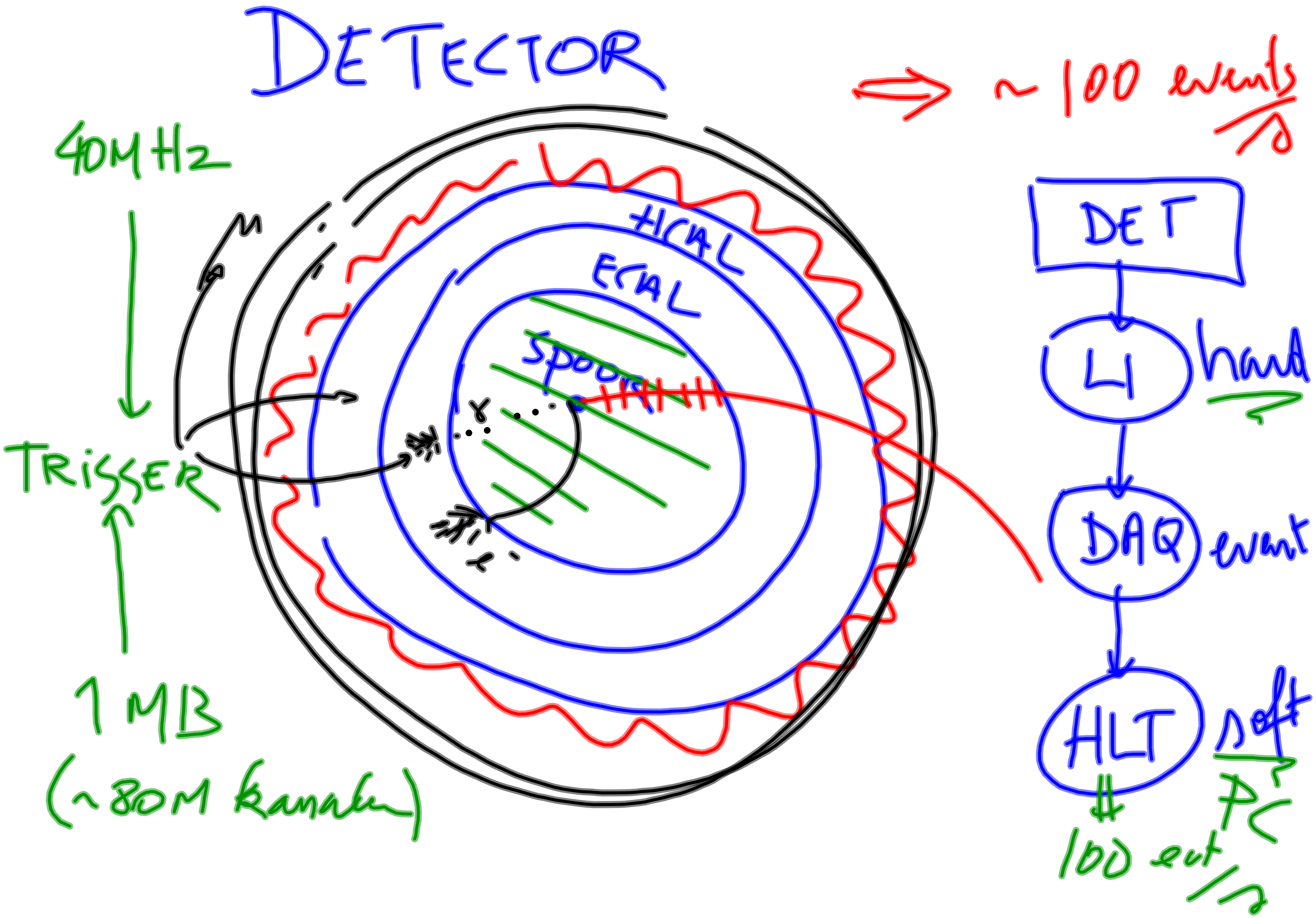
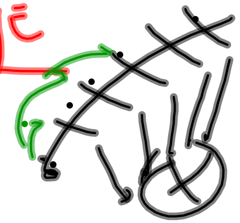

DETECTOR
⇒ ~100 events/s
40MHz
TRIGGER
1MB
(~80M kanalen)
HCAL
ECAL
spoor
μ
γ
e
DET
L1
hard
DAQ
event
HLT
soft
PC
100 evt/s

SPOOR GELADEN DEELTJE



$\vec{B}$ magn. veld.

$$\vec{F} = \overset{e}{q}\,\vec{v} \times \vec{B} = \frac{d\vec{p}}{dt}$$

π^+, π^0, π^-

e^- e^+

μ^- μ^+

$|q| = 1$

$$\vec{p} = m\gamma_0 \vec{v}$$

$$\vec{v} = \frac{d\vec{R}}{dt}$$

$$\Longrightarrow m\gamma_0 \frac{d^2\vec{R}}{dt^2} = e\,\frac{d\vec{R}}{dt} \times \vec{B}$$

$ds = v\,dt$

$$\Longrightarrow m\gamma_0 v^2 \frac{d^2\vec{R}}{ds^2} = e\,\frac{d\vec{R}}{ds} \times \vec{B}$$

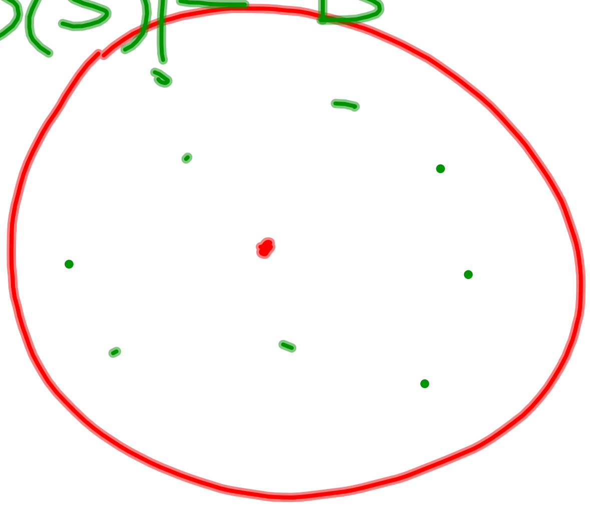
$$\Longrightarrow \boxed{\frac{d^2\vec{R}}{ds^2} = \left(\frac{d\vec{R}}{ds} \times \vec{B}\right)\frac{e}{m\gamma_0 v}}$$

$m\gamma_0 v = p$

$\vec{B} = \vec{B}(s)$

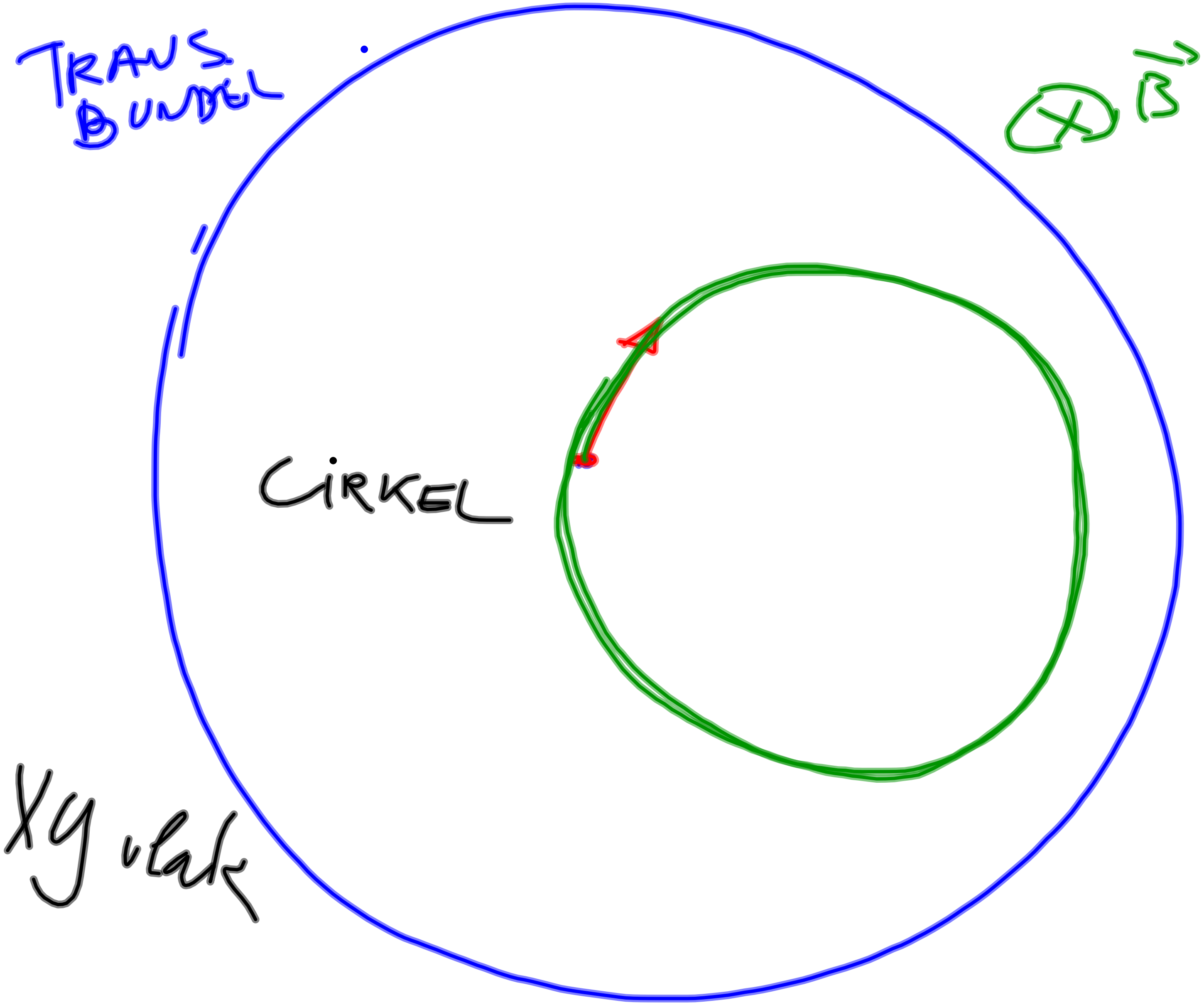
$\Rightarrow \vec{R}(s)$

$|B(s)| = B$

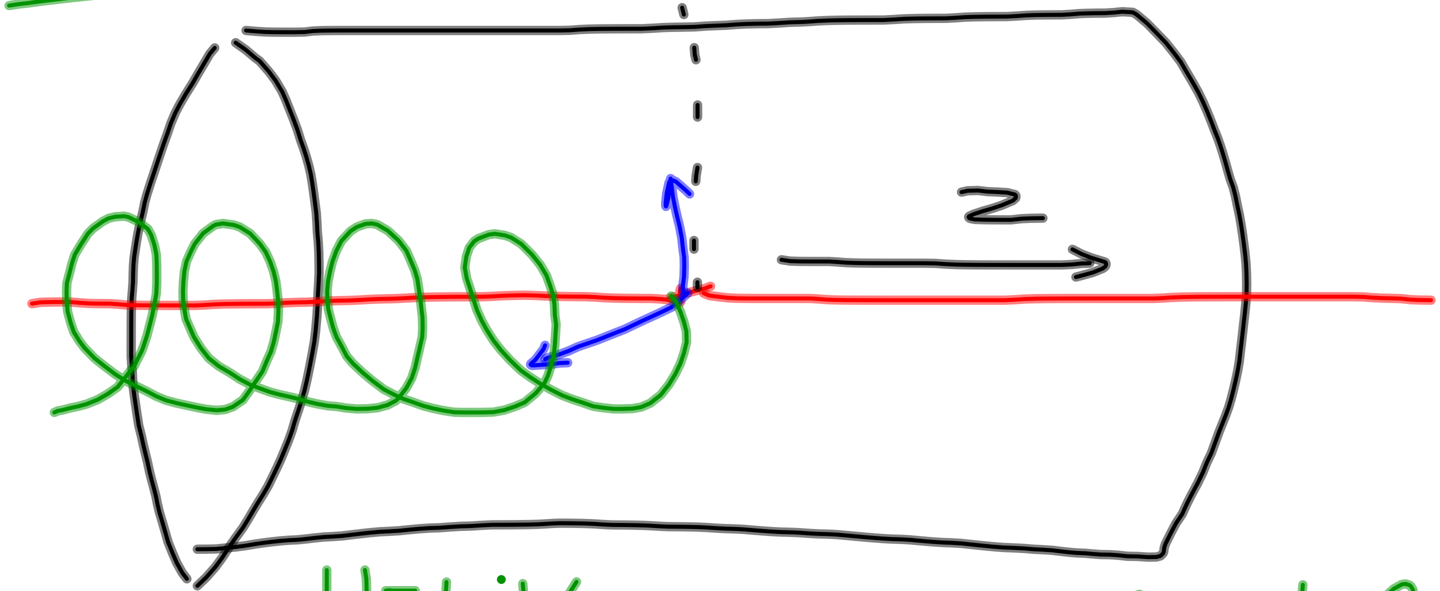


SOLENOÏDE

Uniform $\vec{B}$ veld


TRANS
BUNDEL
$\otimes \vec{B}$
CIRKEL
xy vlak

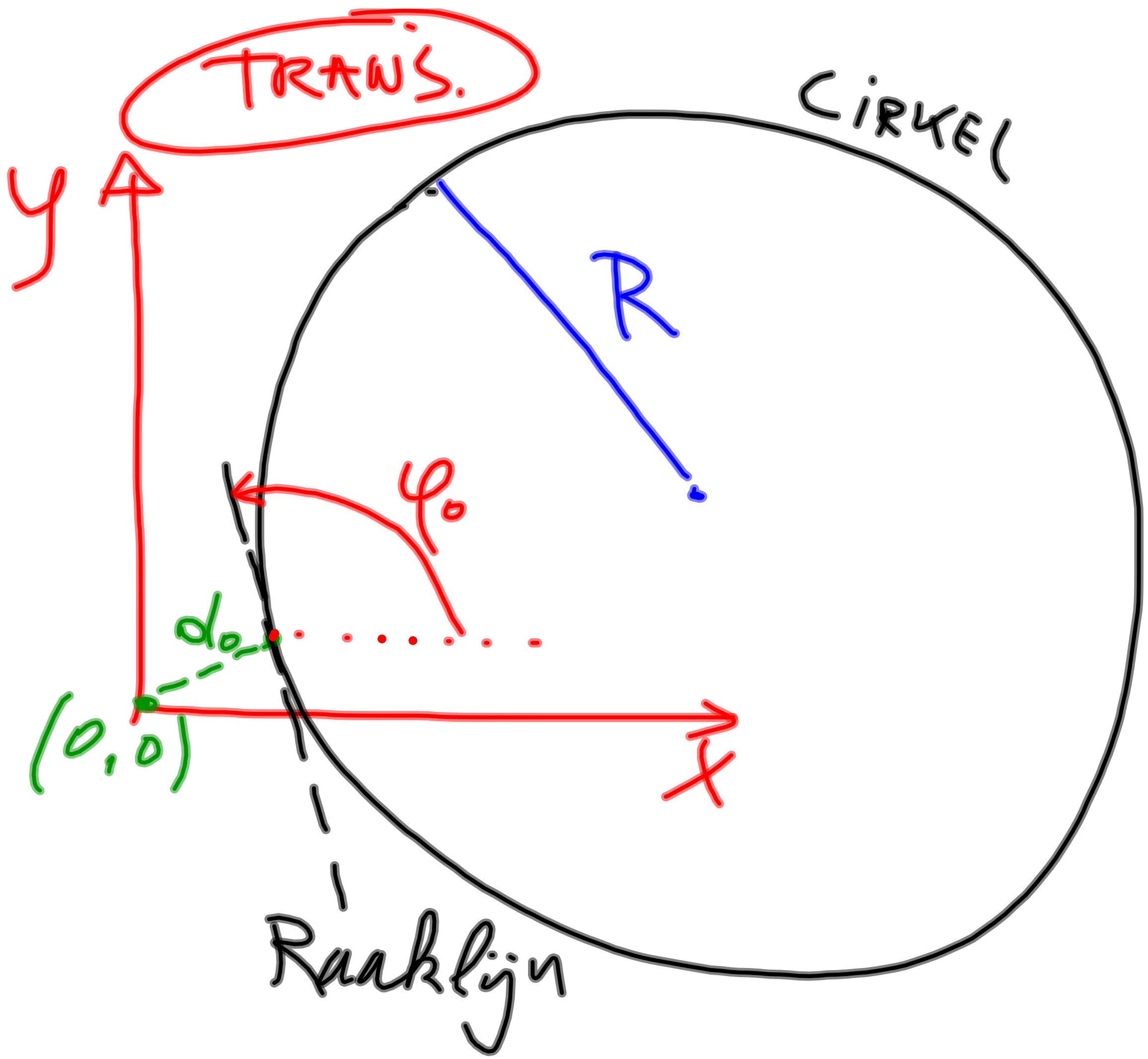
Uniform $\vec{B}$ $\qquad \theta \neq 90^\circ \qquad \vec{B} = B\,\vec{1}_z$

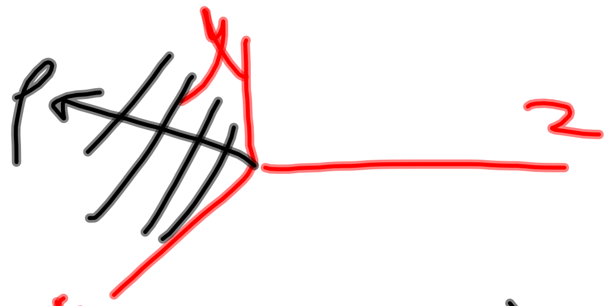


HELIX : pad geladen deeltje

$\vec{R}$

? beschrijven


TRANS.
CIRKEL
y
R
φ_0
d_0
(0, 0)
x
Raaklijn

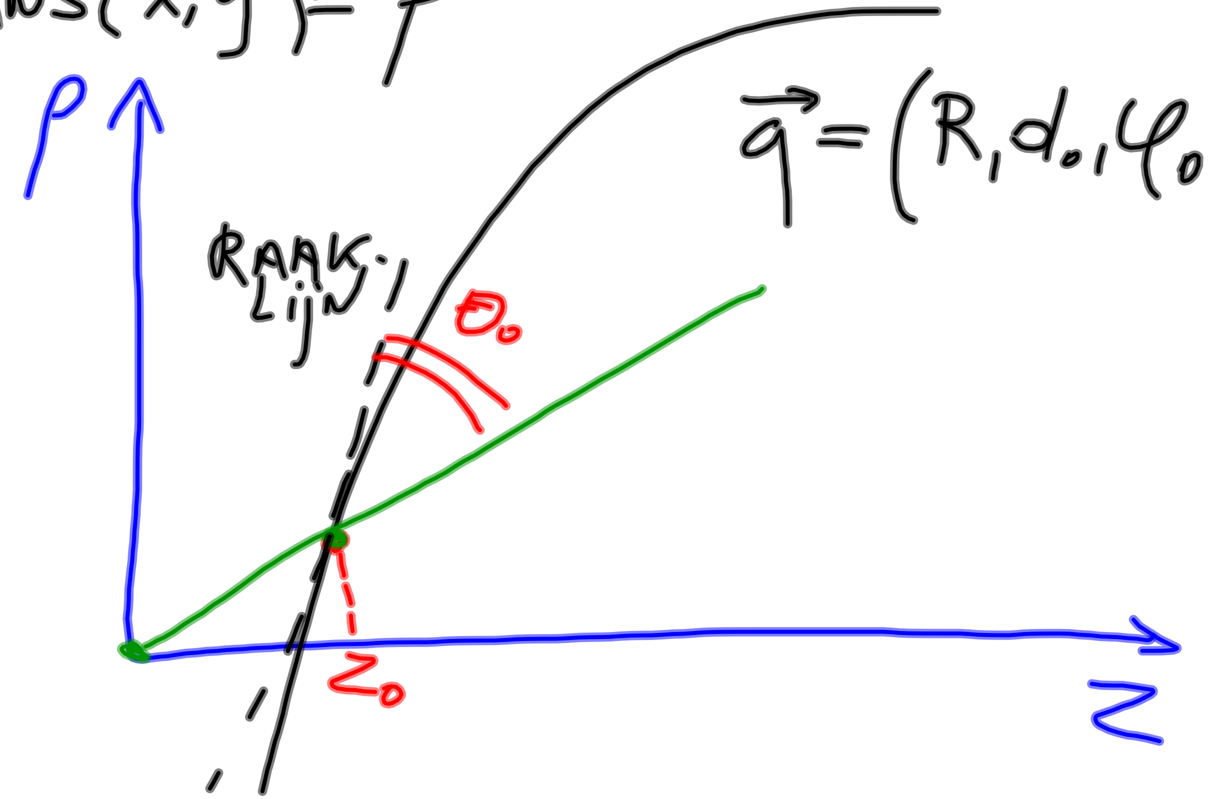


5 parameters track/spoor

"enige" parametrisatie

$\mathrm{TRANS}(x,y) \equiv \rho$

$\vec{q} = (R, d_0, \varphi_0, z_0, \theta_0)$

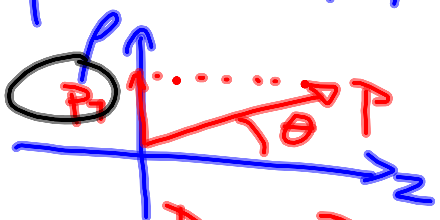


$$\vec{F} = e\,\vec{v} \times \vec{B} \rightarrow e\,\vec{v}_T \times B\,\vec{1}_z$$

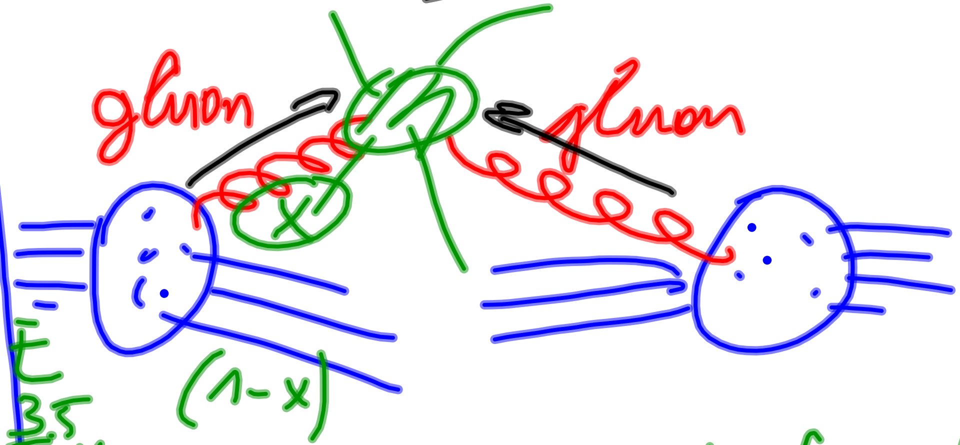
$$\vec{B} = B\,\vec{1}_z$$

$\vec{F}$ lorentz in TRANS vlak

$P \longrightarrow P_T$



$P_T = P \cdot \sin\theta$



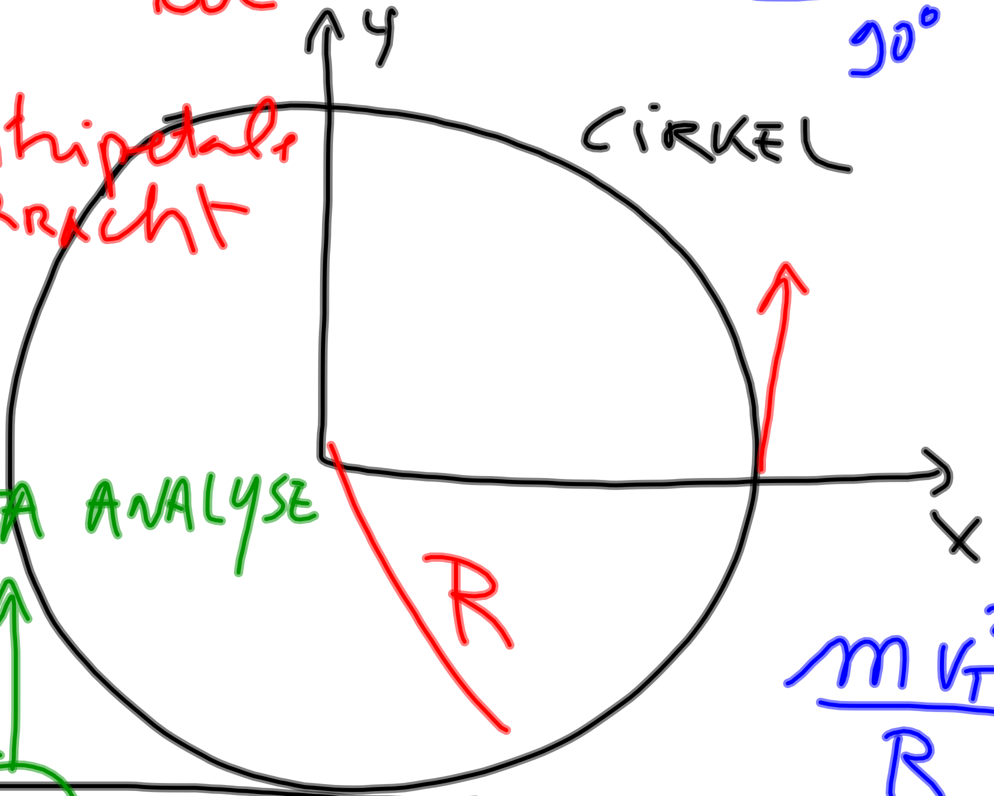
parton density functies
"kans" op deeltje i

$i = u, d, c, s, b,$
$g, \bar{u}, \bar{d}, \bar{c}, \bar{s}, \bar{b}$

$F_i(x, Q^2)$

$$\vec{F} = \frac{d\vec{p}_T}{dt} = e\,\underbrace{\vec{v}_T \times B\,\vec{1}_z}_{90^\circ}$$

centripetale kracht



$$F = \frac{mv^2}{R}$$

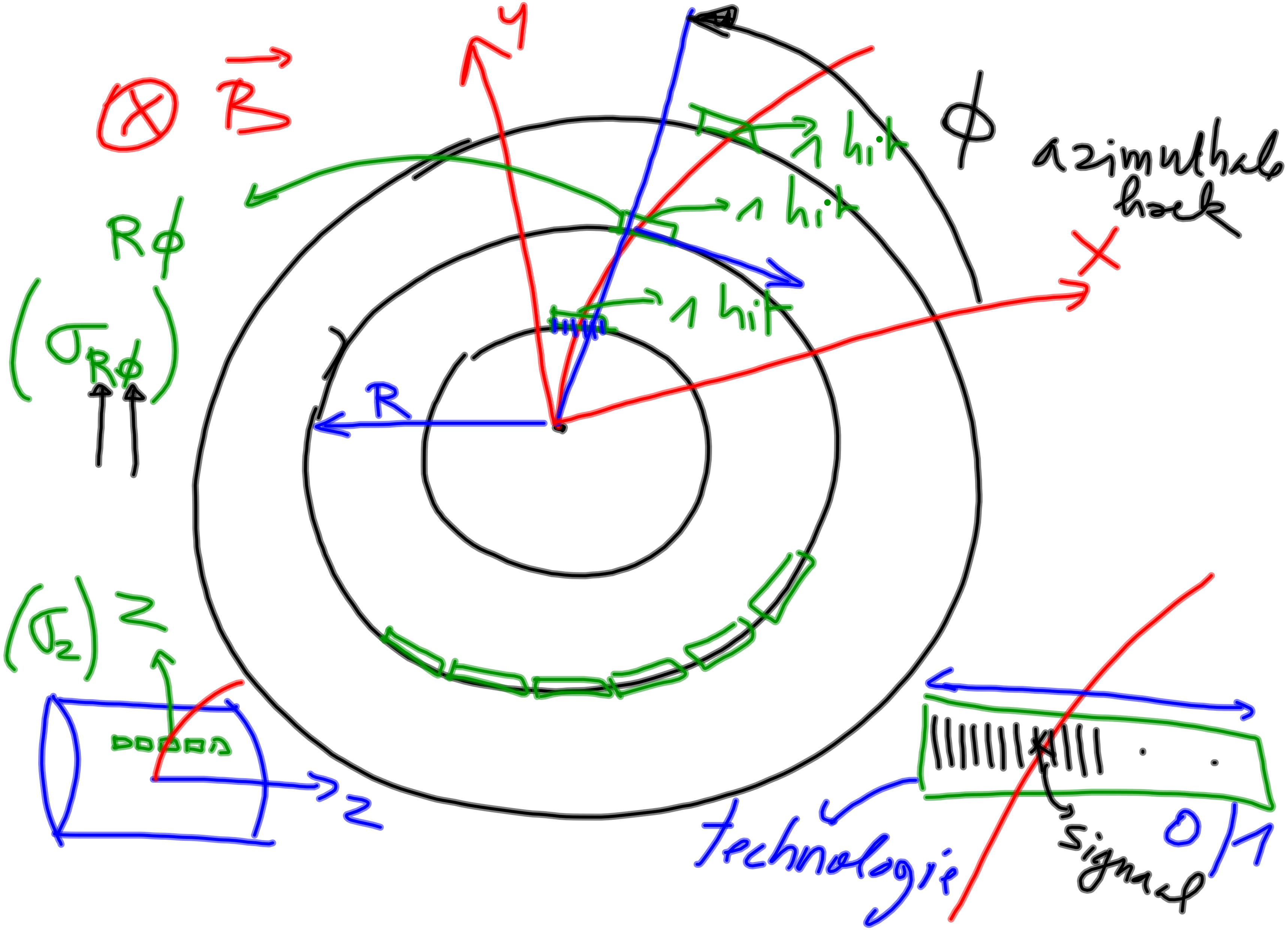
$$\frac{m v_T^2}{R} = e\, v_T\, B$$

$$\Rightarrow R = \frac{mv^2}{e v B} = \frac{mv}{eB}$$

p_T

DATA ANALYSE

$$\boxed{p_T = eRB}$$



y
x
$\vec{B}$
1 hit
1 hit
1 hit
ϕ
azimuthal
track
$R\phi$
$(\sigma_{R\phi})$
R
(σ_z) z
z
technologie
signal
0/1

$$P_T = q R B$$

$$P_T[GeV/c] = 0.3\ \underline{R[\text{meter}]} \cdot B[\text{Tesla}]$$

1 GeV/c → 0.83 m

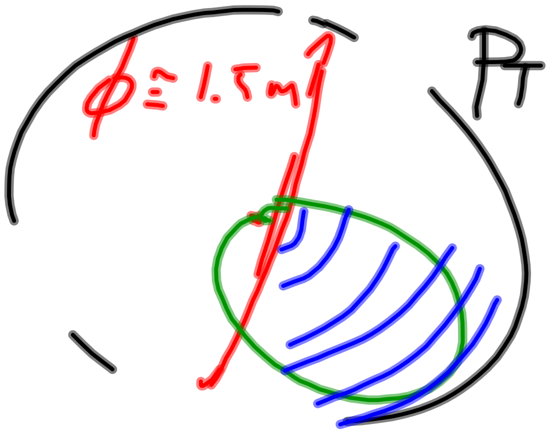
10 GeV/c → 8.3 m

100 GeV/c → 83 m

1000 830

CMS $\sim 4T$

ATLAS $\sim 2T$



$P_T < 1$ GeV

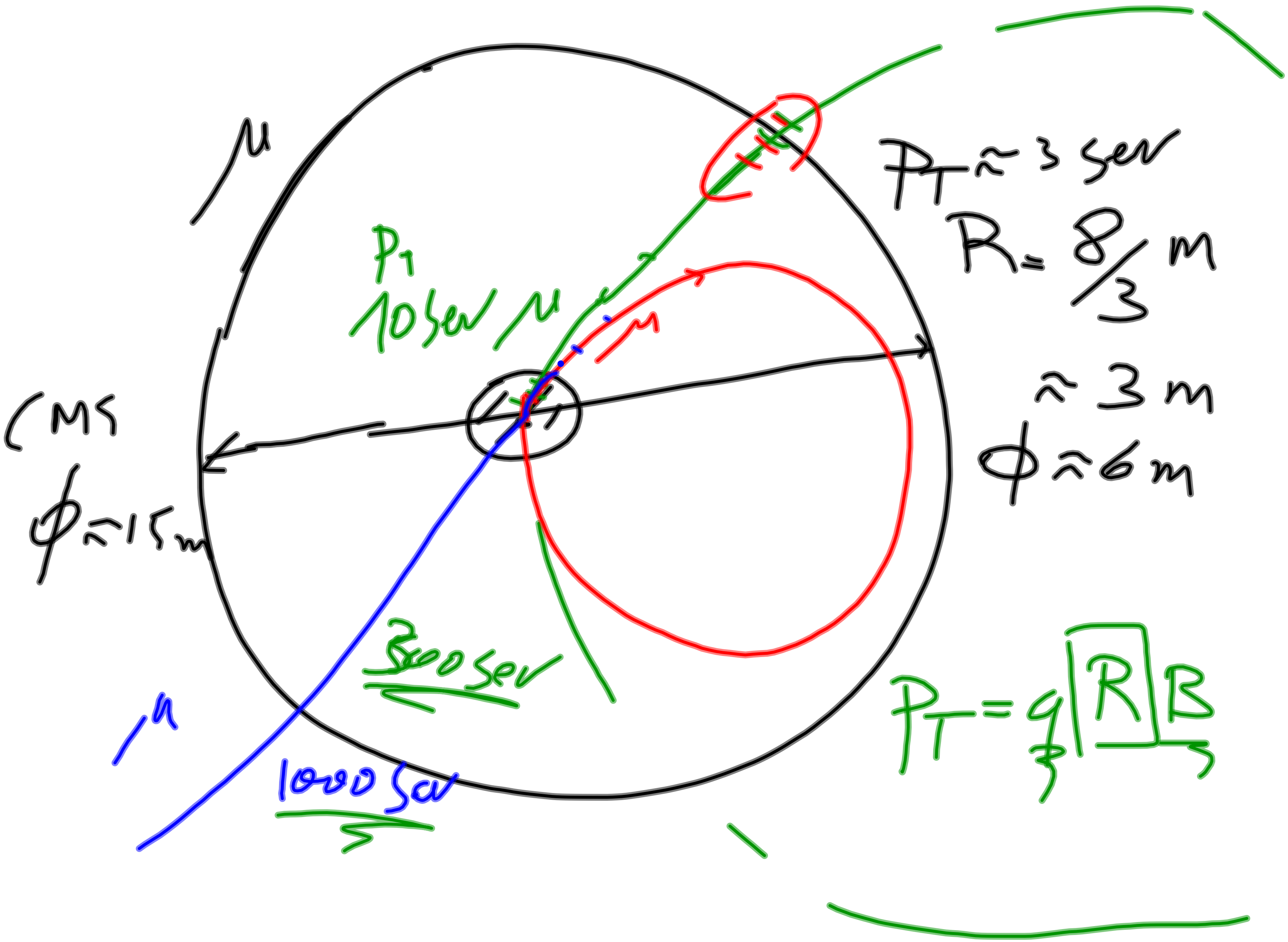
$P_T \lesssim 0.5$ GeV

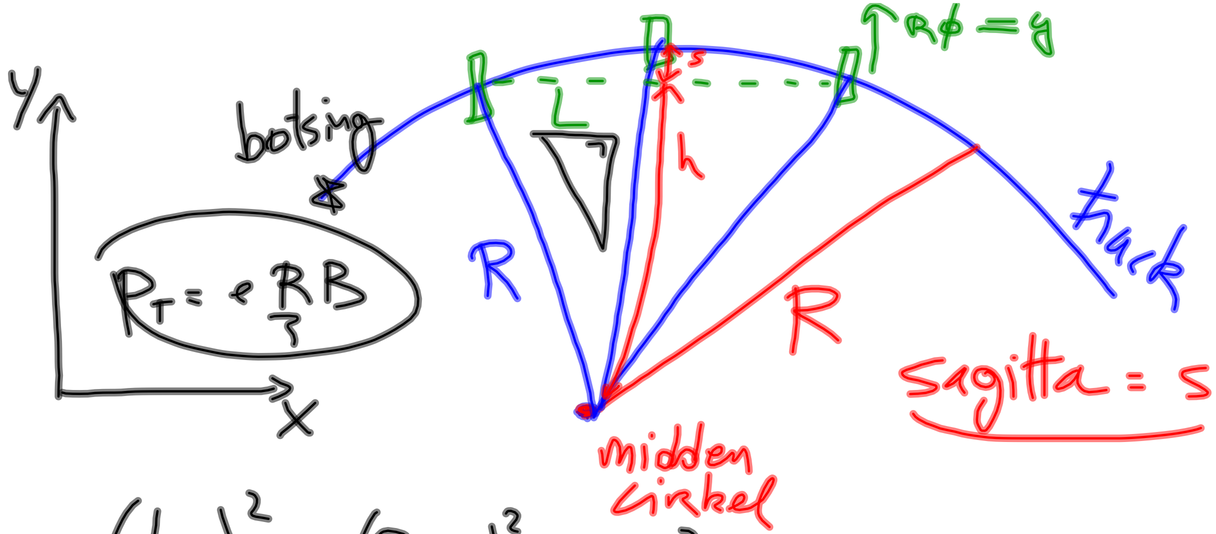
$H \to ZZ \to \mu\mu\mu\mu$

$P_T(\mu) \approx 50$ GeV

$Z' \to \mu\mu$

$m_{Z'} \approx 2$ TeV $\Rightarrow P_T \approx 1$ TeV


μ
P_1
10 GeV μ
μ
CMS
$\phi \approx 15$ m
3000 GeV
μ
1000 GeV
$P_T \approx 3$ GeV
$R = 8/3$ m
≈ 3 m
$\phi \approx 6$ m
$P_T = q\,\boxed{R}\,B$



$$\left(\frac{L}{2}\right)^2 + (R-s)^2 = R^2$$

$$\frac{L^2}{4s} + \cancel{R^2} - 2R\cancel{s} + s^{\cancel{2}} = \cancel{R^2}$$

$$\Rightarrow -\frac{L^2}{4s} + 2R = s$$

$R \gg s$

$$\Rightarrow R \cong \frac{L^2}{8s}$$

$$\Rightarrow \boxed{P_T = e\,\frac{L^2}{8s}\,B}$$

$B = 2\,T$

$L = 1\,m$

$P_T = 10\,GeV$

$\Rightarrow s = 0.7\,cm$

$$p_T = e\,\frac{L^2}{8s}\,B$$

$$\left|\frac{dp_T}{ds}\right| = e\,\frac{L^2}{8s^2}\,B = \frac{p_T}{s}$$

$$\Rightarrow \frac{dp_T}{p_T} = \frac{ds}{s} \Longrightarrow \frac{\Delta p_T}{p_T} = \frac{\Delta s}{s}$$

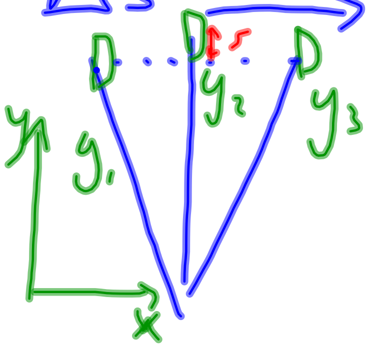
$$\Rightarrow \frac{\Delta p_T}{p_T} = \Delta s\,\frac{1}{s} = \Delta s \cdot \frac{8R}{L^2} = \Delta s\,\frac{p_T}{e\cancel{R}B}\,\frac{8\cancel{R}}{L^2}$$

$$R = \frac{L^2}{8s} \quad \Rightarrow s = \frac{L^2}{8R} \qquad p_T = eRB$$

$$\boxed{\frac{\Delta p_T}{p_T} = \Delta s\,\frac{8}{eBL^2}\,p_T}$$

$$\Delta s \to \sqrt{\tfrac{3}{2}}\,\Delta y \qquad \sigma_{R\phi}$$

$$\Delta s \longrightarrow s = y_2 - \frac{y_1 + y_3}{2}$$



$$(\Delta s)^2 = \sum_i \left(\frac{\partial s}{\partial y_i}\right)^2 (\Delta y_i)^2 = \frac{3}{2}(\Delta y)^2$$

$$= \frac{1}{4}(\Delta y)^2 + (\Delta y)^2 + \frac{1}{4}(\Delta y)^2$$

$$\boxed{\frac{\Delta p_T}{p_T} = \boxed{\sqrt{\frac{3}{2}}}\,\sigma_{R\phi}\,\frac{8}{L^2}\,\frac{p_T}{0.3\,B}}$$

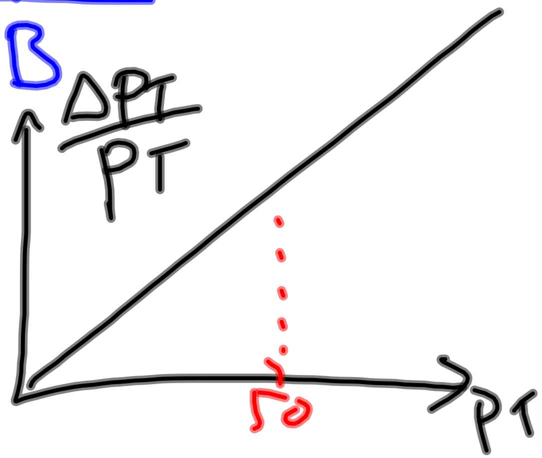
DETECTOR: MET 3 LAGEN
(y_1, y_2, y_3)

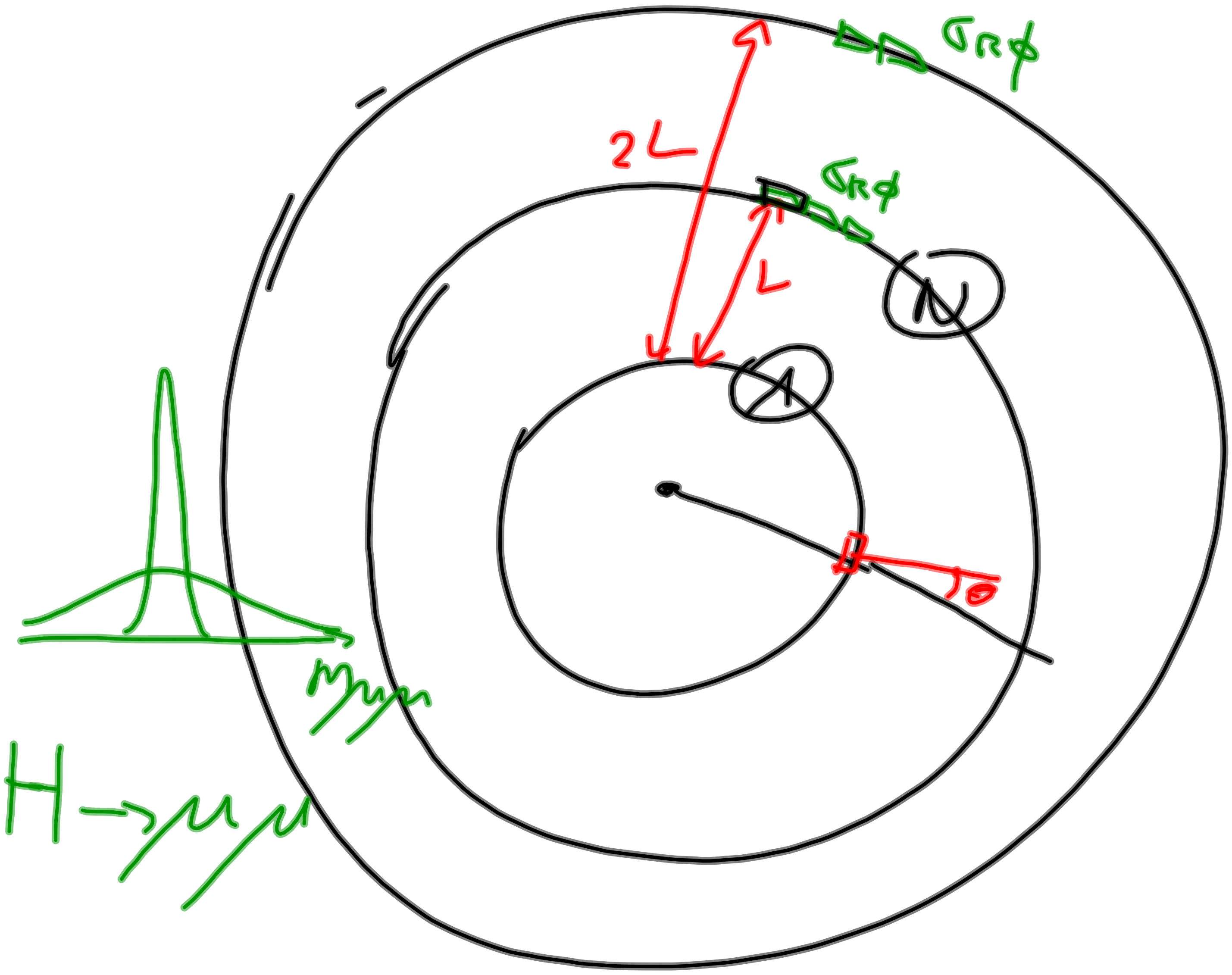
<u>OEFENING</u>: N LAGEN

%

$$\boxed{\frac{\Delta p_T}{p_T}} = a_N\,\frac{\boxed{\sigma_{R\phi}}\;p_T}{0.3\,L^2\;B}$$

$$a_N = \sqrt{\frac{720}{N+4}}$$

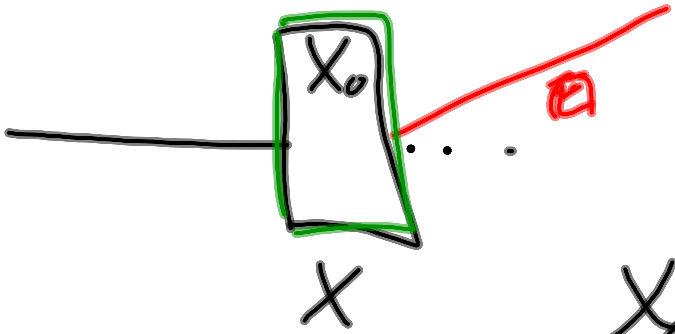



σrφ
2L
σrφ
L
θ
Mμμ
H→μμ

Multiple Scattering

spoordetector

Si $X_0 \sim 9.4\,\mathrm{cm}$



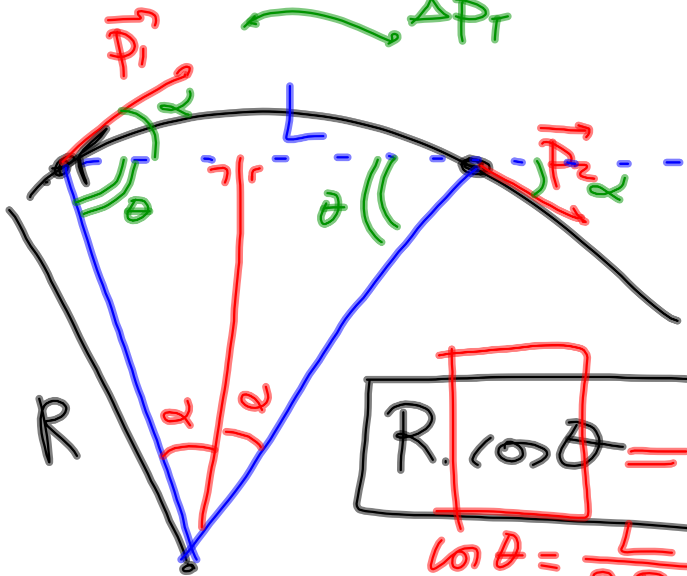
RMS

$$\theta_0 \simeq \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{X}{X_0}}$$

$$E(X) = E(0) \cdot e^{-X/X_0}$$

rms $X \sim 1 \cdot X_0$

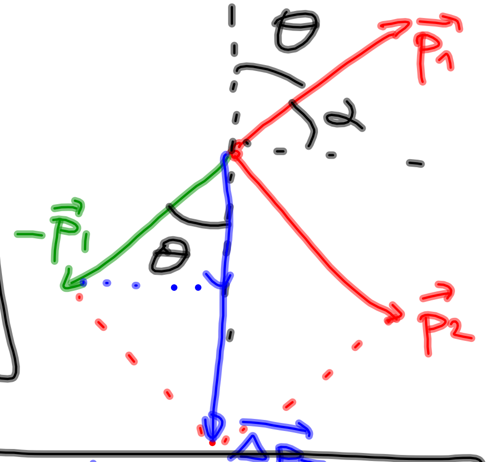
$N \sim 10$



$\vec{B}: |\Delta \vec{p}_T| = |\vec{p}_2 - \vec{p}_1|$

$\boxed{R \cdot \cos\theta = \frac{L}{2}}$

$\cos\theta = \frac{L}{2R}$



$\boxed{|\vec{p}_1| \cdot \cos\theta = \frac{|\Delta \vec{p}_T|}{2}}$

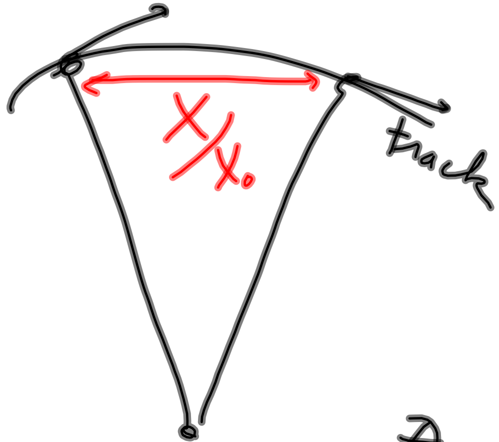
$\theta + \alpha + 90^\circ = 180^\circ$

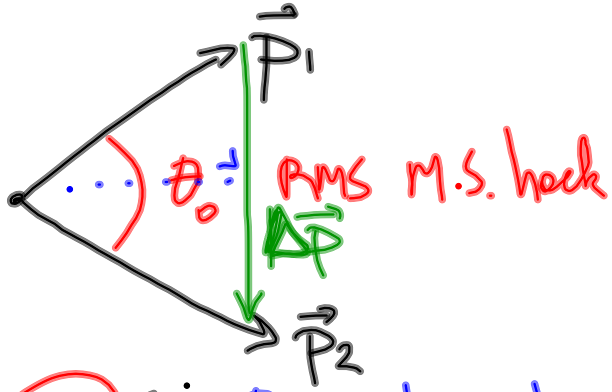
p_T

$\left|\frac{\Delta \vec{p}_T}{\cancel{2}}\right| = |\vec{p}_1| \frac{L}{\cancel{2}R}$

$$|\Delta p_T| = \frac{p_T \cdot L}{R} = \frac{0.3\,\cancel{R} \cdot B \cdot L}{\cancel{R}}$$

$$|\Delta \vec{p}_T|_{M.S.}$$





$$|\vec{p}_1| \cdot \sin\frac{\theta_0}{2} = \frac{|\Delta p_T|}{2}$$

$$|\vec{p}_1| = p_T$$

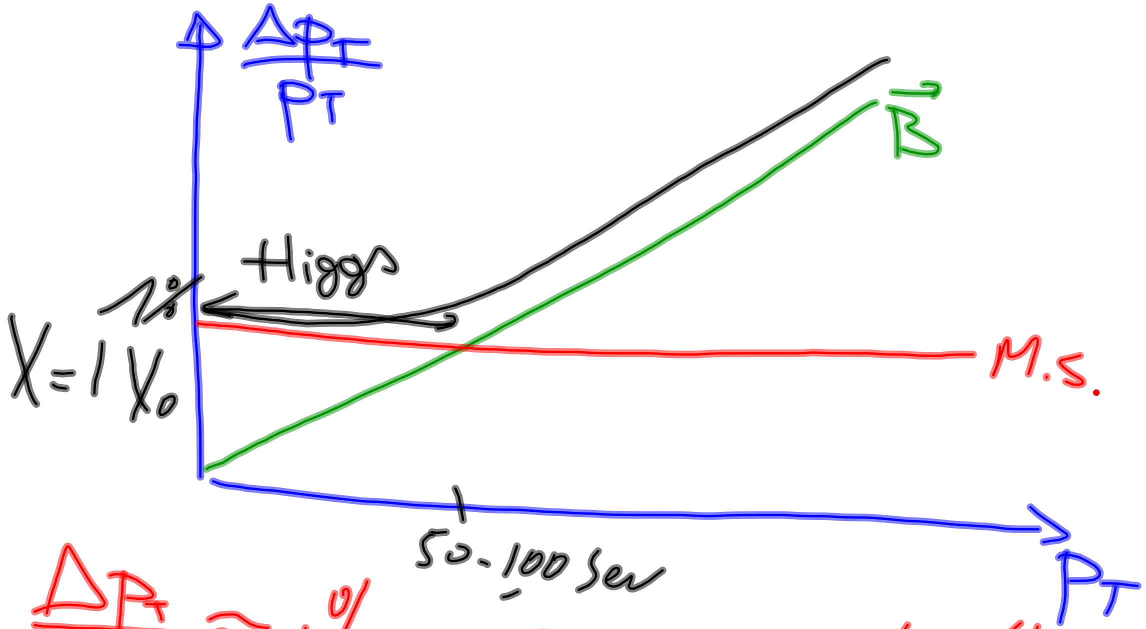
θ_0 kleine hoeken. $\sin\frac{\theta_0}{2} \approx \frac{\theta_0}{2}$

graden. $\rightarrow$ fractie

$$|\Delta p_T|_{M.S.} = p_T \cdot \theta_0$$

$$\Rightarrow \left.\frac{|\Delta p_T|}{p_T}\right|_{M.S.} = \theta_0 \longrightarrow \bar{n} \text{ afh. v. } p_T$$

$$\left(\frac{\Delta p_T}{p_T}\right)^2 = \left(\frac{\Delta p_T}{p_T}\right)^2_{\vec{B}} + \left(\frac{\Delta p_T}{p_T}\right)^2_{M.S.}$$



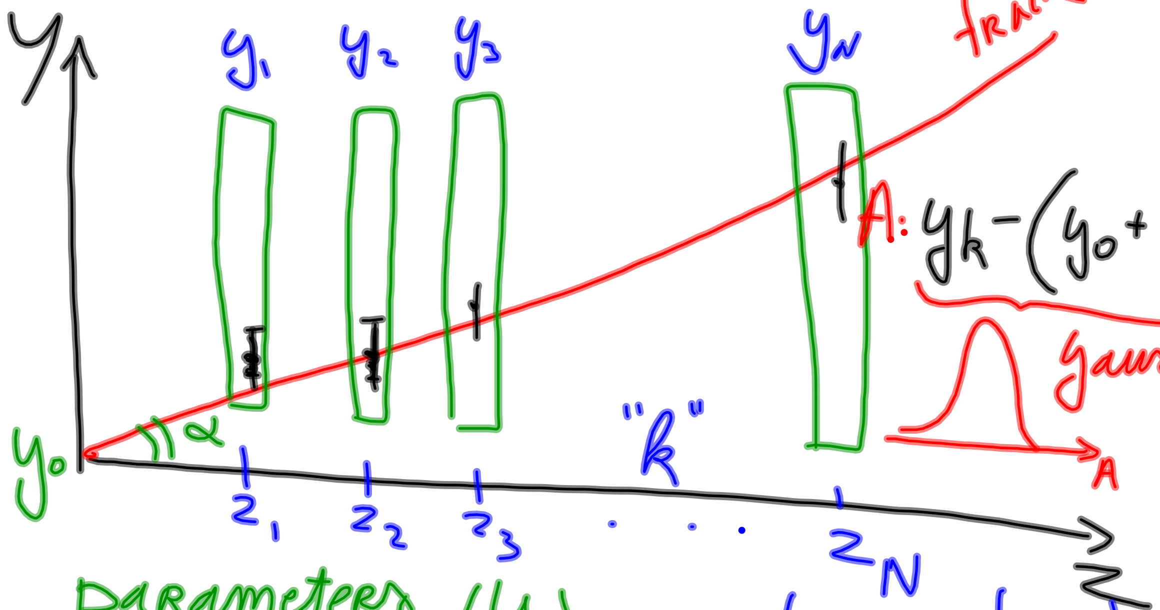
$$\frac{\Delta p_T}{p_T} \simeq 1\% \qquad p_T = 100\ \text{GeV}/c$$

$$N = 10 \qquad L = 1\,\text{m} \qquad B = 4\,\text{T}$$

$$\Longrightarrow \sigma_{R\phi} = 20\,\mu\text{m}$$

TRACK FITTING

$y = y_0 + \alpha(z_k - z_0)$



A: $y_k - (y_0 + \alpha(z_k - z_0))$

parameters $\begin{pmatrix} y_0 \\ \alpha \end{pmatrix}$ metingen $\{y_k\}$

$$\prod_{k=1}^{N} \sigma\left(y_k - \left(y_0 + \alpha(z_k - z_0)\right)\right) = \mathcal{L}$$

likelihood

$$\mathcal{L} = \prod_{k=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma_y} \cdot \exp\left[-\frac{1}{2}\left(\frac{y_k - y_0 - \alpha(z_k - z_0)}{\sigma_y}\right)^2\right]$$

kwadraat

$$\boxed{-\ln\mathcal{L}} = c^{t} + \sum_{k=1}^{N} \frac{1}{2}\left(\frac{y_k - y_0 - \alpha(z_k - z_0)}{\sigma_y}\right)^2$$

Likelihood $\rightarrow$ maximaliseren

$$\mathcal{L}(y_0, \alpha) \longrightarrow \ln\mathcal{L} \quad \max$$

$$\longrightarrow -\ln\mathcal{L} \quad \min.$$

$$\Longrightarrow \chi^2(y_0,\alpha) = \sum_{k=1}^{N}\left(\frac{y_k - y_0 - \alpha(z_k - z_0)}{\sigma_y}\right)^2 \quad \underline{\min}$$

$$\begin{cases} \dfrac{\partial \chi^2(y_0,\alpha)}{\partial y_0} = 0 \\[2ex] \dfrac{\partial \chi^2(y_0,\alpha)}{\partial \alpha} = 0 \end{cases} \Longrightarrow (\hat{y}_0, \hat{\alpha})$$

$$\vec{q} = (R, d_0, \theta_0, \varphi_0, z_0) \longrightarrow 5 \text{ parameters.}$$

$$\vec{y} = (y_1, y_2, \dots, y_N) \longrightarrow N \text{ metingen}$$

$y_k = f_k(\vec{q})$ "transformatie" (helix model) ↳ niet-lineair

⟹ Lineariseren!

$$y_k = f_k(\vec{q}_A) + \sum_{i=1}^{5} \left.\frac{\partial f_k(\vec{q})}{\partial q_i}\right|_{\vec{q}=\vec{q}_A} \cdot (\vec{q}_i - \vec{q}_{A_i})$$

Rond $\vec{q} = \vec{q}_A$

$$\sum_{k=1}^{N} \left(\frac{y_k - f_k(\vec{q}_A) - \sum_{i=1}^{5} \left.\frac{\partial f_k(\vec{q})}{\partial q_i}\right|_{\vec{q}=\vec{q}_A} (\vec{q}_i - \vec{q}_{A_i})}{\sigma_{y_k}} \right)^2$$

$\chi^2(\vec{q})$

$$A_{ki} \equiv \left.\frac{\partial f_k(\vec{q})}{\partial q_i}\right|_{\vec{q}=\vec{q}_A}$$

$$\chi^2 = \sum_{k=1}^{N} \left(\frac{y_k - f_k(\vec{q}_A) - A_{ki}(\vec{q}_i - \vec{q}_{iA})}{\sigma_{y_k}} \right)^2$$

$$\chi^2 = \left[\vec{y} - \vec{f}(\vec{q}_A) - \tilde{A}\cdot(\vec{q}-\vec{q}_A)\right]^T \cdot \tilde{V}_y^{-1} \cdot \left[\vec{y} - \vec{f}(\vec{q}_A) - \tilde{A}\cdot(\vec{q}-\vec{q}_A)\right]$$

matrix

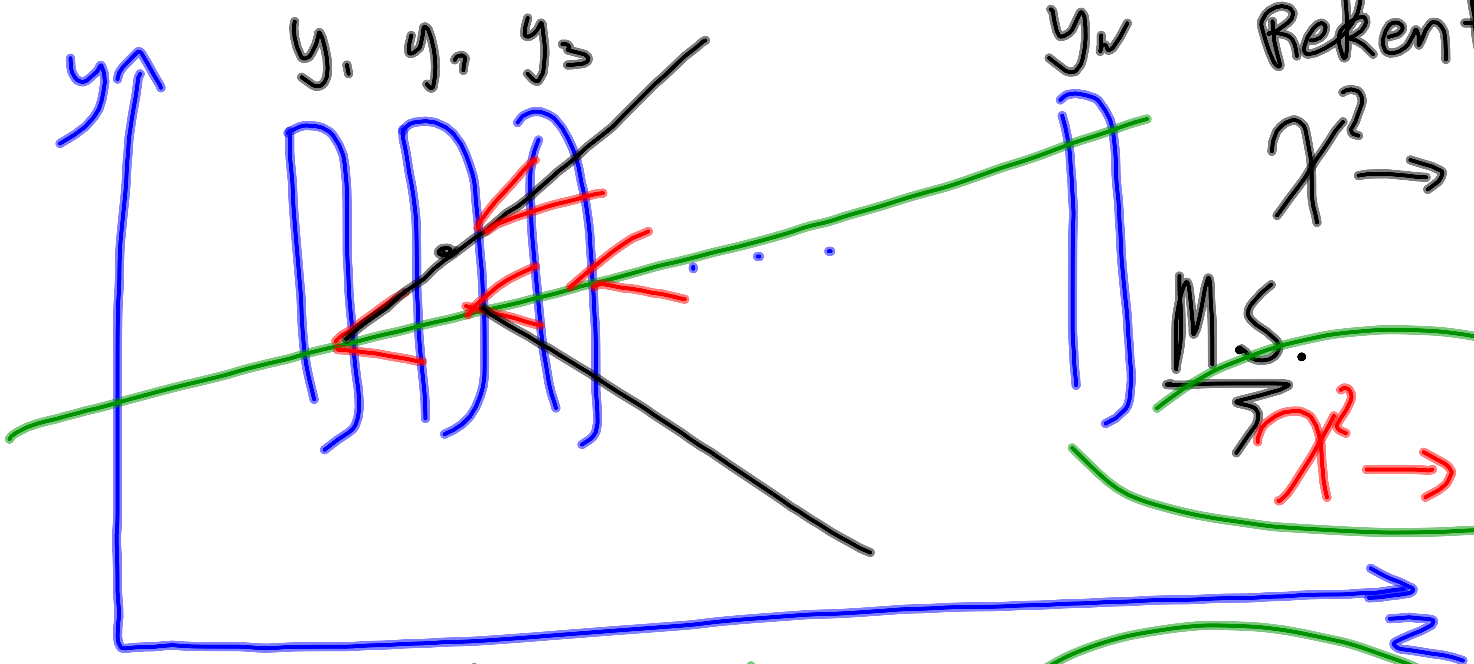
$$\tilde{V}_y^{-1} = \begin{pmatrix} 1/\sigma_{y_1}^2 & 0 & & \\ 0 & 1/\sigma_{y_2}^2 & & \\ & & \ddots & \\ & & & 1/\sigma_{y_N}^2 \end{pmatrix}$$

$$\Delta\vec{y} \equiv \vec{y} - \vec{f}(\vec{q}_A)$$

$$\chi^2 = \left[\Delta\vec{y} - \tilde{A}\cdot(\vec{q}-\vec{q}_A)\right]^T \cdot \tilde{V}_y^{-1} \left[\Delta\vec{y} - \tilde{A}(\vec{q}-\vec{q}_A)\right]$$

$$\left\{ \frac{\partial \chi^2(\vec{q})}{\partial q_i} = 0 \right\} \Longrightarrow \boxed{\vec{q} = \vec{q}_A + \left(\tilde{A}^T \tilde{V}_y^{-1} \tilde{A}\right)^{-1} \cdot \tilde{A}^T \tilde{V}_y^{-1} \cdot \Delta\vec{y}}$$

$$\tilde{V}_q = \left(\tilde{A}^T \tilde{V}_y^{-1} \tilde{A}\right)^{-1}$$



Rekentijd

$\chi^2 \to N$

M.S.

$\chi^2 \to N^3$

$$\bar{V}_y = \begin{pmatrix} x & & \cancel{0} & \\ \cancel{0} & y & & \\ & & y & \\ & & & x \end{pmatrix}$$

M.S.

$y_k = f_k(\vec{p})$

"HELIX"

KALMAN FILTER

Hoe q deeltje beweegt in $\vec{B}$

precisie v.d. meting

M.S. (θ_0)

voorspellen (y_1 & y_2)

vergelijken

meting y_3

? voorspelling aanpassen ?

TRACK MODEL

(HELIX)

$\vec{q}(z_k) \equiv \vec{q}_k$ "state v.d. vector"

$$\boxed{\vec{q}_k = f_{k-1}(\vec{q}_{k-1}) + \vec{\omega}_{k-1}}$$

stochastisch "Random"

effect M.S

$E[\vec{\omega}] = 0$

MEASUREMENT MODEL

$(R\phi, z)$ $\boxed{\vec{m}_k = h_k(\vec{q}_k) + \vec{\varepsilon}_k}$ → detector Resolutie

transformatie

$E[\vec{\varepsilon}] = 0$

indien f_k & h_k lineair

$\Longrightarrow$ lineair dyn. system

$\Longrightarrow$ OPTIMAAL !

$\Longrightarrow$ onzekerheid op $\vec{q}$ zo klein mogelijk (M.V.B.)

$$f \longrightarrow \tilde{F} \; (5\times5)$$
$$h \longrightarrow \tilde{H} \; (2\times5)$$

MODEL

$$\boxed{\begin{aligned}\vec{q}_k &= \tilde{F}_{k-1}\,\vec{q}_{k-1} + \vec{w}_{k-1}\\ \vec{m}_k &= \tilde{H}_k\,\vec{q}_k + \vec{\varepsilon}_k\end{aligned}}$$

→ M.S.

→ detector Res.

$$\mathrm{cov}(\vec{w}_k) = \tilde{Q}_k$$
$$\mathrm{cov}(\vec{\varepsilon}_k) = \tilde{V}_k$$

$$\vec{q}_{k,t} = \tilde{F}_{k-1}\,\vec{q}_{k-1,t} + \vec{w}_{k-1}$$
$$\vec{m}_k = \tilde{H}_k\,\vec{q}_{k,t} + \vec{\varepsilon}_k$$

t : true

$\vec{q}_k^{\,i}$: parameters $\vec{q}$ op positie k, rekening houdend met meting tot detector i

. . . .

$$\mathrm{cov}(\underbrace{\vec{q}_{k,t} - \vec{q}_k^{\,i}}_{\text{ERROR}}) = \tilde{C}_k^i$$

$$\vec{R}_k^i = \underbrace{\vec{m}_k}_{\text{meting}} - \underbrace{\tilde{H}_k \vec{q}_k^{\,i}}_{\text{voorspelling}} \qquad \text{"residual"}$$

$$\hookrightarrow \mathrm{cov}(\vec{m}_k - \tilde{H}_k \vec{q}_k^{\,i}) = \tilde{R}_k^i$$

KALMAN FILTER

① VOORSPELLING. $(k-1 \longrightarrow k)$

$$\vec{q}_k^{\,k-1} = \tilde{F}_{k-1}\,\vec{q}_{k-1} \Longrightarrow \tilde{C}_k^{k-1} = \tilde{F}_{k-1}\,\tilde{C}_{k-1}\,\tilde{F}_{k-1}^T + \tilde{Q}_{k-1}$$

$$\vec{R}_k^{\,k-1} = \boxed{\vec{m}_k} - \tilde{H}_k\,\vec{q}_k^{\,k-1}$$

informatie nog niet gebruikt !

$$\Longrightarrow \tilde{R}_k^{k-1} = \tilde{H}_k\,\tilde{C}_k^{k-1}\,\tilde{H}_k^T + \tilde{V}_k$$

② filtering

KALMAN GAIN MATRIX

$$\underline{\vec{q}_k^{\,k}} = \boxed{\underline{\vec{q}_k^{\,k-1}}} + \boxed{\tilde{K}_k} \, \underline{(\vec{m}_k - \tilde{H}_k \vec{q}_k^{\,k-1})}$$

$$\tilde{K}_k \equiv \underbrace{\tilde{C}_k^{k-1}}_{} \boxed{\tilde{H}_k^T} \underbrace{(\tilde{V}_k + \tilde{H}_k \tilde{C}_k^{k-1} \tilde{H}_k^T)}_{}^{-1}$$

⟹ Min. Var. Bound.

(kleinste variantie voor $\vec{q}$)

"onzekerheid op de voorspelling"

"onzekerheid op $\vec{R}_k^{k-1}$"

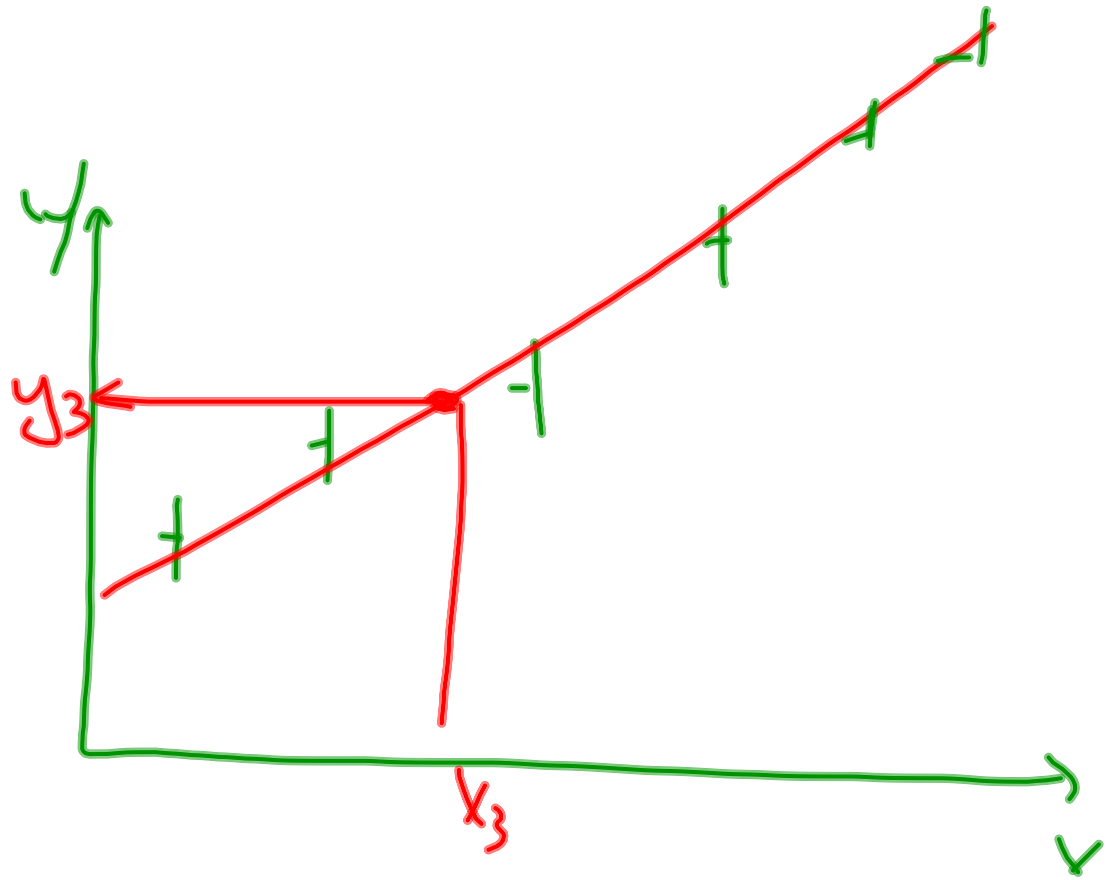
$$\underbrace{\tilde{C}_k^k}_{\{1\ldots k\}} = (\mathbb{1} - \tilde{K}_k \tilde{H}_k) \cdot \underbrace{\tilde{C}_k^{k-1}}_{\text{unz. } \vec{q}}$$

$$\{1\ldots k\} \qquad \{1\ldots(k-1)\}$$

$$\boxed{\vec{R}_k^k = \vec{m}_k - \tilde{H}_k \vec{q}_k^{\,k}}$$

$$= (\mathbb{1} - \tilde{H}_k \tilde{K}_k)\, \vec{R}_k^{k-1}$$

$$\tilde{R}_k^k = (\mathbb{1} - \tilde{H}_k \tilde{K}_k) \cdot \tilde{V}_k$$



y
y_3
x_3
x