quarks/gluons → jet (hadrons)

$\pi^+ \ \pi^0 \ \pi^-$

jet algo's (calorimeter)

40 MHz

bunch crossing

p p

10^{11} 10^{11}

3 μs

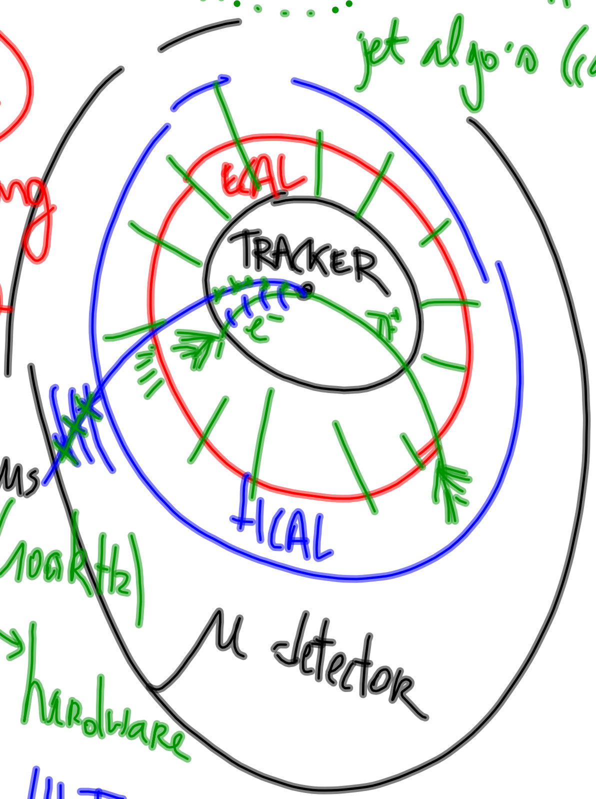
L1 trigger (100 kHz)

hardware

DAQ

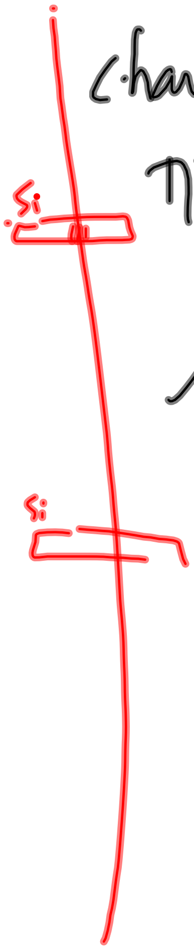
HLT (software) → ~100 Hz (~1MB)

1 event



TRACKS

* particle detector
* method to combine hits



charged particles

$\pi^+ \; \cancel{\pi^0} \; \pi^-$

$e^+ e^-$

$\mu^+ \mu^-$

$\vdots$

Vacuum

trajectory? $\vec{E}$ $\vec{B}$

Lorentz: $\vec{F} = \underset{\displaystyle e}{q}\,(\underset{\displaystyle 0}{\vec{E}} + \vec{v} \times \vec{B})$

$\vec{F} = \frac{d\vec{p}}{dt}$ $\quad \vec{p} = m_0 \gamma \vec{v} = m \vec{v}$

$\vec{v} = \frac{d\vec{R}}{dt}$

$\Rightarrow \vec{F} = m_0 \gamma \frac{d\vec{v}}{dt} = m_0 \gamma \frac{d^2\vec{R}}{dt^2}$

$\Rightarrow m_0 \gamma \frac{d^2\vec{R}}{dt^2} = e \frac{d\vec{R}}{dt} \times \vec{B} \Leftrightarrow m_0 \gamma \frac{d^2\vec{R}}{ds^2} v^{\cancel{2}} = e \cancel{v} \frac{d\vec{R}}{ds} \times \vec{B}$

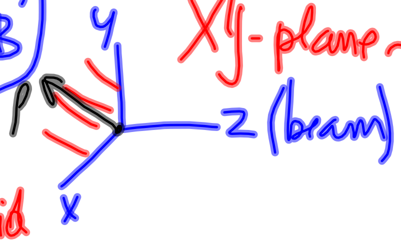
$ds = v\,dt$

$\Rightarrow dt = \frac{ds}{v}$

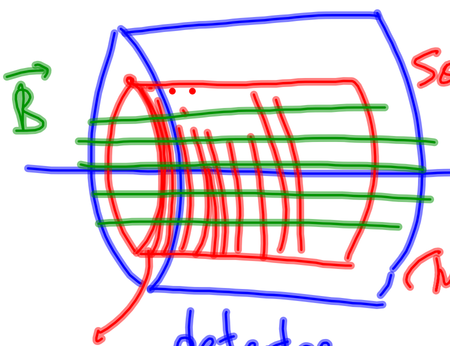
$|\vec{p}| \equiv p$

$$\boxed{\frac{d^2\vec{R}}{ds^2} = \frac{e}{p} \cdot \frac{d\vec{R}}{ds} \times \vec{B}(\vec{R})} \longrightarrow \text{diff. eq.}$$

$$\boxed{\frac{d^2\vec{R}}{dt^2} = \frac{e}{p}\,\frac{d\vec{R}}{dt}\times\vec{B}}$$



xy-plane → transverse plane (ρ)

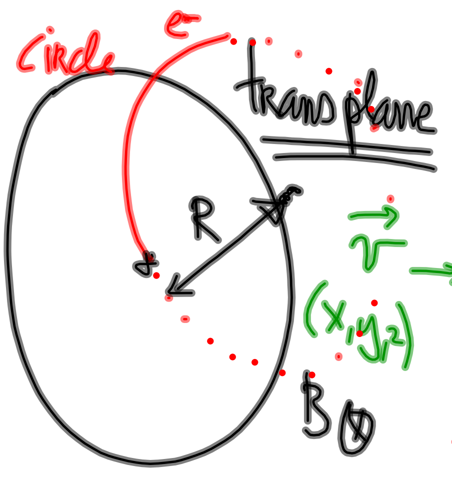


CMS $|\vec{B}| = 4$ Tesla

magn. field homogeneous! $\vec{B}(\vec{R}) = B\,\vec{1}_z$ — unit vector

$$\frac{d^2\vec{R}}{ds^2} = \frac{e}{p}\,\frac{d\vec{R}}{ds}\times B\,\vec{1}_z$$

$$\vec{F} = q\,\vec{v}_T\times\vec{B} = \frac{d\vec{p}_T}{dt}$$



$\vec{v} \rightarrow \vec{v}_T$

(x,y,z) (x,y)

$$F = \frac{mv^2}{R}$$

centripetal force

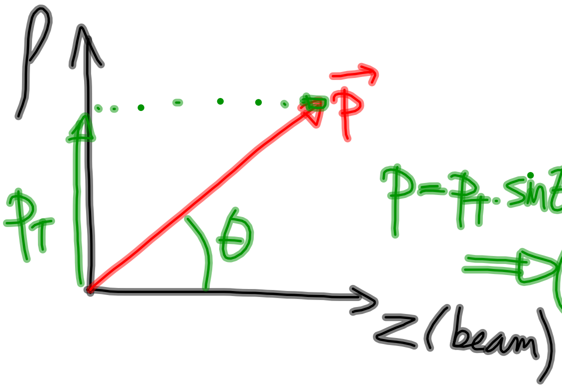
$$\Rightarrow\ q\,v_T\cdot B = \frac{m v_T^2}{R}$$

$$R = \frac{m v_T}{qB} \Rightarrow \boxed{p_T = qRB}$$

$p_T = qRB$

$\bar{p} = (E, p_x, p_y, p_z)$

mass $m^2 = E^2 - |\vec{p}|^2$



$p = p_T \cdot \sin\theta$

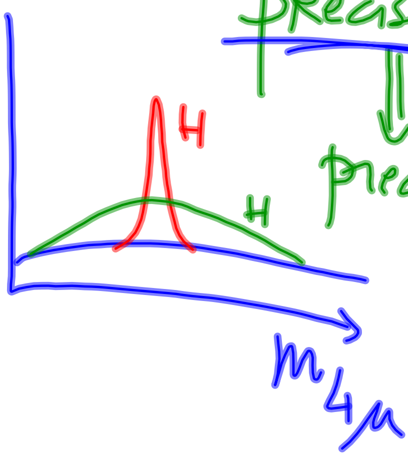
$\Longrightarrow p \xRightarrow[\text{mass}]{} \bar{p}$

$H \to ZZ \to \mu\mu\mu\mu$

$\bar{p}(H)$

$\Downarrow$

mass H



Precise $p_T(\mu)$

$\Downarrow$

precise $m_{4\mu}$

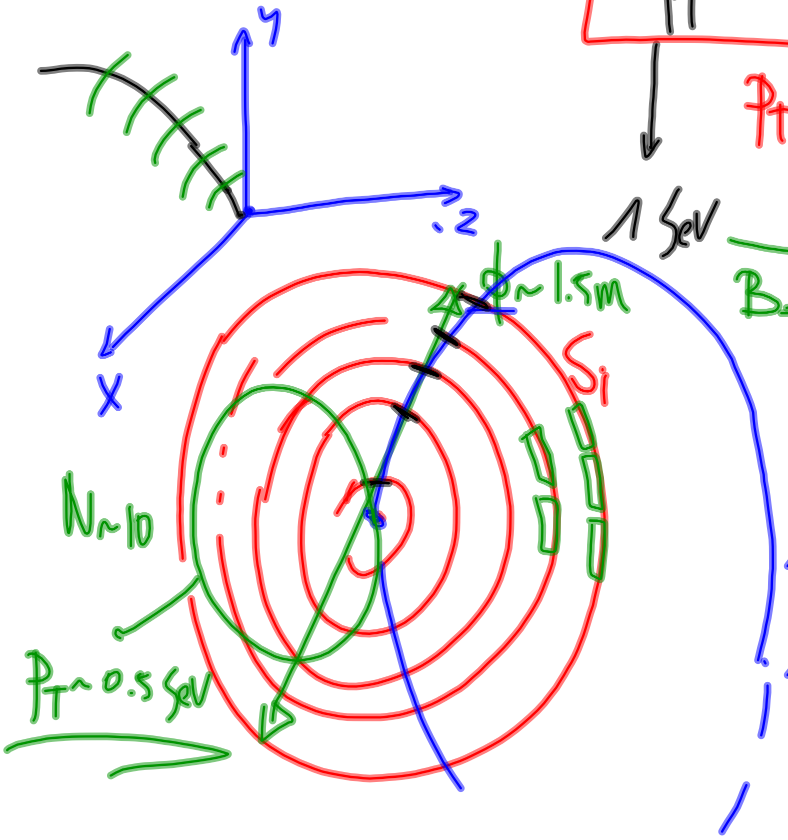
$\Downarrow$

narrow H

$$\boxed{p_T = q\,\boxed{R}\,B}$$

$p_T[\mathrm{GeV}/c] = 0.3\, R[\mathrm{m}]\, B[\mathrm{T}]$

$|q| = 1$



1 GeV $\longrightarrow$ $R = 0.83\,\mathrm{m}$

B = 4 Tesla

$\phi = 1.67\,\mathrm{m}$

10 GeV	8.3 m
100 GeV	83 m
1000 GeV	830 m

$pp \rightarrow \underset{3\,\mathrm{TeV}}{Z'} \rightarrow \mu^-\mu^-$

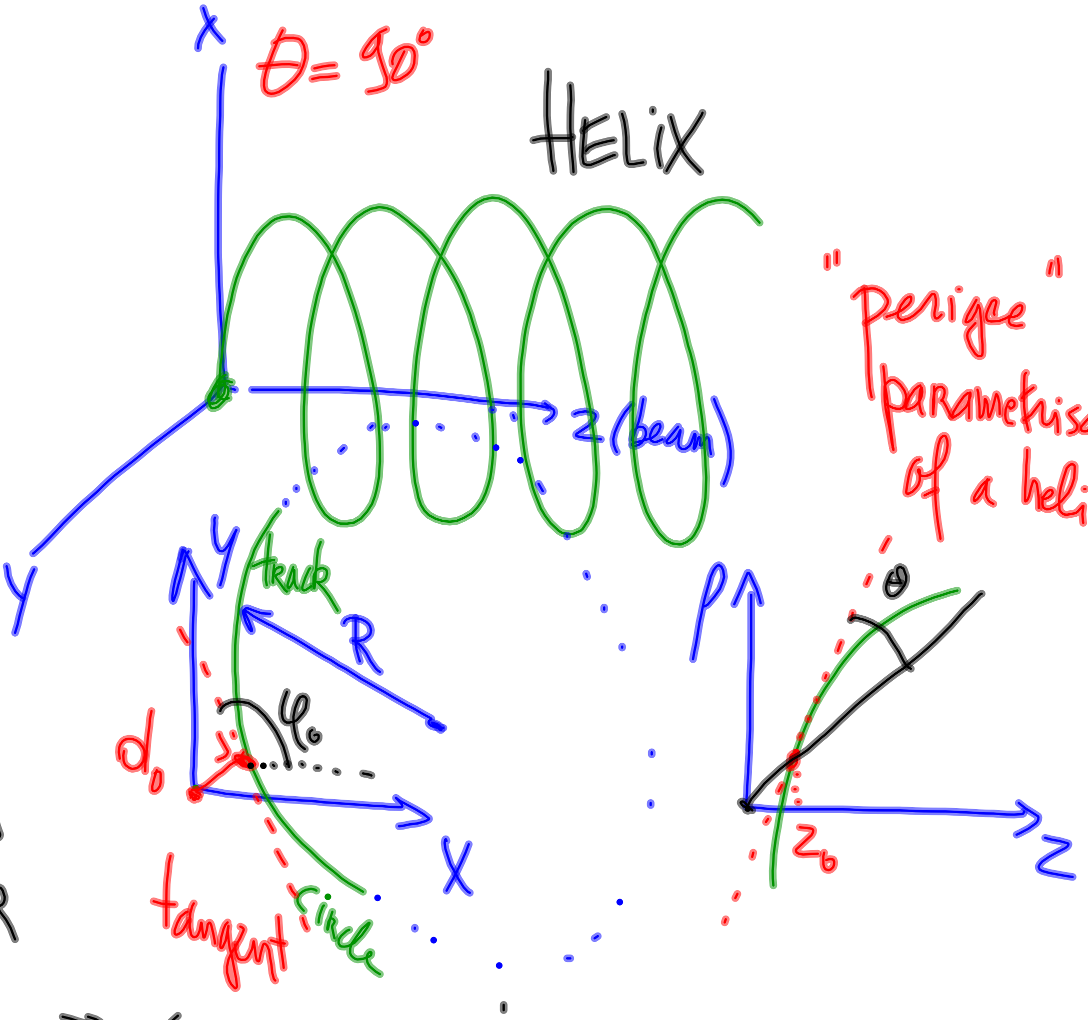
$\vec{R}$

$x =$

$y =$

$z =$

HELIX

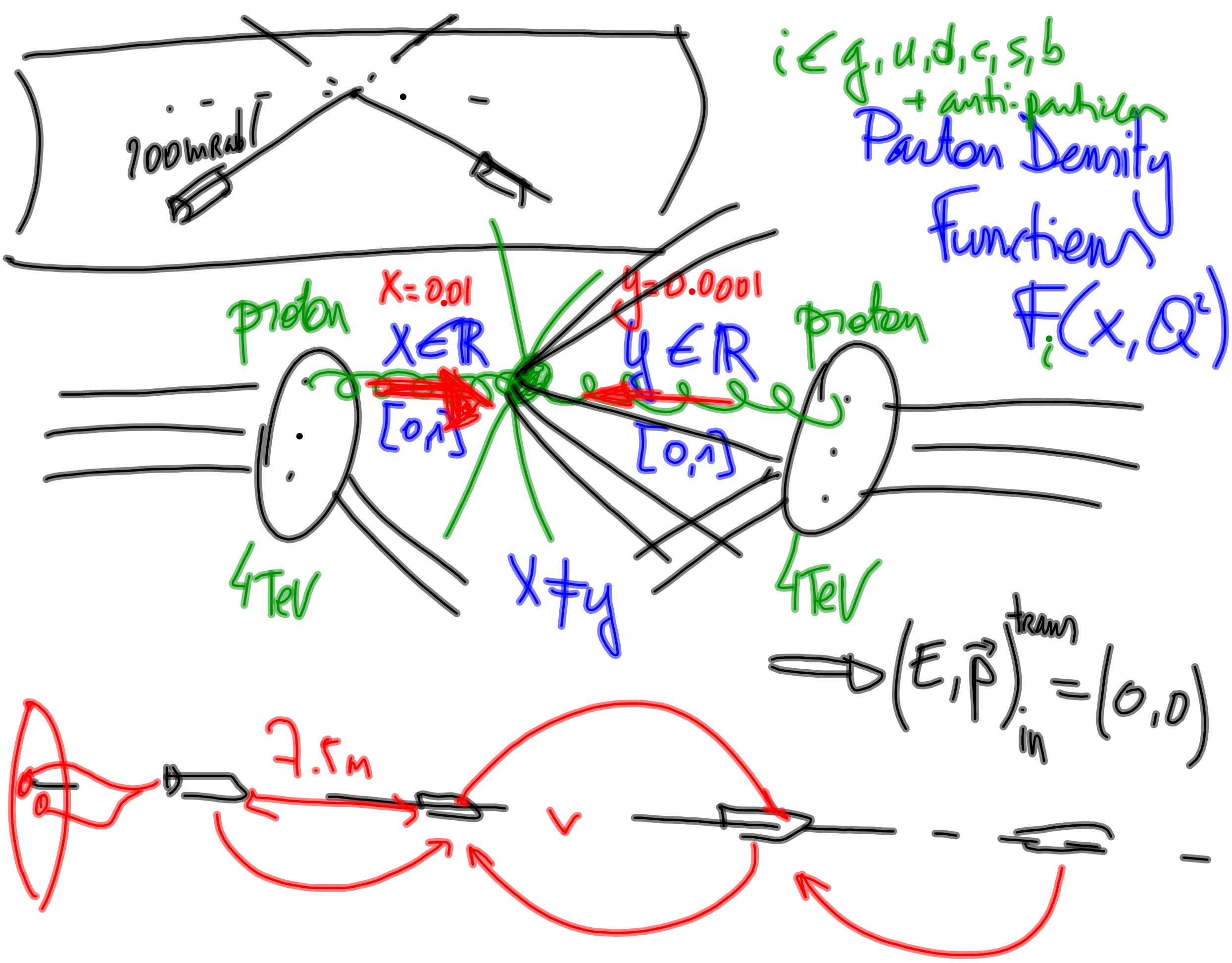


"perigee" parametrisation of a helix

$\boxed{\vec{q}}$

vector

$\vec{q} = (d_0, \varphi_0, z_0, \theta, R)$



900 mrad
$i \in g, u, d, c, s, b$
+ anti-particles
Parton Density Functions
$F_i(x, Q^2)$
proton
proton
x = 0.01
y = 0.0001
$x \in \mathbb{R}$ [0,1]
$y \in \mathbb{R}$ [0,1]
4 TeV
4 TeV
$x \neq y$
$\Rightarrow (E, \vec{p})^{trans}_{in} = (0, 0)$
7.5 m

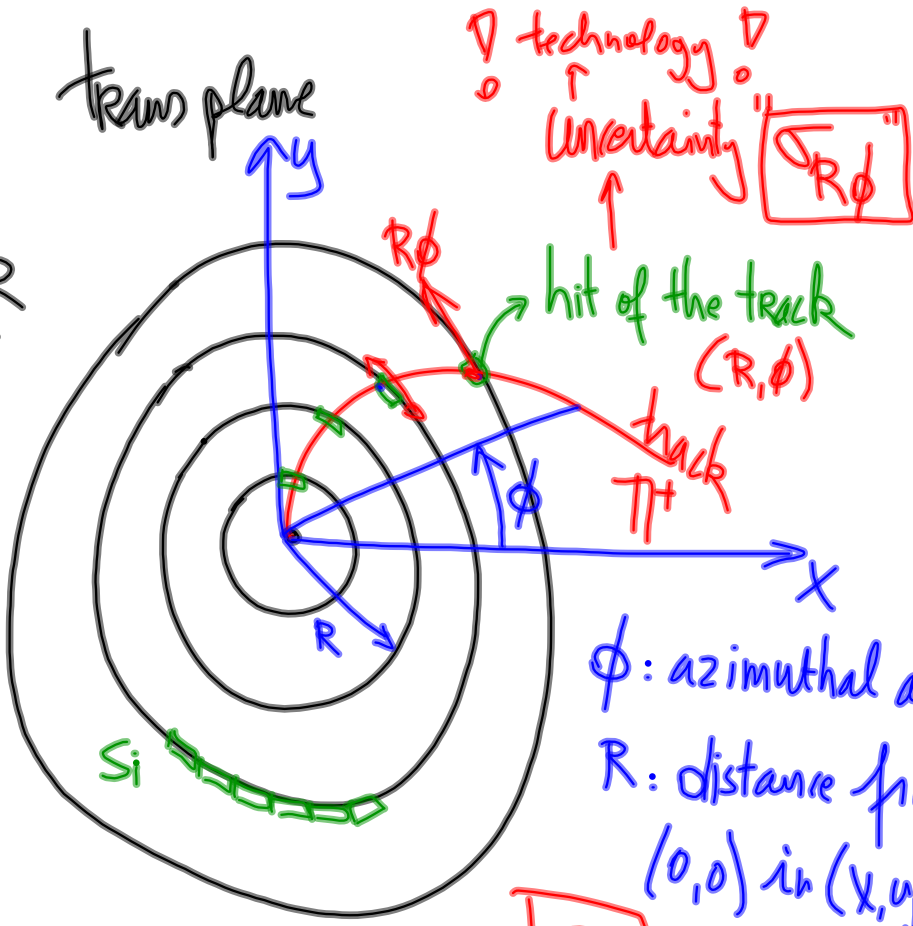
$\frac{\Delta\phi_T}{p_T}$

TRACKER

layer N

(N~10)

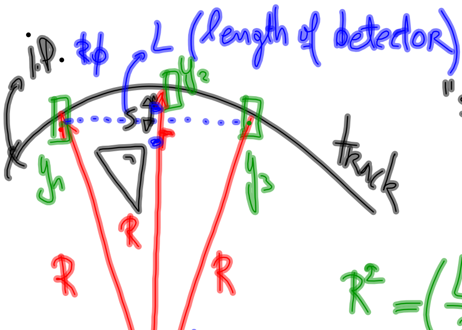
(x,y,z)



φ : azimuthal angle

R : distance from (0,0) in (x,y)-plane

$\boxed{\sigma_z}$



"sagitta" (s) → deviation from straight line at "middle" of detector

$$s = y_2 - \frac{y_1+y_3}{2}$$

center of circle of the track

$$R^2 = \left(\frac{L}{2}\right)^2 + (R-s)^2$$

$$\Rightarrow \cancel{R^2} = \frac{L^2}{4} + \cancel{R^2} - 2sR + s^2$$

$$\Rightarrow R = \frac{L^2}{8s} + \cancel{\frac{s}{2}}$$

high momentum $R \gg s$

$$\Rightarrow R \approx \frac{L^2}{8s}$$

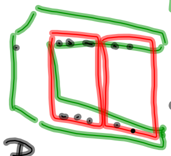
$$\rightarrow \; p_T = q \cdot \frac{L^2}{8s} \cdot B$$

$B = 4T$ $L = 1\,m$

$p_T = 100\,GeV$

$\Rightarrow s \approx 0.07\,cm$

$700\,\mu m$



$$\left|\frac{dp_T}{ds}\right| = q \cdot \frac{L^2}{8s^2} B = \frac{p_T}{s}$$

$$\rightarrow \; \frac{\Delta p_T}{p_T} = \frac{\Delta s}{s} = 8 \boxed{\Delta s}\, p_T \frac{1}{qL^2B}$$

$$s = \frac{qL^2B}{8p_T}$$

$$s = y_2 - \frac{y_1+y_3}{2}$$

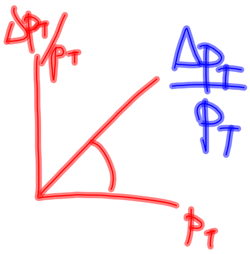
$$\Delta y \equiv \Delta y_1 = \Delta y_2 = \Delta y_3$$

$$(\Delta s)^2 = \sum_{i=1}^{3} \left(\frac{\partial s}{\partial y_i}\right)^2 (\Delta y_i)^2$$

$$\Rightarrow (\Delta s)^2 = \frac{1}{4}(\Delta y)^2 + (\Delta y)^2 + \frac{1}{4}(\Delta y)^2$$

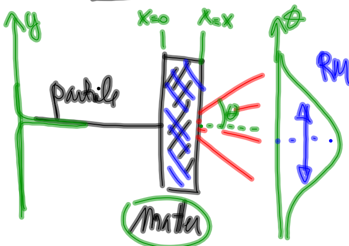
$$= \frac{3}{2}(\Delta y)^2 \Rightarrow \Delta s = \sqrt{\frac{3}{2}} \cdot \sigma_{R\phi}$$

$$\boxed{\frac{\Delta p_T}{p_T} = \boxed{8\sqrt{\frac{3}{2}}}\, \boxed{\sigma_{R\phi}} \cdot p_T \frac{1}{qL^2B}}$$



$$\frac{\Delta p_T}{p_T} = a_N \frac{\sigma_{R\phi}\, p_T}{0.3\, B\, L^2} \quad \text{with } a_N = \sqrt{\frac{720}{N+4}}$$

Multiple Scattering (M.S.)



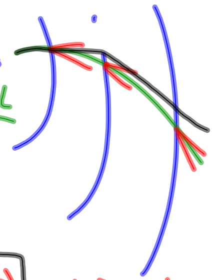
$$RMS(\theta) \sim \theta_0 \simeq \frac{13.6\,MeV}{\beta c p} z \sqrt{\frac{X}{X_0}}$$

$$E[\theta] = 0$$

X_0 : radiation length. SMALL

LARGE

$$E(X) = E(X=0) \cdot e^{-X/X_0}$$

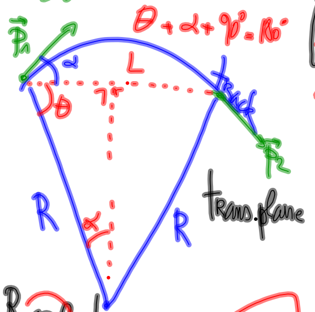


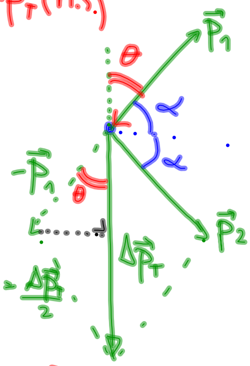
$X_0(\text{air}) \sim 300\,m$ $\quad X_0(Si) \sim 9\,cm$

$\theta + \alpha + 90° = 180°$

$\boxed{\Delta\vec{p}_T(\vec{B})} = |\vec{p}_2 - \vec{p}_1| - |\vec{p}_2 - \vec{p}_1|$

$\Delta\vec{p}_T(M.S.)$





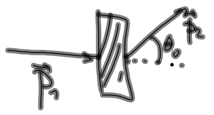
$R \cos\theta = \frac{L}{2}$

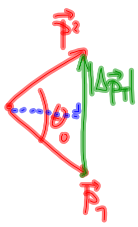
$\vec{p}_1 \cos\theta = \frac{\Delta\vec{p}}{2}$

$q.R.B$ $\quad p_T$

$\rightarrow p_T \cdot \frac{L}{\cancel{2}R} = \frac{\Delta p_T}{\cancel{2}} = q \cdot B \cdot L \rightarrow \boxed{\Delta p_T(\vec{B}) = qBL}$

$\boxed{\Delta\vec{p}_T(M.S.)} = |\vec{p}_2 - \vec{p}_1|$





p_T

$|\vec{p}_1| \cdot \sin\frac{\theta_0}{2} = \frac{\Delta p_T}{2}$

$\theta_0 \approx$ small.

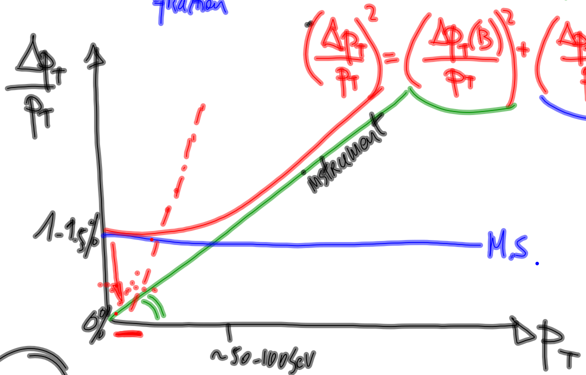
$\sin\theta \approx \theta$

fraction

$\Rightarrow p_T \cdot \frac{\theta_0}{\cancel{2}} = \frac{\Delta p_T}{\cancel{2}}$

$\Rightarrow \Delta p_T(M.S.) = \boxed{p_T} \cdot \theta_0$

$$\left(\frac{\Delta p_T}{p_T}\right)^2 = \left(\frac{\Delta p_T(B)}{p_T}\right)^2 + \left(\frac{\Delta p_T(MS)}{p_T}\right)^2$$

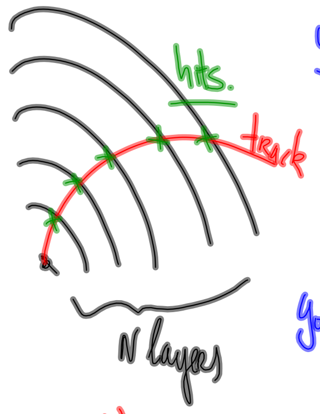


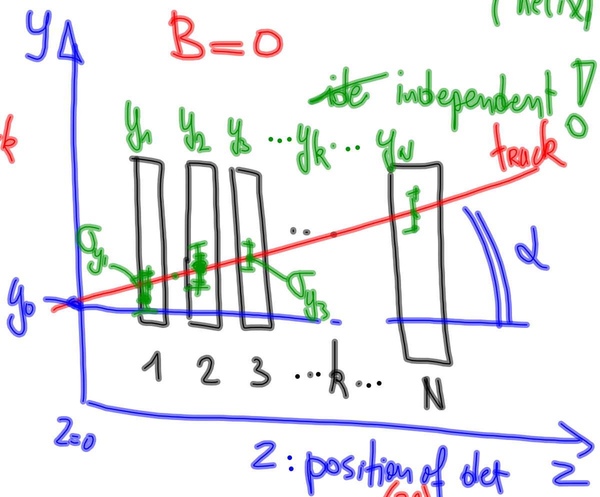
$\frac{\Delta p_T}{p_T} \sim 1\%$ $\quad (p_T \sim 100\,GeV/c) \longrightarrow N = 10 \quad L = 1\,m \quad B = 4\,T$

$\Longrightarrow \sigma_{R\phi} \approx 20\,\mu m$

TRACK FITTING

estimate track parameters. $\vec{q} \longrightarrow$ 5 par. (helix)





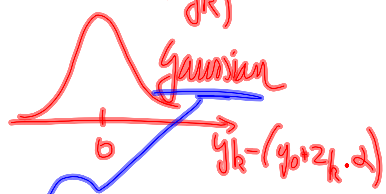
z: position of det (z_k)

y: measurement position (y_k)

$\vec{q} = \begin{pmatrix} y_0 \\ \alpha \end{pmatrix}$

Straight line: $y = y_0 + (z_k - \overset{0}{z_0})\cdot\alpha$

distribution $d\Big(\underbrace{y_k - (y_0 + z_k\cdot\alpha)}_{\text{error}}\Big)$.



$\Longrightarrow$ likelihood $\mathcal{L}(y_0,\alpha) = \prod_{k=1}^{N} d\big(y_k - (y_0 + z_k\cdot\alpha)\big)$

$$\Longrightarrow \underbrace{\mathcal{L}(y_0,\alpha)}_{\text{maximize}} = \prod_{k=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma_{y_k}}\cdot\exp\left[-\frac{1}{2}\left(\frac{y_k-(y_0+z_k\cdot\alpha)}{\sigma_{y_k}}\right)^2\right]$$

$$\Longrightarrow \underbrace{-\ln\mathcal{L}(y_0,\alpha)}_{\text{minimize}} = c^{te} + \underbrace{\sum_{k=1}^{N}\frac{1}{2}\left(\frac{y_k-(y_0+z_k\cdot\alpha)}{\sigma_{y_k}}\right)^2}_{\text{minimize}}$$

$$\Longrightarrow \chi^2(\underbrace{y_0,\alpha}_{\text{estimate}}) = \sum_{k=1}^{N}\left(\frac{y_k-(y_0+z_k\cdot\alpha)}{\sigma_{y_k}}\right)^2$$

$$\begin{cases} \left.\dfrac{\partial\chi^2(y_0,\alpha)}{\partial y_0}\right|_{y_0,\alpha=\hat{y}_0,\hat{\alpha}} = 0 \\[2ex] \left.\dfrac{\partial\chi^2(y_0,\alpha)}{\partial\alpha}\right|_{y_0,\alpha=\hat{y}_0,\hat{\alpha}} = 0 \end{cases} \Longrightarrow \begin{cases} \hat{y}_0 = \ldots \\ \hat{\alpha} = \ldots \end{cases}$$

$\hat{x}$: estimator of x

$\vec{q}$ (helix) $\longrightarrow$ N measurements y_k

M parameter ($M=5$)

relation : $y_k = f_k(\vec{q})$ $\quad k \in \{1 \dots N\}$

$\hookrightarrow$ helix (non-linear)

$\vec{q} = \vec{q}_A$ (number) $\Longrightarrow$ linearize it (TAYLOR)

$$\Rightarrow y_k = f_k(\vec{q}_A) + \sum_{i=1}^{5} \underbrace{\left(\frac{\partial f_k(\vec{q})}{\partial q_i}\right)_{\vec{q}=\vec{q}_A}}_{\text{note } A_{ki}} \cdot (q_i - q_{A_i}) + \dots$$

$$\Rightarrow \chi^2 = \sum_{k=1}^{N} \left(\frac{y_k - f_k(\vec{q}_A) - \sum_{i=1}^{5} A_{ki} \cdot (q_i - q_{A_i})}{\sigma_{y_k}} \right)^2$$

$\chi^2(\vec{q})$

$$\longrightarrow \chi^2 = [\underbrace{\vec{y} - \vec{f}(\vec{q}_A)}_{\Delta\vec{y}} - \tilde{A}(\vec{q}-\vec{q}_A)]^T \tilde{V}_y^{-1} [\vec{y} - \vec{f}(\vec{q}_A) - \tilde{A}(\vec{q}-\vec{q}_A)]$$

matrix

covariance matrix

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

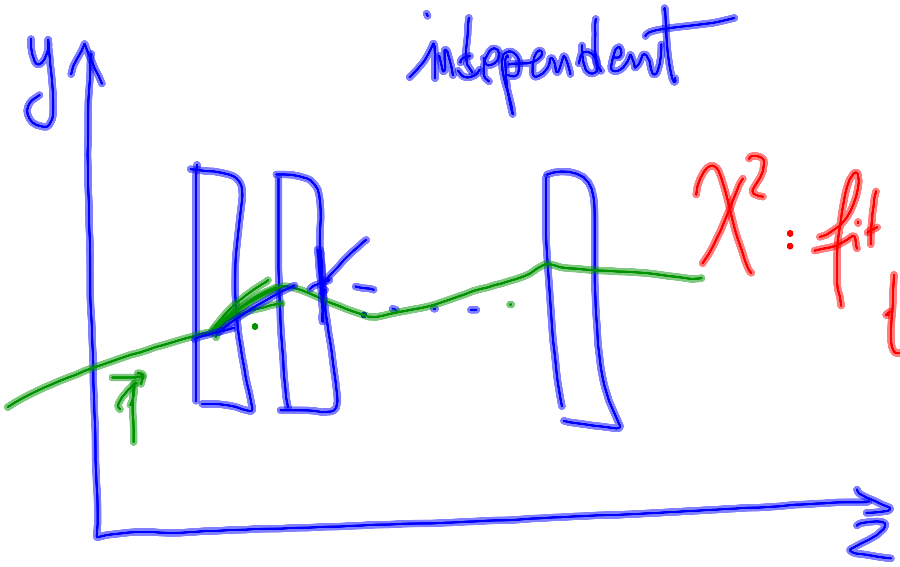
$$\tilde{V}_y^{-1} = \begin{pmatrix} 1/\sigma_{y_1}^2 & & 0 \\ & 1/\sigma_{y_2}^2 & \\ 0 & & \ddots \quad 1/\sigma_{y_N}^2 \end{pmatrix}$$

diagonal

$$\longrightarrow \chi^2(\vec{q}) = [\Delta\vec{y} - \tilde{A}(\vec{q}-\vec{q}_A)]^T \tilde{V}_y^{-1} [\Delta\vec{y} - \tilde{A}(\vec{q}-\vec{q}_A)]$$

$$\left\{ \frac{\partial \chi^2(\vec{q})}{\partial q_i} = 0 \right\}_{(5)} \quad \overset{\dots}{\Longrightarrow} \quad \hat{\vec{q}} = \vec{q}_A + (\tilde{A}^T \tilde{V}_y^{-1} \tilde{A})^{-1} \tilde{A}^T \tilde{V}_y^{-1} \Delta\vec{y}$$

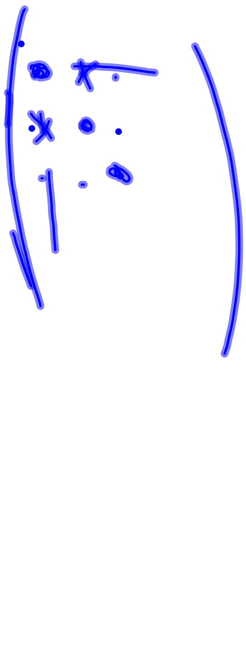
$$\tilde{V}_{\hat{q}} = (\tilde{A}^T \tilde{V}_y^{-1} \tilde{A})^{-1}$$



χ^2: fit everything together.

$\Downarrow$

global fit

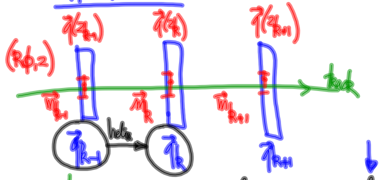


$$\tilde{V}_y = \tilde{V}_{meas.} + \tilde{V}_{MS.}$$

$\tilde{V}_{meas.}$ $\downarrow$ indep.

$\tilde{V}_{MS.}$ ↳ non-diagonal!!

KALMAN FILTER →



$E[\vec{\varepsilon}]=0$

$\sigma_{R\phi}, \sigma_z$

measurement model: $\vec{m}_k = h_k(\vec{q}_k) + \vec{\varepsilon}_k$

track model (system): $\vec{q}_k = f_{k-1}(\vec{q}_{k-1}) + \vec{\omega}_{k-1}$

M.S. $E[\vec{\omega}]=0$

$f \to \tilde{F}(5\times5)$

$h \to \tilde{H}(2\times5)$ ⇒

$$\boxed{\begin{aligned}\vec{m}_k &= \tilde{H}_k \vec{q}_k + \vec{\varepsilon}_k \\ \vec{q}_k &= \tilde{F}_{k-1}\vec{q}_{k-1} + \vec{\omega}_{k-1}\end{aligned}}$$

$\mathrm{cov}(\vec{\varepsilon}_k) = \tilde{V}_k$ $\quad \mathrm{cov}(\vec{\omega}_k) \equiv \tilde{Q}_k$

1. predict $\vec{q}_{k-1} \longrightarrow \vec{q}_k$
2. $\boxed{\text{filter}}$ $\vec{q}_k \to \vec{m}_k \leftrightarrow \vec{q}_k$
3. smoothing

$\vec{q}_k$ up to measure. i

true $\vec{q}_{k,t} = \tilde{F}_{k-1}\vec{q}_{k-1,t} + \vec{\omega}_{k-1}$ ⇒ $\tilde{C}_k^i = \mathrm{cov}(\underbrace{\vec{q}_k^{\,i} - \vec{q}_{k,t}}_{\text{error}})$

$\hookrightarrow \vec{m}_k = \tilde{H}_k \vec{q}_{k,t} + \vec{\varepsilon}_k$

Residuals: $\vec{R}_k^i = \vec{m}_k - \tilde{H}_k \vec{q}_k^{\,i}$

$\mathrm{cov}(\vec{R}_k^i) \equiv \tilde{R}_k^i$

1 PREDICTION $(k-1) \longrightarrow (k)$

$\vec{q}_k^{\,k-1} = \tilde{F}_{k-1}\vec{q}_{k-1}^{\,k-1} + 0$ (extrapolate $\vec{q}$)

$\tilde{C}_k^{k-1} = \tilde{F}_{k-1}\tilde{C}_{k-1}\tilde{F}_{k-1}^T + \tilde{Q}_{k-1}$ (extrapol. of $\tilde{C}$)

$\vec{R}_k^{k-1} = \boxed{\vec{m}_k} - \tilde{H}_k \vec{q}_k^{\,k-1}$

$\tilde{R}_k^{k-1} = \boxed{\tilde{V}_k} + \tilde{H}_k \tilde{C}_k^{k-1}\tilde{H}_k^T$

2. FILTERING

$\vec{q}_k^{\,k} = \vec{q}_k^{\,k-1} + \tilde{K}_k\left(\underbrace{\vec{m}_k - \tilde{H}_k\vec{q}_k^{\,k-1}}_{\text{correction proposed}}\right)$

prediction

KALMAN GAIN MATRIX

$\tilde{K}_k = \underbrace{\tilde{C}_k^{k-1}}_{\text{Unc. prediction}} \tilde{H}_k^T \underbrace{(\tilde{V}_k + \tilde{H}_k \tilde{C}_k^{k-1}\tilde{H}_k^T)^{-1}}_{\text{Unc. on correction}}$

Unc. prediction ! Unc. on correction !

Min. Variance Bound. $\hat{\theta} \to \{x_i\}$ ⇒ smallest variance an estimator of θ can have

$\{x_i\} \longrightarrow \theta$

$\hat{\theta} = f(\{x_i\}) \to \mathrm{Var}[\hat{\theta}]$

$\tilde{C}_k^k = (\mathbb{1} - \tilde{K}_k\tilde{H}_k)\tilde{C}_k^{k-1}$

$\vec{R}_k^k = \vec{m}_k - \tilde{H}_k \underset{\text{posted}}{\vec{q}_k^{\,k}} = \vec{m}_k - \tilde{H}_k(\vec{q}_k^{\,k-1} + \tilde{K}_k\vec{R}_k^{k-1})$

$\vec{R}_k^{k-1} - \tilde{H}_k\tilde{K}_k \cdot \vec{R}_k^{k-1}$

$= (\mathbb{1} - \tilde{H}_k\tilde{K}_k)\cdot\vec{R}_k^{k-1}$

$\tilde{R}_k^k = (\mathbb{1} - \tilde{H}_k\tilde{K}_k)\cdot\tilde{R}_k^{k-1}$

3. SMOOTHING (skip)