpha_3/\alpha_2/\alpha_1$

LSP is always the gravitino (also possible in SUGRA)

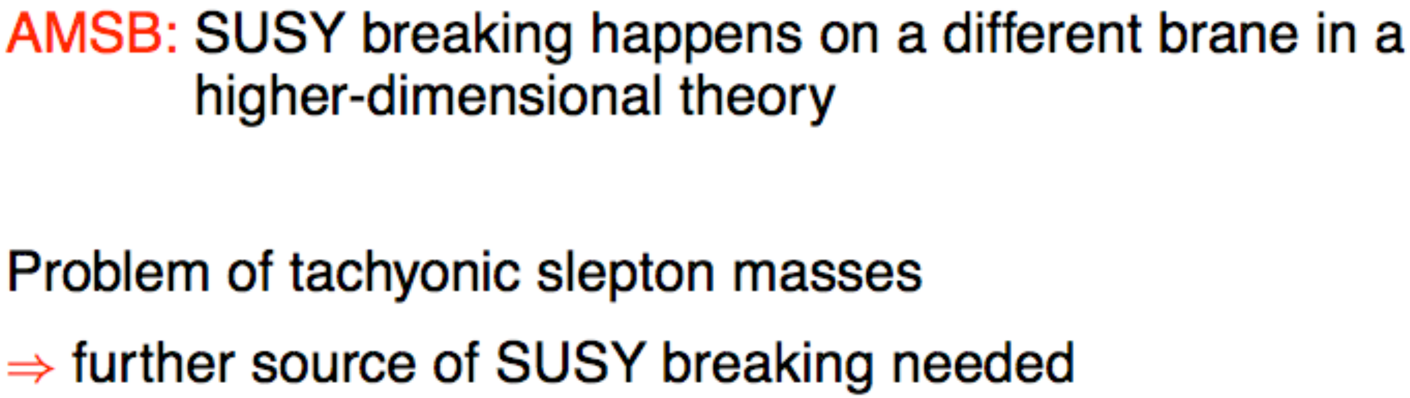
next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector

GMSB scenario with $\tilde{\tau}$ NLSP (SPS 7 benchmark scen.):



AMSB



AMSB: SUSY breaking happens on a different brane in a higher-dimensional theory

Problem of tachyonic slepton masses

⇒ further source of SUSY breaking needed

Summary on SUSY-breaking scenarios:

"We are still far away from a Standard Model of SUSY breaking"

How heavy?

How heavy are the sparticles?

independent arguments for 1 TeV scale:

- Gauge coupling unification
- Hierarchy solution
- Dark matter (?)
- EWSB relation

Higgs potential in MSSM

MSSM Higgs potential contains two Higgs doublets:

$$V = \left(|\mu|^2 + m^2_{H_u}\right)\left(|h_u^0|^2 + |h_u^+|^2\right) + \left(|\mu|^2 + m^2_{H_d}\right)\left(|h_d^0|^2 + |h_d^-|^2\right)$$

$$+ \left[b\left(h_u^+ h_d^- - h_u^0 h_d^0\right) + \text{h.c.}\right]$$

$$+ \underbrace{\frac{g^2 + g'^2}{8}}\left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2\right)^2 + \underbrace{\frac{g'^2}{2}}\left|h_u^+ h_d^{0*} + h_u^0 h_d^{-*}\right|^2$$

gauge couplings, in contrast to the SM

Five physical states: $h^0, H^0, A^0, H^\pm$

Parameters (besides g, g'):

μ: mixing term of the two Higgs doublets in superpotential, $\mu H_d H_u$

m_{H_u}, m_{H_d}, b: soft SUSY-breaking parameters

Higgs parameters in MSSM

$$v_d,\ v_u,\ (|\mu|^2 + m_{H_u}^2)\,,\ (|\mu|^2 + m_{H_d}^2)\,,\ b$$

Relation for M_{W}^2, M_{Z}^2 yields 1 condition:

$$M_{\mathrm{W}}^2 = \frac{1}{2} g'^2 (v_d^2 + v_u^2), \quad M_{\mathrm{Z}}^2 = \frac{1}{2} (g^2 + g'^2)(v_d^2 + v_u^2)$$

Minimization of V w.r.t. neutral Higgs fields h_d^0, h_u^0

⇒ 2 conditions

⇒ only two free parameters remain in the Higgs potential, conventionally chosen as

$$\tan\beta \equiv \frac{v_u}{v_d}, \qquad M_{\mathrm{A}}^2 = -b(\tan\beta + \cot\beta)$$

⇒ M_{h}, M_{H}, mixing angle α, $M_{\mathrm{H}^\pm}$: derived quantities can be predicted

E.g., lowest-order prediction: $M_{\mathrm{H}^\pm}^2 = M_{\mathrm{A}}^2 + M_{\mathrm{W}}^2$

Higgs mass bounds

Prediction for M_h, M_H, $\ldots$

Tree-level result for M_h, M_H:

$$M^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_\mathrm{A}^2 + M_\mathrm{Z}^2 \pm \sqrt{(M_\mathrm{A}^2 + M_\mathrm{Z}^2)^2 - 4M_\mathrm{Z}^2 M_\mathrm{A}^2 \cos^2 2\beta}\,\right]$$

$\Rightarrow$ $M_\mathrm{h} \leq M_\mathrm{Z}$ at tree level

MSSM tree-level bound (gauge sector): excluded by LEP!

Large radiative corrections (Yukawa sector, $\ldots$):

Yukawa couplings: $\frac{\mathrm{e}\, m_\mathrm{t}}{2M_\mathrm{W} s_\mathrm{W}}$, $\frac{\mathrm{e}\, m_\mathrm{t}^2}{M_\mathrm{W} s_\mathrm{W}}$, $\ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_\mathrm{t}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_\mathrm{t}^2}\right)$, $\quad \mathcal{O}(100\%)$!

Higgs mass radiative corrections

- Top loop corrections: 1-loop leading log approximation

$$\Delta(m_h^2) = \frac{3m_t^4}{4\pi^2 v^2}\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

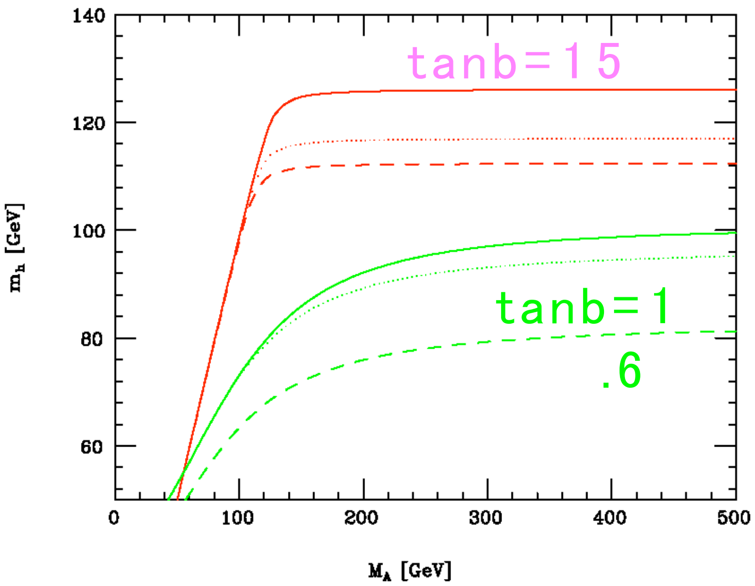
  - Introduces a dependence on top and stop masses
  - More accurate calculation:also on stop mixing $X_t = A_t - \mu\cot\beta$
- In MSSM, **m_h^0 has upper bound**
  - Increases with tanβ
  - Increases from min $X_t/M_{SUSY}=0$

    To max $(X_t/M_{SUSY})^2=6$

$$\boxed{m_h \leq 130\ \text{GeV}}$$

(for M_{SUSY} = 1 TeV, m_t=175 GeV)

➔ Lower than preferred SM range



MSSM Higgs sector

- MSSM contains 2 Higgs doublets, therefore 5 physical Higgs states: h^0, $\underbrace{H^0, A^0, H^\pm}$

~ degenerate in mass for high m_A

looks like H_{SM} (but $m_h < 130$ GeV)

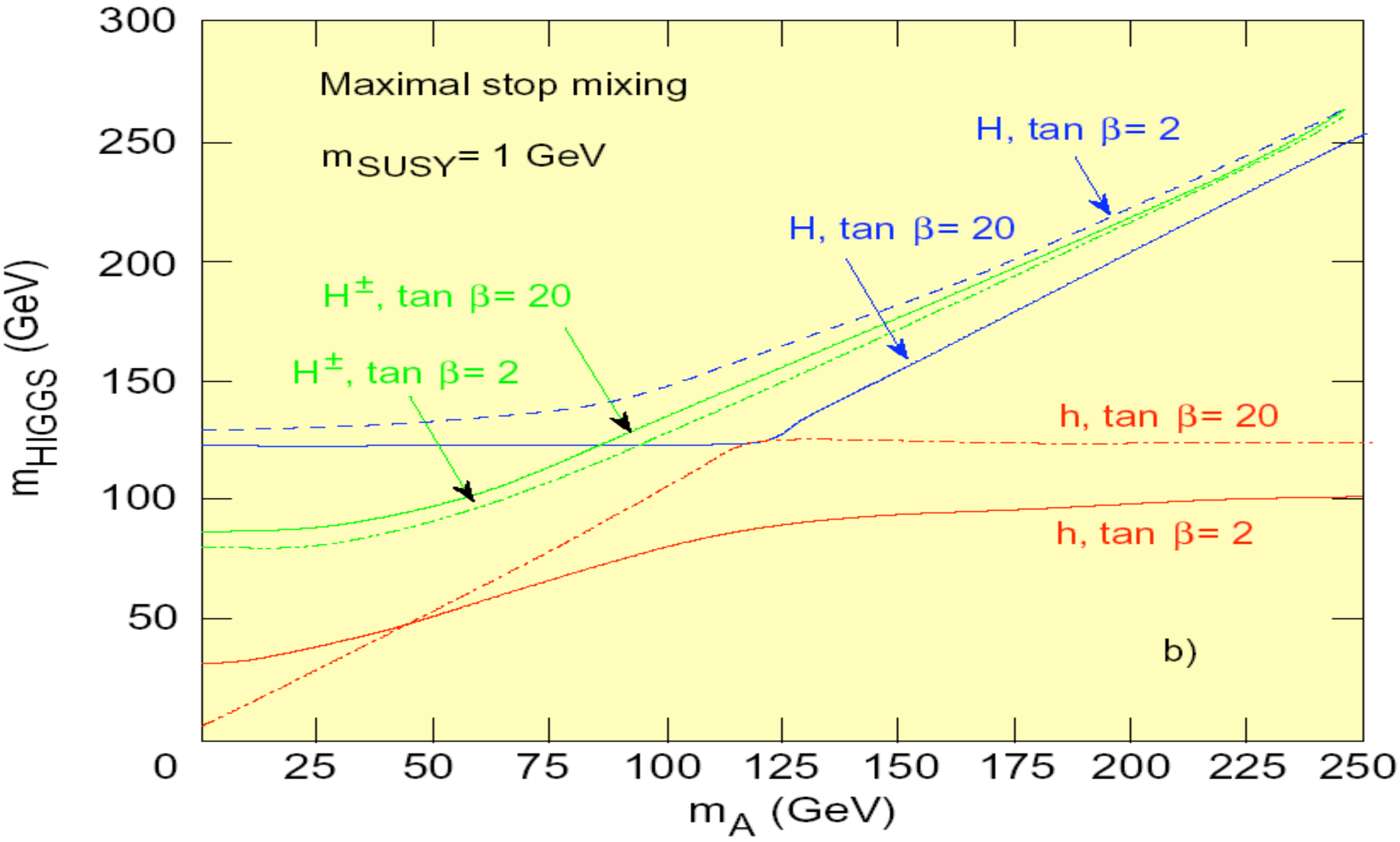
- masses & couplings depend at tree level only on 2 parameters, say m_A & tanβ: (1< tanβ <60)

$$m^2_{H^\pm} = m^2_{A^0} + m^2_{W^\pm}$$

$$m^2_{h^0,H^0} = \tfrac{1}{2}\left(m^2_{A^0} + m^2_{Z^0} \mp \sqrt{(m^2_{A^0} + m^2_{Z^0})^2 - 4m^2_{Z^0}m^2_{A^0}\cos^2 2\beta}\right)$$

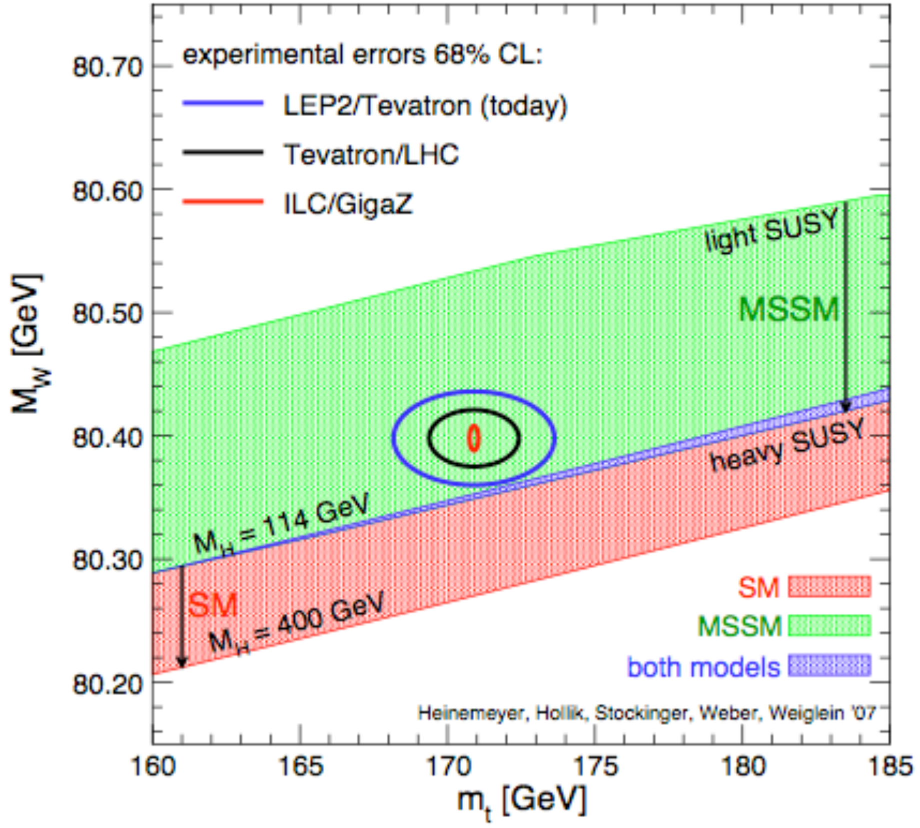
- radiative corrections can be important (e.g. for h^0 !!)

MSSM Higgs masses, summary



SM versus MSSM

Prediction for M_W in the SM and the MSSM:



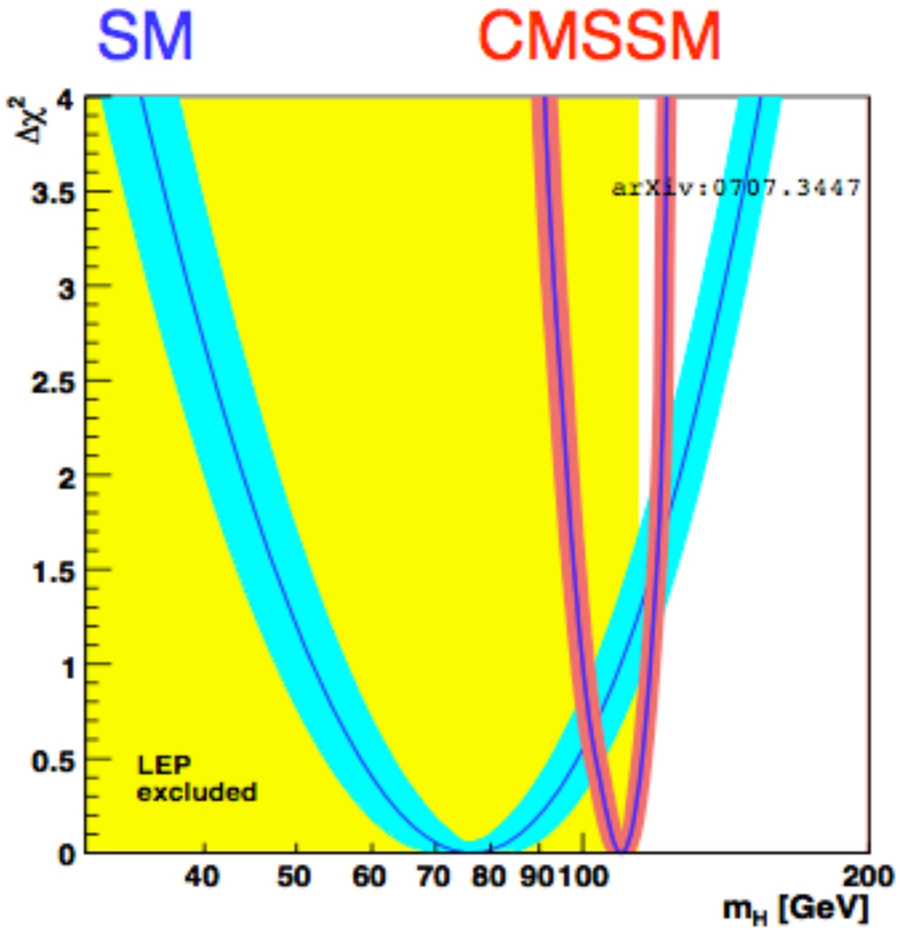
MSSM: SUSY parameters varied

SM: M_H varied

⇒ Slight preference for MSSM over SM

SM versus CMSSM(2)

χ^2 fit for M_h, without imposing direct search limit



Electroweak precision observables in CMSSM

Less tension than in SM

Why SUSY is a good idea

One of the most appealing extensions of the Standard Model:

TeV-scale supersymmetry

[= a symmetry between fermions and bosons,
duplicates the SM particle spectrum, but not the couplings]

Solves several problems at once:

- hierarchy problem
- opening towards a theory of gravity
- unification of gauge couplings
- dark matter candidate (=lightest susy particle or LSP)
- allows to explain why the Higgs mechanism works (radiative EWSB)