Extensions of the Standard Model

(part 2)

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Content:

- **The Higgs sector of the Standard Model and extensions**
- **Theoretical constraints on the Higgs boson mass**
- **Searching for the Higgs boson**
- **The hierarchy problem in the Standard Model**
- **Introduction to the phenomenology of Supersymmetry**

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

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Lecture 1

Important note: whenever we note "Higgs boson" (or mechanism), we mean the well-known Brout-Englert-Higgs boson or mechanism.

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How far can we stretch our theory?

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Content:

- **Short reminder of the spontaneous ElectroWeak symmetry breaking**
- **One missing piece: the mass of the Brout-Englert-Higgs scalar**
- **Theoretical constraints on the Higgs boson mass**
  - Pertubativity or unitarity constraint
  - Triviality bound and stability bound
  - Fine-tuning
- **Methods can be applied to models beyond the Standard Model**
- **What about more then one Higgs doublet…**

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

Spontaneously broken QED theory

Let us illustrate the "Higgs" mechanism with a massive U(1) theory before going to the symmetry group $SU(2)_L \times U(1)_Y$. The Lagrangian of QED is:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu D_\mu - m)\psi$$

This is invariant under the U(1) gauge transformation

$$\psi \rightarrow e^{-i\alpha(x)}\psi$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

Now we wish to give the photon a mass by adding the term

$$\mathcal{L}_{mass} = \frac{m_A^2}{2}A_\mu A^\mu$$

Which breaks the initial U(1) gauge symmetry. Hence need to invoke a mechanism which introduces a mass without breaking the symmetry.

Spontaneously broken QED theory

Introduce a complex scalar field Φ as

$$\mathcal{L} = \mathcal{L}_{QED} + (D_\mu \Phi)^*(D^\mu \Phi) - V(\Phi)$$

with the potential V defined as $V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$ which is symmetric under the transformation $\Phi \to -\Phi$

We can choose a parametrization as

$$\Phi = \frac{1}{\sqrt{2}}\phi(x)e^{i\xi(x)}$$

where both fields ϕ and ξ are real fields.

The potential becomes

$$V(\Phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

where the Higgs self-coupling term should be positive (λ>0) to get a potential bound from below. When $\mu^2 < 0$ a non-zero vacuum expectation value is obtained.

$$< 0|\phi^2|0 >= \phi_0^2 = \frac{-\mu^2}{\lambda} = v^2$$

Spontaneously broken QED theory

Therefore we can normalize the field $\xi(x)$ as $\frac{\xi(x)}{\phi_0}$.

We can choose the unitary gauge transformation

$$\alpha(x) = -\frac{\xi(x)}{\phi_0}$$

and then Φ becomes real-valued everywhere. The kinetic term in the Lagrangian becomes

$$(D_\mu \Phi)^* (D^\mu \Phi) \rightarrow \frac{1}{2}\partial_\mu \psi \partial^\mu \psi + \frac{e^2}{2} A_\mu A^\mu \psi^2$$

The Lagrangian can be expanded around its minimum ϕ_0 by introducing a degree of freedom h (a new field). The potential becomes

$$V(\phi \rightarrow \phi_0 + h) = + \frac{m_h^2}{2} h^2 + \frac{\mu'}{3!} h^3 + \frac{\eta}{4!} h^4$$

with $m_h^2 = 2\lambda\phi_0^2$ **and** $\mu' = \frac{3m_h^2}{\phi_0}$ **and** $\eta = 6\lambda = 3\frac{m_h^2}{\phi_0^2}$

Spontaneously broken QED theory

The kinetic term becomes

$$\frac{1}{2}\partial_\mu(\phi_0 + h)\partial^\mu(\phi_0 + h) + \frac{e^2}{2}A_\mu A^\mu(\phi_0 + h)^2$$

and with $\partial_\mu \phi_0 = 0$ this becomes

$$\frac{e^2}{2}A_\mu A^\mu \phi_0^2 + e^2 A_\mu A^\mu \phi_0 h + \frac{e^2}{2}A_\mu A^\mu h^2 + \frac{1}{2}(\partial_\mu h)(\partial^\mu h)$$

where the first term provides a mass to the photon $m_A^2 = e^2\phi_0^2$,
the second term gives the interaction strength of the coupling A-A-h,
the third term the interaction strength of the coupling A-A-h-h

In the new potential term $V(\phi_0 + h)$ also cubic terms appear which break the reflexion symmetry $\phi \longrightarrow -\phi$.

This U(1) example is the most trivial example of a spontaneous broken symmetry.

Spontaneously broken SU(2)xU(1) theory

The bosonic part of the Lagrangian is

$$\mathcal{L}_{bosonic} = |D_\mu \Phi|^2 - \mu^2|\Phi|^2 - \lambda|\Phi|^4 - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}$$

with Φ a doublet field consisting out of two complex scalar fields or components

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$$

We need at least 3 massive gauge bosons, hence need at least 2 complex fields (cfr. Goldstone theorem).

$$\begin{aligned} D_\mu\Phi &= \left(\partial_\mu + ig\frac{\tau^a}{2}W^a_\mu + ig'\frac{Y}{2}B_\mu\right)\Phi \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^a_{\mu\nu} &= \partial_\nu W^a_\nu - \partial_\nu W^a_\mu - gf^{abc}W^b_\mu W^c_\nu \end{aligned}$$

with τ^a the Pauli matrices and f^{abc} the structure constants of the $SU(2)_L$ group.

Spontaneously broken SU(2)xU(1) theory

The B_μ field corresponds to the generator Y of the $U(1)_Y$ group and the three W_μ^a fields to the generators T^a of the $SU(2)_L$ group.

$$T^a = \frac{1}{2}\tau^a$$
$$\left[T^a, T^b\right] = i f^{abc} T^c$$
$$Tr\left[T^a T^b\right] = \frac{\delta_{ab}}{2}$$

When $\mu^2<0$ the vacuum expectation value of Φ is non-zero.

$$< 0|\Phi|0 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad with \qquad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

The VEV will carry the hypercharge and the weak charge into the vacuum, but the electric charge remains unbroken, hence $Q = T^3 + \frac{Y}{2}$ and we break $SU(2)_L x U(1)_Y$ to $U(1)_Q$ with only one generator.

Expending the terms in the Lagrangian around the minimum of the potential gives

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Spontaneously broken SU(2)xU(1) theory

We obtain

$$|D_\mu \Phi|^2 \rightarrow \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{8} g^2 (v+h)^2 |W_\mu^{(1)} + iW_\mu^{(2)}|^2 + \frac{1}{8}(v+h)^2 |gW_\mu^{(3)} - g' B_\mu|^2$$

and define the following fields

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}\left(W_\mu^{(1)} \mp iW_\mu^{(2)}\right) \\ Z_\mu &= \frac{gW_\mu^{(3)} - g' B_\mu}{\sqrt{g^2 + g'^2}} \\ A_\mu &= \frac{gW_\mu^{(3)} + g' B_\mu}{\sqrt{g^2 + g'^2}} \end{aligned}$$

which we can transform into expressions for $B_\mu \quad and \quad W_\mu^{(i)}$
and put this in the above equation for $|D_\mu \Phi|^2$ **and isolate the Higgs boson interaction terms**

$$\mathcal{L}_{Higgs\ int} = \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)\left(1 + \frac{h}{v}\right)^2 - \frac{m_h^2}{2} - \frac{\xi}{3!} h^3 - \frac{\eta}{4!} h^4$$

Spontaneously broken SU(2)xU(1) theory

...

$$\mathcal{L}_{Higgs\ int} = \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)\left(1 + \frac{h}{v}\right)^2 - \frac{m_h^2}{2} - \frac{\xi}{3!}h^3 - \frac{\eta}{4!}h^4$$

with

$$\begin{aligned} m_W^2 &= \tfrac{1}{4}g^2v^2 \\ m_Z^2 &= \tfrac{1}{4}(g^2 + g'^2)v^2 \\ m_h^2 &= 2\lambda v^2 \\ \xi &= 3\frac{m_h^2}{v} \\ \eta &= 6\lambda = 3\frac{m_h^2}{v^2} \end{aligned}$$

where it is convenient to define the Weinberg mixing angle θ_W

$$tan\theta_W = \frac{g'}{g}$$

and therefore

$$\frac{m_W^2}{m_Z^2} = 1 - sin^2\theta_W$$

Spontaneously broken SU(2)xU(1) theory

From experiment we know

$$\begin{aligned} m_W &\simeq 80 GeV \\ m_Z &\simeq 91 GeV \\ g &\simeq 0.65 \\ g' &\simeq 0.35 \end{aligned}$$

Hence we obtain $v \simeq 246\ GeV$

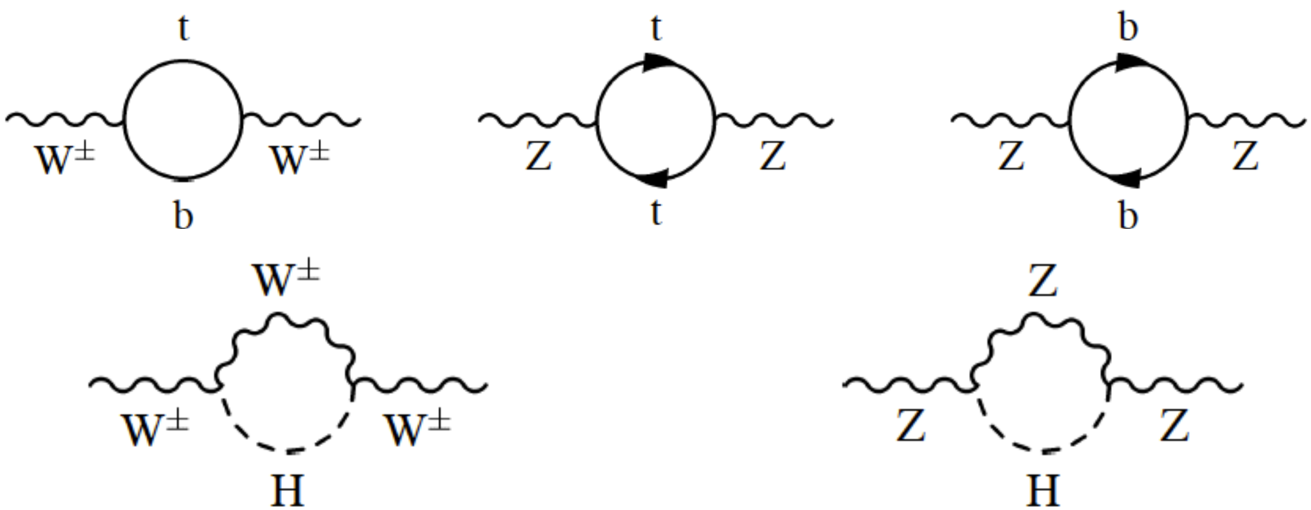
And for the couplings between V=W/Z bosons and the Higgs boson

$$\begin{aligned} g_{hVV} &= 2\frac{m_V^2}{v} \\ g_{hhVV} &= 2\frac{m_V^2}{v^2} \\ g_{hhh} &= 3\frac{m_h^2}{v} \\ g_{hhhh} &= 3\frac{m_h^2}{v^2} \end{aligned}$$

We observe that the Higgs sector in the Standard Model is completely determined from the mass of the Higgs boson.

Experimental constraints on the Higgs boson mass

Radiative corrections on the propagators of bosons in the theory



References:

- **"A combination of preliminary Electroweak Measurements and Constraints on the Standard Model", hep-ex/0612034 (and recent updates)**
- **"Precision Electroweak measurements on the Z boson resonance", hep-ex/0509008**

The free parameters in the fit

- The Standard Model gives a unified description of Electro-Magnetic & Weak interactions, hence the weak coupling is related to the EM coupling → only 2 coupling constants remain independent
  1) α : EM interactions (fine structure constant)
  2) α_s : strong interactions

- Among the fermion masses only the top quark mass plays an important role (all others are well enough determined and can be assumed fixed) as they have $m_f << m_Z$ and do not influence the observations at high energies significant: m_t

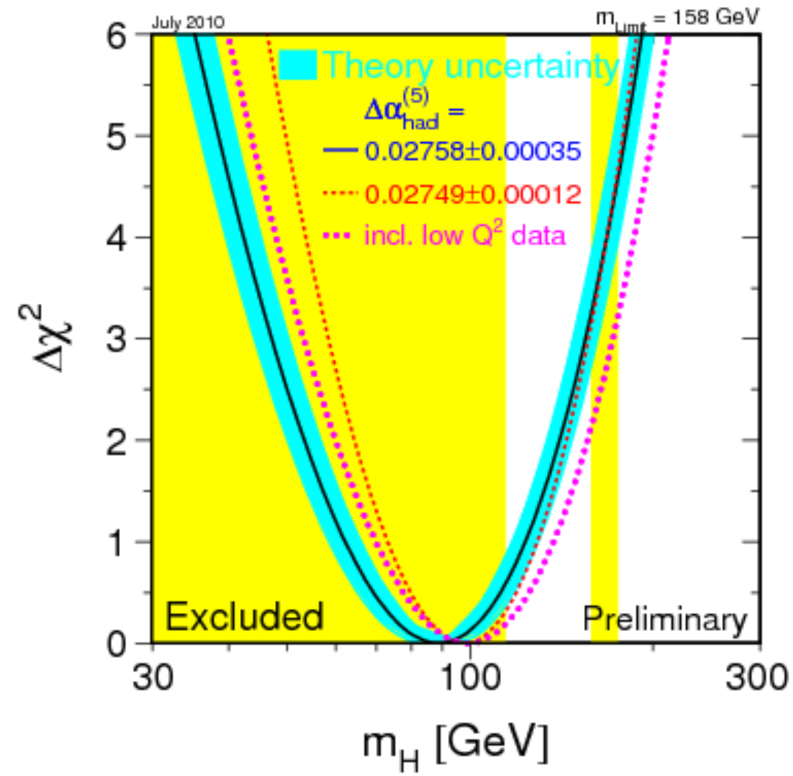
- Among the boson masses the Z boson mass (m_Z) is very well measured while the W boson mass not that presice. The free parameter m_W has been replaced by G_F, hence m_W becomes a quantity derived from the SM relations or the EW fit.

- The Higgs boson mass (m_H).

→ the free parameters are $\alpha_s(m_Z^2)$, $\alpha(m_Z^2)$, m_Z, m_t, m_H, G_F

The ElectroWeak fit: the result

- Five relevant input parameters of the Standard Model relations
  $\alpha_s(m_Z^2)$, $\alpha(m_Z^2)$, m_Z, m_t, m_H, G_F
- Given these parameters we can obtain indirect measurements of the observables measured directly by LEP, SLC, Tevatron.



- These predictions go through radiative corrections calculated to some precision
  → blueband in the plot
  (eg. 2-loop fermionic and bosonic corrections for the calculation of m_W)
- On each of the input parameters there is some uncertainty, hence we derive a confidence interval where the observed quantity should have its value give the SM relations
  → reflected in the $\Delta\chi^2(m_H)$

Theoretical constraints on the Higgs boson mass

Aim:

- Get a feeling how one can test if a theory is consistent
- How far can we stretch the EW theory until it does not make sense anymore?
- Example for the yet unobserved Higgs sector in the Standard Model, but techniques can be applied elsewhere

Content:

- Perturbativity & unitarity
- The triviality bound
- The vacuum stability bound
- The fine tuning constraints

Perturbative constraint & unitarity

The scattering of vector bosons at high energies is divergent due to their longitudinal polarization. Take V = W or Z traveling in the z-direction with 3-momentum magnitude k.

$$k^\mu = (E_k; \vec{k}) = (E_k; 0, 0, k)$$

with

$$E_k^2 = k^2 - m_V^2$$

The three polarization vectors are (resp. right handed, left handed and longitudinal):

$$\begin{aligned}
\epsilon_+^\mu(\vec{k}) &= \frac{1}{\sqrt{2}}(0; 1, i, 0) \\
\epsilon_-^\mu(\vec{k}) &= \frac{1}{\sqrt{2}}(0; 1, -i, 0) \\
\epsilon_L^\mu(\vec{k}) &= \frac{1}{m_V}(k; 0, 0, E_k)
\end{aligned}$$

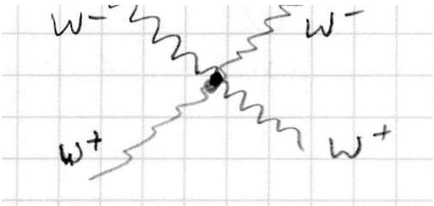
which satisfy (a,b = +, - , L)

$$\begin{aligned}
k_\mu \epsilon_a^\mu(\vec{k}) &= 0 \\
\epsilon_a^\mu(\vec{k}) \epsilon_{b\mu}^*(\vec{k}) &= -\delta_{ab}
\end{aligned}$$

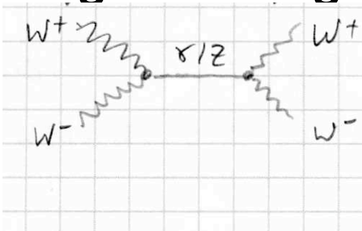
Perturbative constraint & unitarity

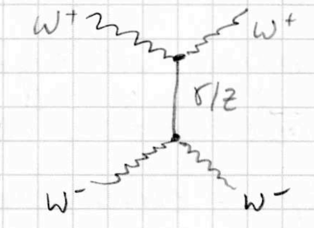
When E_k >> m_V the longitudinal polarization is divergent. Diagrams with external vector bosons have divergent cross sections.
Consider the process $W_L^+ W_L^- \to W_L^+ W_L^-$

(i) Four point interaction

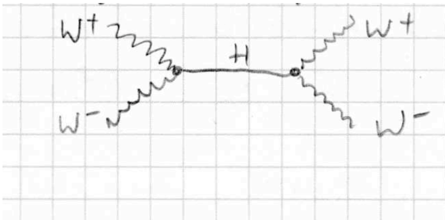


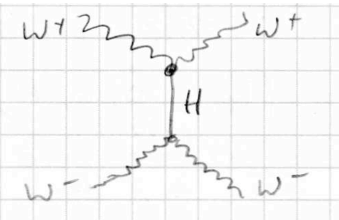
(ii) Gauge exchange of photon/Z in the s- and t-channel





(iii) Higgs exchange in the s- and t-channel





Perturbative constraint & unitarity

The amplitude can be written as (S.Weinberg, Vol.1, sec 3.7)

$$\mathcal{A} = \mathcal{A}^{(2)} s^2 + \mathcal{A}^{(1)} s + \mathcal{A}^{(0)}$$

s = Mandelstam variable (square sum of initial momenta)
From computations we learn that (when $s, t >> m_V^2, m_h^2$)

$$\begin{aligned} \mathcal{A}^{(2)} &\longrightarrow 0 \\ \mathcal{A}^{(1)} &\longrightarrow 0 \\ \mathcal{A}^{(0)} &\longrightarrow -\frac{2m_h^2}{v^2} \simeq -4\lambda \end{aligned}$$

Perfect cancellation between the diagrams. But the amplitude remains proportional to the Higgs boson mass. If the Higgs boson mass is too large the theory becomes strongly interacting and we cannot perform expansions versus λ.

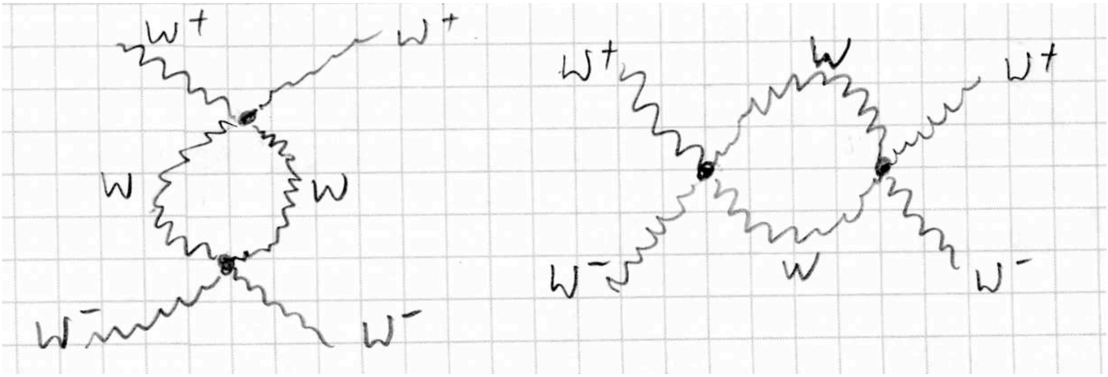
Perturbative constraint & unitarity

At the loop level the process

$$W^+W^- \rightarrow (WW)_{loop} \rightarrow W^+W^-$$

has an amplitude of

$$\frac{2\lambda^2}{16\pi^2}$$



The one-loop amplitude becomes equal to the tree-level amplitude when $\lambda \sim 32\pi^2$, hence the Electro-Weak theory should break down when $m_h > 6$ TeV.

More rigorous via partial wave analysis: $m_h < 870$ GeV
When taking also the WW→ZZ process into account: $m_h < 710$ GeV

The triviality bound

The couplings should remain finite at all energy scales Q.

$$g_i = (0.41; 0.64; 1.2)$$
$$y_t = \sqrt{2}\frac{m_t}{v} \simeq 1$$
$$\lambda = \frac{m_h^2}{2v^2}$$

Via the renormalization group equations we can evolve the couplings to higher scales Q.

$$\frac{dg_1}{dt} = \frac{41}{10}\frac{g_1^3}{16\pi^2}$$
$$\frac{dg_2}{dt} = -\frac{19}{6}\frac{g_2^3}{16\pi^2}$$
$$\frac{dg_3}{dt} = -7\frac{g_3^3}{16\pi^2}$$
$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(-\frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \frac{9}{2}y_t^2\right)$$
$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left(24\lambda^2 - \lambda\left(\frac{9}{5}g_1^2 + 9g_2^2 + 12y_t^2\right) + \frac{9}{8}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) - 6y_t^4\right)$$

$$t = ln\left(\frac{Q}{Q_0}\right)$$

For large Higgs boson masses the term λ^2 dominates and after integration one obtains Landau pole or a limit on the value of Q for which the theory is still valid.

$$Q_{LP} = m_h exp\left(\frac{4\pi^2v^2}{3m_h^2}\right)$$

The vacuum stability bound

When the Higgs boson mass is light the term $-6y_t^4$ will dominate:

$$\frac{d\lambda}{dt} \simeq -\frac{1}{16\pi^2} 6y_t^4$$

hence for higher scales Q the value of λ could become negative, hence the vacuum instable (V<0). With the constraint λ(Q)>0 for all values of Q we obtain a lower limit on the Higgs boson mass. After integrating the part of the RGE which is λ independent from Q_0 to Q we obtain:

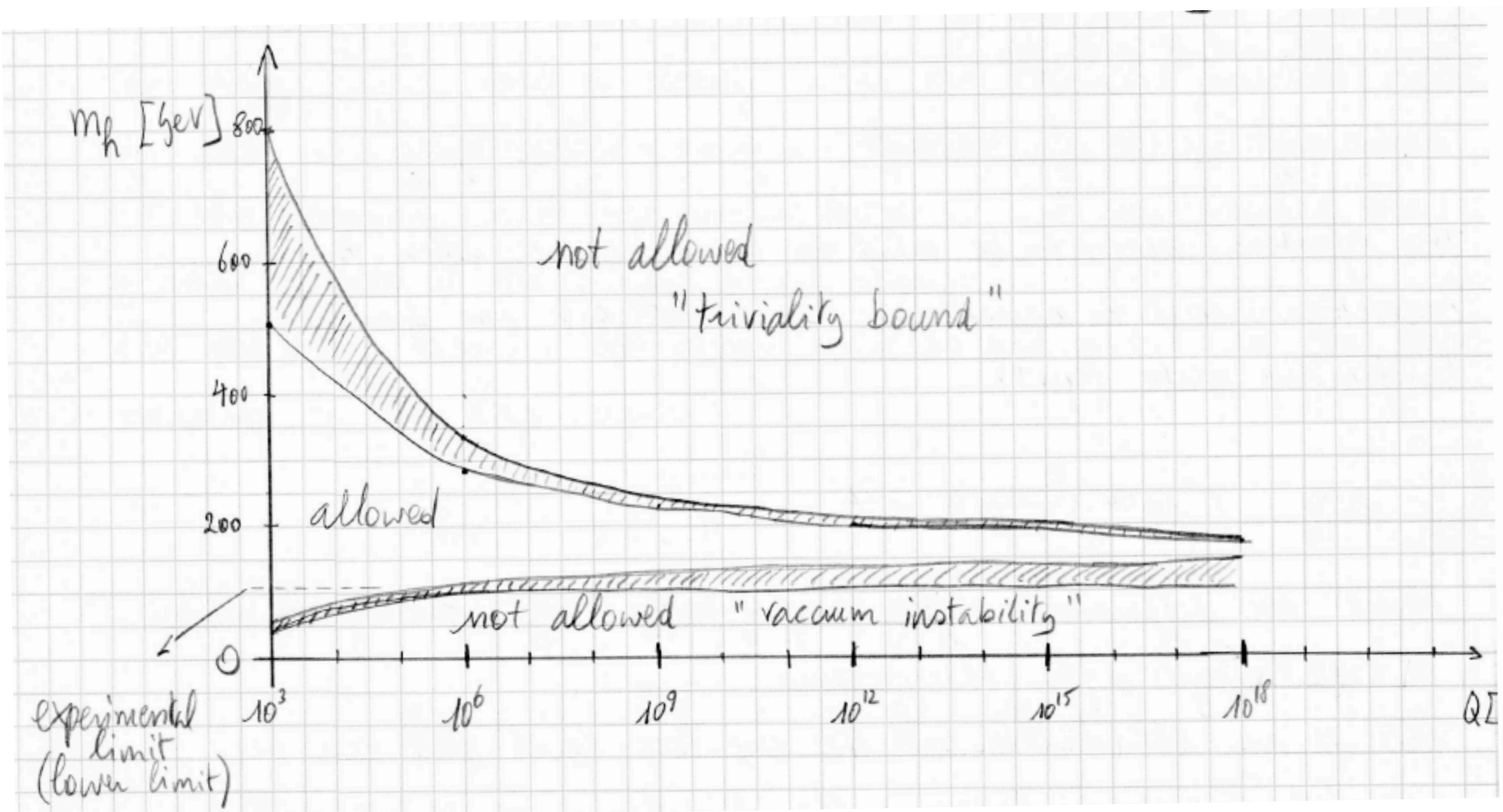
$$m_h^2 > \frac{v^2}{8\pi^2} \left(\frac{9}{8} \left(\frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) - 6y_t^4 \right) ln \left(\frac{Q}{Q_0} \right)$$

Hence a lower limit for the Higgs boson mass for a given Q scale to keep the vacuum stable (without the presence of new physics phenomena beyond the Standard Model).

The full calculations at higher order (more loops) is done.

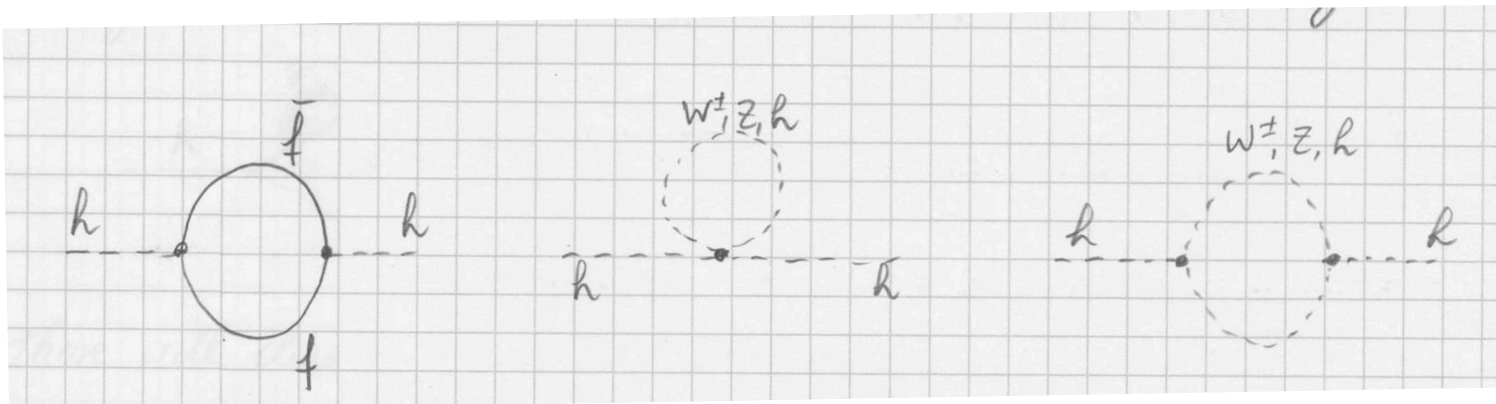
All together: theoretical bounds on the Higgs boson mass

If the Higgs boson is to be found at 60 GeV then this means the vacuum is instable in the absence of new physics. Only when the mass is between 130-180 GeV the vacuum can remain stable up to the Planck scale.



The fine-tuning constraint

The radiative corrections to the Higgs boson mass induce a fine tuning problem. At one loop



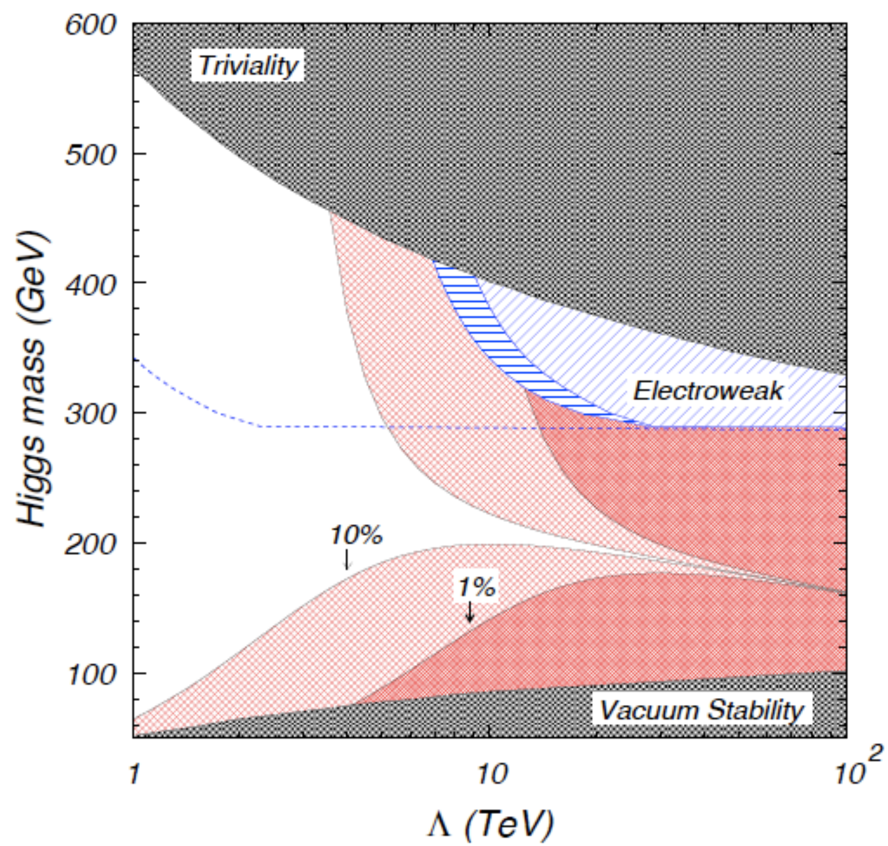
The integral can be cut-off at a momentum scale Λ

$$m_h^2 = (m_h^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)$$

hence to cancel this we need $m_h^2 \sim (320\ \mathrm{GeV})^2$
To cancel the radiative terms up to the GUT scale $\Lambda \sim 10^{16}$ GeV we need to cancel up to 32 digits after the comma.

The fine-tuning constraint

Requesting up to 10% (or 1%) fine-tuning the allowed range for the validity of the Standard Model is reduced.



The fine-tuning constraint

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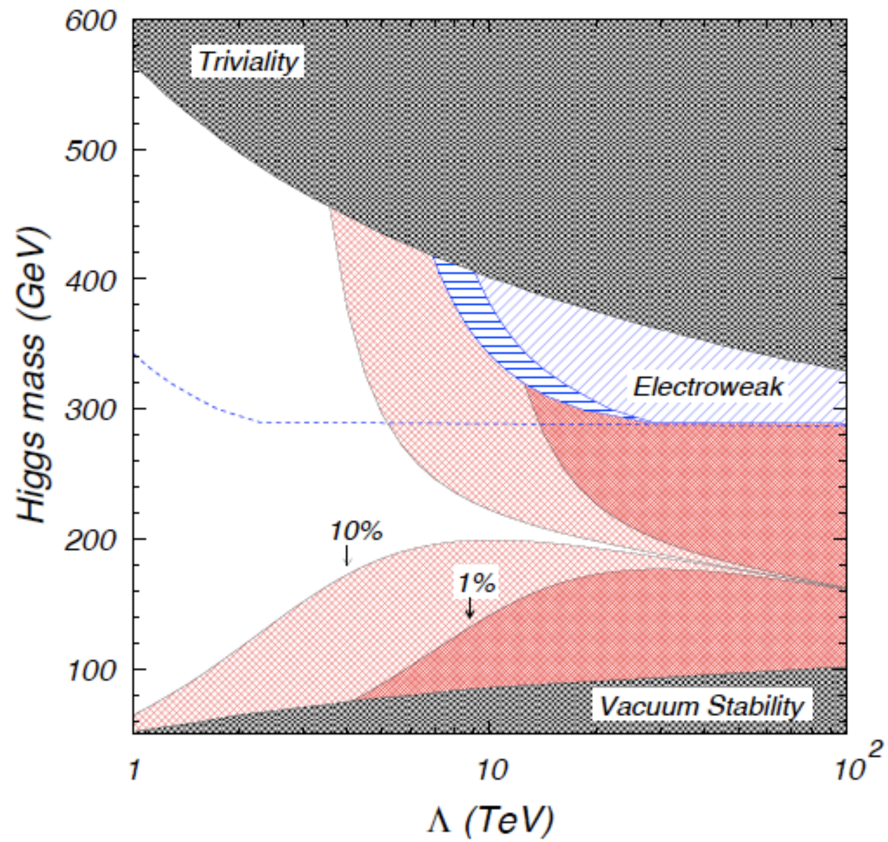


… why should there be only one Higgs doublet ?

Two Higgs Doublet Models (2HDM)

In the Standard Model we have introduced only one complex Higgs doublet resulting into one physical Higgs boson field and masses for 3 vector bosons. There is however no experimental reason why we cannot have more then one Higgs doublet.

Let us introduce two complex Higgs doublet fields ϕ_1 and ϕ_2.
The most general Higgs potential V which spontaneously breaks $SU(2)_L \times U(1)_Y$ into $U(1)_{EM}$ is

$$
\begin{aligned}
V(\phi_1,\phi_2) \quad = \quad & \lambda_1(|\phi_1|^2 - v_1^2)^2 + \lambda_2(|\phi_2|^2 - v_2^2)^2 \\
& +\lambda_3\left[(|\phi_1|^2 - v_1^2) + (|\phi_2|^2 - v_2^2)\right]^2 \\
& +\lambda_4\left[|\phi_1|^2|\phi_2|^2 - (\phi_1^{*T}\phi_2)(\phi_2^{*T}\phi_1)\right] \\
& +\lambda_5[Re(\phi_1^{*T}\phi_2) - v_1v_2cos\xi]^2 \\
& +\lambda_5[Im(\phi_1^{*T}\phi_2) - v_1v_2sin\xi]^2
\end{aligned}
$$

where the λ_i values are real and the ϕ_i's are the Higgs fields with a minimum of the potential appearing at

$$
\phi_1 = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}_2 = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}
$$

Two Higgs Doublet Models (2HDM)

When $sin\xi = 0$ there is no CP violation in the Higgs sector, which we will force. The last two terms can be combined when λ_5=λ_6 into

$$\left|\phi_1^{*T}\phi_2 - v_1 v_2 e^{i\xi}\right|^2$$

where the $e^{i\xi}$ term can be rotated away by redefining one of the ϕ fields.

We develop the two doublets around the minimum of the potential. For this we parameterize the fields as

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ v_1 + \eta_1 + i\chi_1 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \eta_2 + i\chi_2 \end{pmatrix}$$

where η_i is the CP-even part and χ_i the CP-odd part. We put these fields in the potential and the mass terms appear.

Two Higgs Doublet Models (2HDM)

This results in the following relevant terms, grouped according to the CP-even, CP-odd and charged Higgs sectors:

$$(\eta_1 \eta_2) \begin{pmatrix} 4(\lambda_1+\lambda_3)v_1^2+\lambda_5 v_2^2 & (4\lambda_3+\lambda_5)v_1 v_2 \\ (4\lambda_3+\lambda_5)v_1 v_2 & 4(\lambda_2+\lambda_3)v_2^2+\lambda_5 v_1^2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

$$\lambda_6\,(\chi_1 \chi_2) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\lambda_4\,(\phi_1^- \phi_2^-) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

where these squared-mass terms can be obtained from

$$\mathcal{M}_{ij}^2 = \frac{1}{2}\frac{\partial^2 V(\phi_1,\phi_2)}{\partial\psi_i \partial\psi_j}$$ **with** $\psi_i \in \{\phi_1^\pm, \phi_2^\pm, \eta_1, \eta_2, \chi_1, \chi_2\}$

Two Higgs Doublet Models (2HDM)

The physical eigenstates of the Higgs fields are obtained from a mixing between these fields $\psi_i \in \{\phi_1^{\pm}, \phi_2^{\pm}, \eta_1, \eta_2, \chi_1, \chi_2\}$
With a rotation of the eigenstates the squared-mass matrices can be diagonalized and the masses of the physical eigenstates can be determined. Express the potential in terms of the real fields.

For the CP-odd Higgs (mixing angle β with $\tan\beta = v_2/v_1$)

$$M_A^2 = \lambda_6(v_1^2 + v_2^2) \qquad M_{G^0}^2 = 0$$

For the CP-even Higgs (mixing angle α)

$$M_{H^0,h^0}^2 = \frac{1}{2}\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}$$

For the charged Higgs (mixing angle β)

$$M_{H^\pm}^2 = \lambda_4(v_1^2 + v_2^2) \qquad M_{G^\pm}^2 = 0$$

3 Goldstone bosons to give masses to W and Z bosons

Two Higgs Doublet Models (2HDM)

These relations depend on the values of λ and the mixing angles, hence they can be inverted to write the λ values as a functions of the masses and mixing angles.

These will fully define the potential. Hence this non-CP violating 2HDM Higgs sector has 6 free parameters:

$$M_{H^{\pm}}, M_{H^0}, M_{h^0}, m_{A^0}, tan\beta, \alpha$$

The fermions can couple to these two Higgs doublet field in different ways:

- **Type-I 2HDM**: the field ϕ_2 couples couples to both the up- and down-type fermions
- **Type-II 2HDM**: the field ϕ_1 generates the masses for the down-type fermions, while the field ϕ_2 generates the masses for the up-type quarks (this is the basis of the Higgs sector in the MSSM)

The couplings between the fermions and the neutral Higgs bosons are defined from the mixing angles α and β.

Lecture summary

- Reminder of the mechanism of spontaneous symmetry breaking
- Applied on the EW symmetry of SU(2)xU(1)
- The yet to be observed Higgs sector of the Standard Model depends only on one parameter, the mass of the Higgs boson

- Diverse arguments can be invoked to put theoretically constraints on the Higgs boson mass ($m_H < 1$ TeV)
- For the theory to be valid up to the Planck scale, the allowed range of the Higgs boson mass is very limited ($m_H \sim 130\text{-}180$ GeV)
- When you do not "believe" that Nature has fine-tuned the parameters of the model, the allowed range is even vanishing or new physics has to appear at scale below $\Lambda \sim 10\text{-}100$ TeV

- Maybe one Higgs doublet is not enough…
- 2-Higgs Doublet Models are the basis of supersymmetric models
- We have walked through the techniques needed to calculate the mass spectrum of the Higgs sector in a general 2HDM

Extensions of the Standard Model

(part 2)

Prof. Jorgen D'Hondt
Vrije Universiteit Brussel
Inter-university Institute for High Energies

Lecture 2

http://w3.iihe.ac.be/~jdhondt/Website/BeyondTheStandardModel.html

Phenomenology of the Standard Model Higgs boson

Aim:

- Main decay properties of the Higgs boson
- Be able to calculate towards the phenomenology of Higgs physics

Content:

- Decays to quarks & leptons
- Decays to Electro-Weak gauge bosons
- Loop induced decays into photons and gluons

The decay of Higgs bosons

The Higgs boson couplings are directly proportional to the mass of the particles involved, hence it tends to decay to the heaviest particle allowed by phase-space.

For the vector bosons we have the hVV term in the Lagrangian

$$\mathcal{L}_{hVV} = \sqrt{\sqrt{2}G_F} m_V^2 h V^\mu V_\mu$$

While for the fermions the couplings are given as

$$g_{hf\overline{f}} \sim \frac{m_f}{v} = \sqrt{\sqrt{2}G_F} m_f$$

with

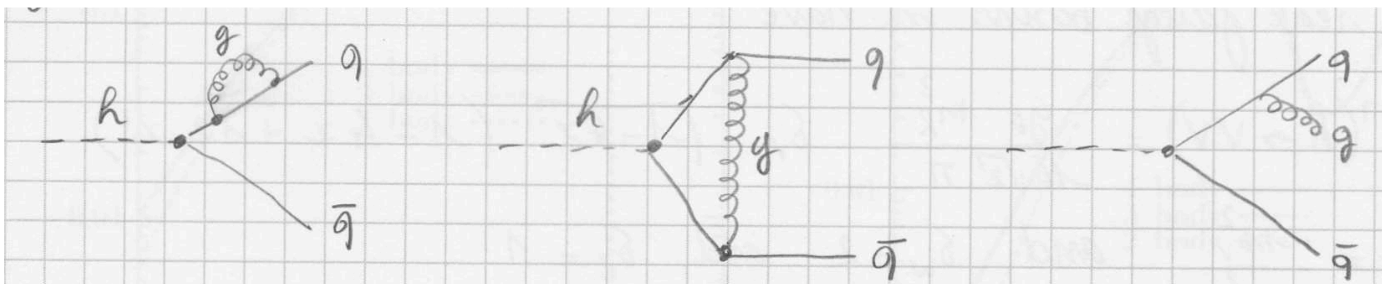
$$G_F = \frac{g^2}{\sqrt{32} m_W^2}$$

The decay of Higgs bosons into quarks & leptons

Born approximation:

$$\Gamma_{Born}(h \to f\overline{f}) = \frac{G_F N_C}{4\sqrt{2}\pi} m_h m_f^2 \beta_f^3 \qquad \beta_f = \sqrt{1 - \frac{4m_f^2}{m_h^2}}$$

with N_c a color factor.
There are loop corrections to this from diagrams like



The decay width becomes

$$\Gamma_{NLO}(h \to q\overline{q}) = \frac{3G_F}{4\sqrt{2}\pi} m_h m_q^2 \left[1 + \frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{9}{4} + \frac{3}{2} log \frac{m_q^2}{m_h^2}\right)\right]$$

Absorb the large logarithms into a redefinition of the quark masses, MSbar scheme

$$m_q \longrightarrow \overline{m_q}(m_h)$$

The decay of Higgs bosons into quarks & leptons

After QCD radiative corrections up to 3rd order

$$\Gamma(h \to q\overline{q}) = \frac{3G_F}{4\sqrt{2}\pi} m_h \overline{m}_q^2(m_h)\left[1 + \Delta_{q\overline{q}} + \Delta_h^2\right]$$

With

$$\Delta_{q\overline{q}} = 5.67\frac{\overline{\alpha}_s}{\pi} + (35.94 - 1.36N_f)\frac{\overline{\alpha}_s^2}{\pi^2} + \left(164.14 - 25.77N_f + 0.26N_f^2\right)\frac{\overline{\alpha}_s^3}{\pi^3}$$

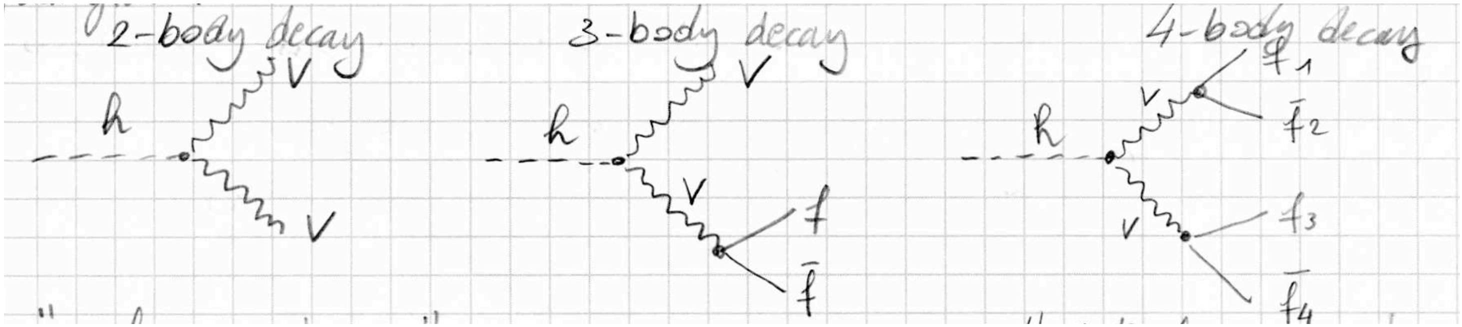
$$\Delta_h^2 = \frac{\overline{\alpha}_s^2}{\pi^2}\left[1.57 - \frac{2}{3}log\left(\frac{m_h^2}{m_t^2}\right) + \frac{1}{9}log^2\left(\frac{\overline{m}_q^2}{m_h^2}\right)\right]$$

$$\overline{\alpha}_s = \alpha_s(m_h)$$

N_f the number of accessible fermion flavours

The decay of Higgs bosons into Electro-Weak gauge bosons

The decay widths are directly proportional to the hVV terms in the Lagrangian. Difference between "real" and "virtual" gauge bosons:



For two real bosons

$$\Gamma(h \to VV) = \frac{G_F m_h^2}{16\sqrt{2}\pi} \delta_V \sqrt{1-4x}(1-4x+12x^2)$$

$$x = \frac{m_V^2}{m_h^2} \qquad \delta_W = 2 \qquad \delta_Z = 1$$

Hence when the Higgs boson mass is much larger than the mass of the vector bosons, we have

$$\Gamma(h \to WW) \simeq 2 \cdot \Gamma(h \to ZZ)$$

The decay of Higgs bosons into Electro-Weak gauge bosons

For large Higgs boson masses

$$\Gamma(h \to WW + ZZ) \simeq 0.5TeV \left(\frac{m_h}{1TeV}\right)^3$$

the width becomes similar to the mass itself around m_h=1.4 TeV. When there is a 3-body decay one of the vector bosons is off-shell, hence the branching ratio can be non-zero below the kinematic threshold

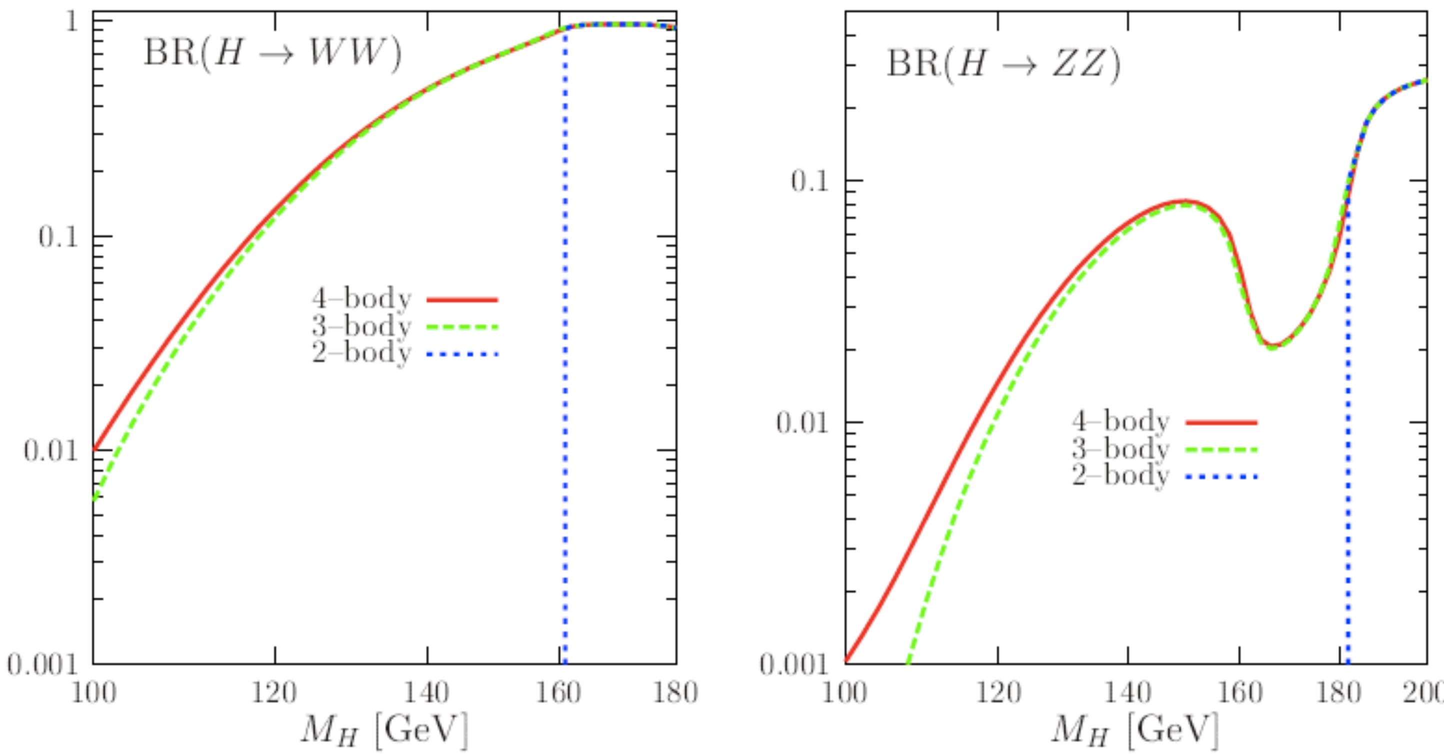
$$\Gamma(h \to VV^*) = \frac{3G_F^2 m_V^4}{16\pi^3} m_h \delta'_V R_T(x)$$

with

$$\delta'_W = 1 \qquad \delta'_Z = \frac{7}{12} - \frac{10}{9} sin^2\theta_W + \frac{40}{9} sin^4\theta_W$$

$$R_T(x) = \frac{3(1-8x+20x^2)}{\sqrt{4x-1}} arccos\left(\frac{3x-1}{2x^{3/2}}\right) - \frac{1-x}{2x}(2-13x+47x^2) - \frac{3}{2}(1-6x+4x^2)log(x)$$

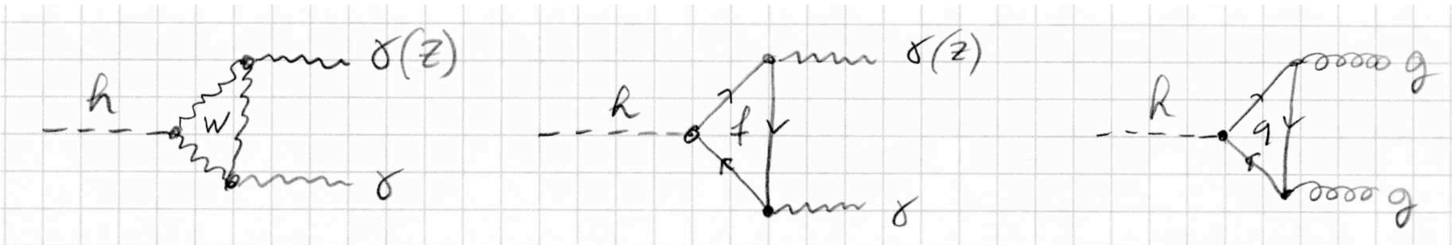
The decay of Higgs bosons into Electro-Weak gauge bosons



Reference for these and following plots: arXiv:hep-ph/0503172v2

Loop induced decays into photons and gluons

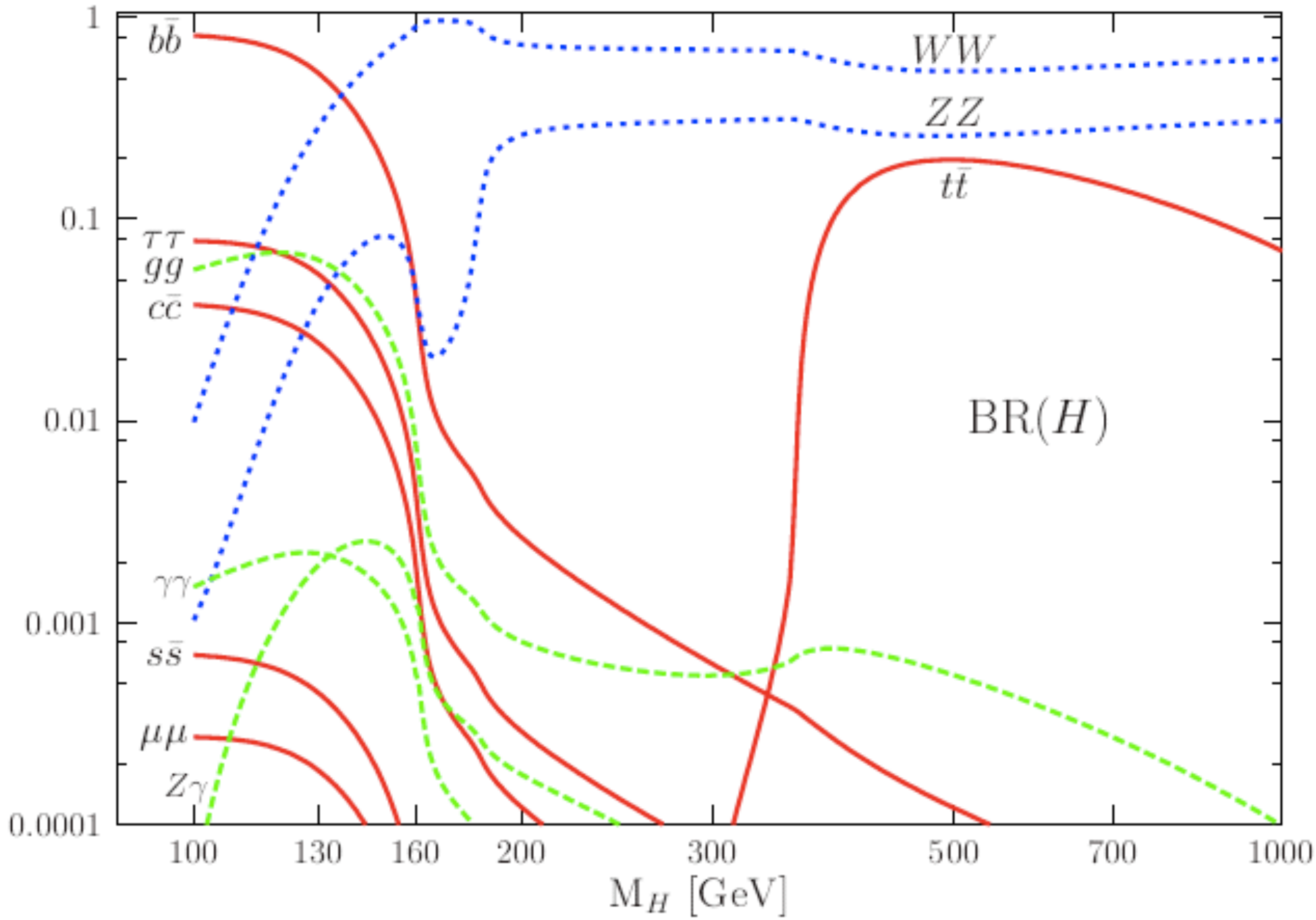
Photons and gluons are massless hence do not couple to the Higgs boson, nevertheless they can appear in loops:



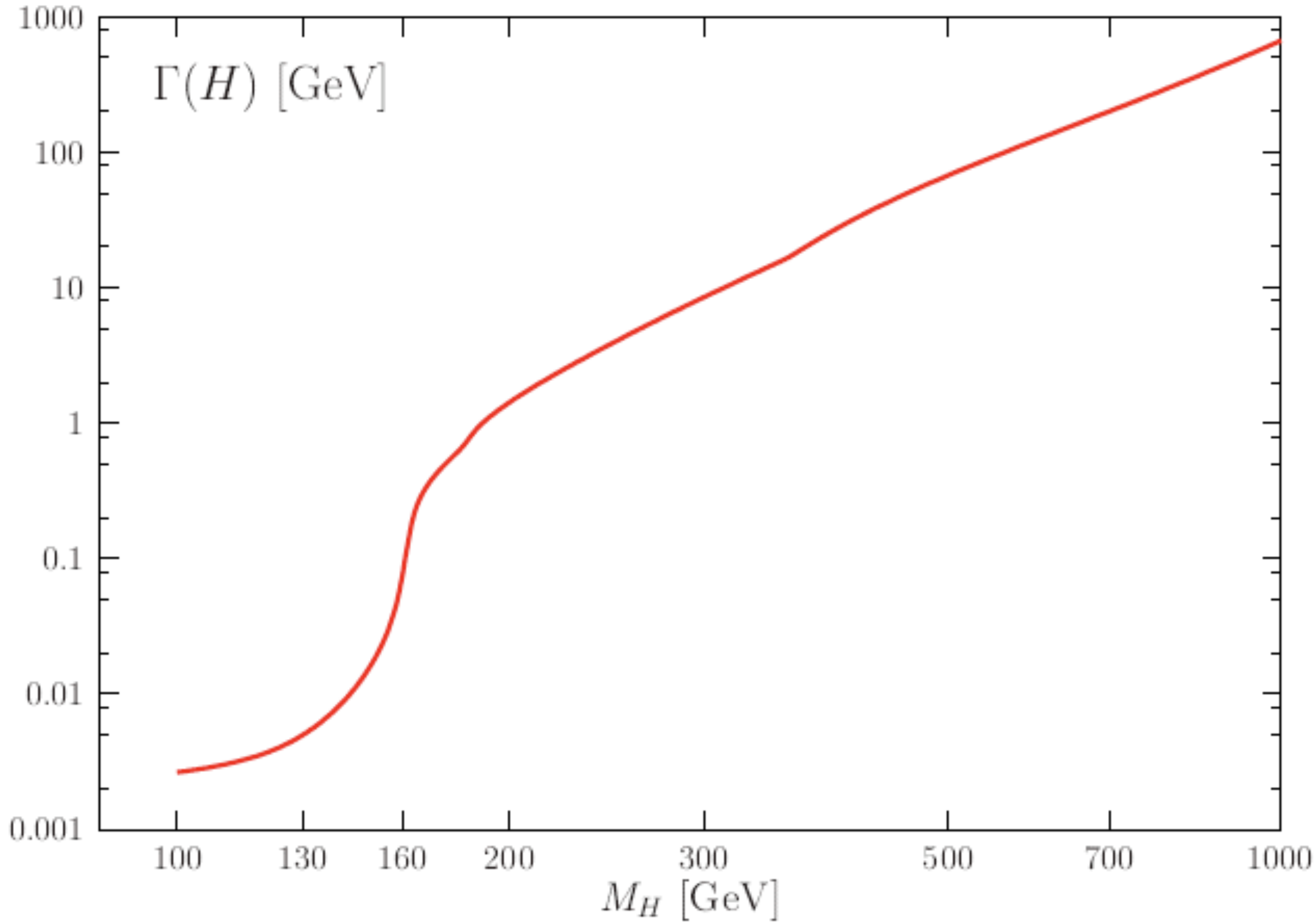
These diagrams contribute according to the mass of the particles in these loops. Hence we can probe physics far beyond the scale of the mass of the Higgs boson. For example the new heavy charged or colored particles appearing in models beyond the Standard Model.

Looks like these diagrams are only relevant when m_h < 130 GeV.

All together: branching ratios



All together: total decay width



All together: zoom into the branching ratios

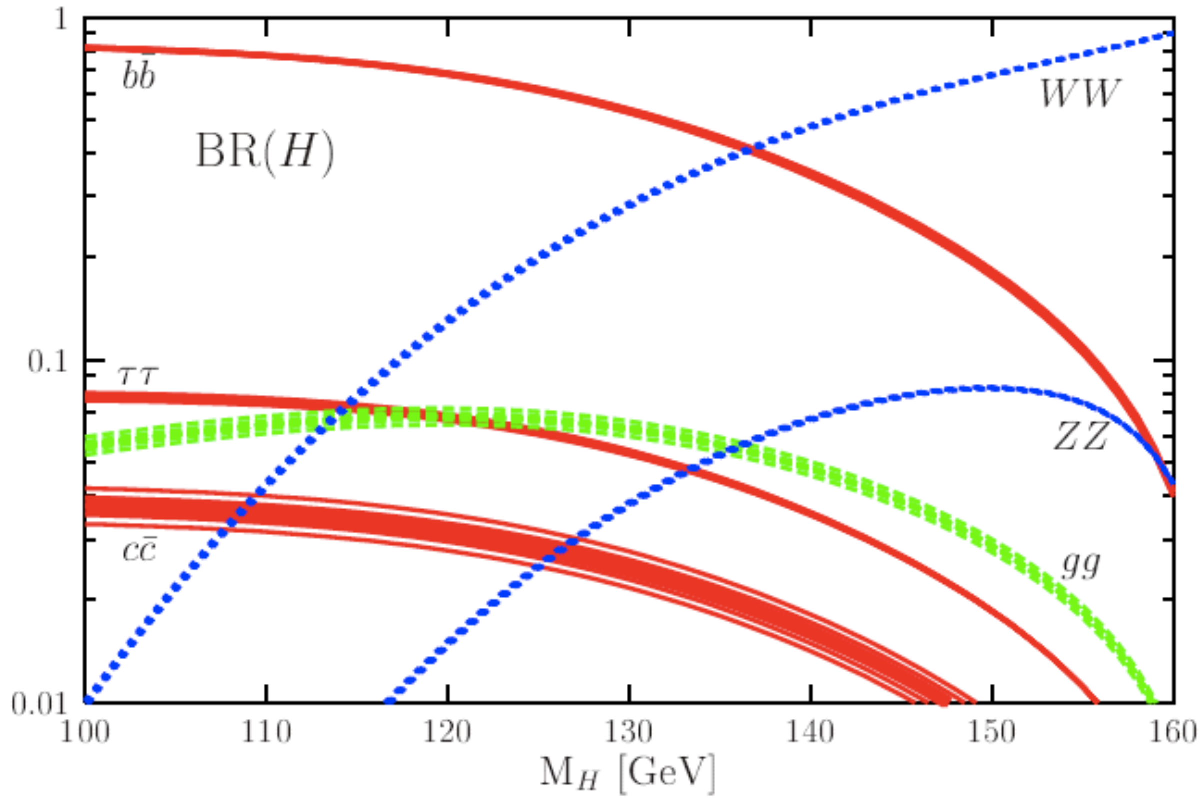


Figure 2.27: The SM Higgs boson decay branching ratios in the low and intermediate Higgs mass range including the uncertainties from the quark masses $m_t = 178 \pm 4.3$ *GeV*, $m_b = 4.88 \pm 0.07$ *GeV and* $m_c = 1.64 \pm 0.07$ *GeV as well as from* $\alpha_s(M_Z) = 0.1172 \pm 0.002$.

Searching for the Standard Model Higgs boson

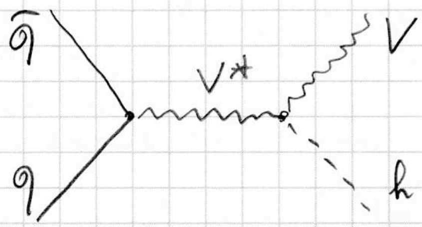
Aim:

- Learn how with a phenomenological approach one can identify the relevant experimental signatures to search for (in this case) Higgs bosons
- Experiments are designed to discover phenomena (eg. the Higgs boson), hence we have to be able to judge on the experimental design parameters needed to make this discovery possible

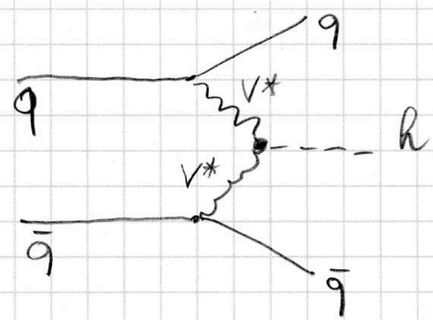
Exercise (typical exam question):

Can we discover the Standard Model Higgs boson at the LHC (14 TeV) in the pp→ttH channel? Try to estimate the significance of this process after some event selection enhancing this signal. Which integrated luminosity is needed to have a significance larger than 5. Motivate your arguments.

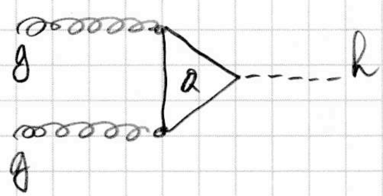
Main Higgs boson production processes



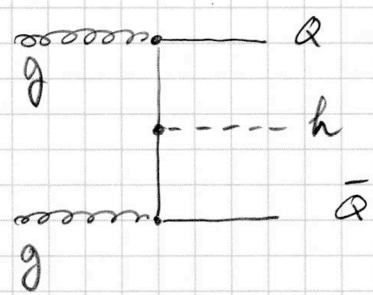
"associated production with W/Z bosons"



"weak vector boson fusion production"



"gluon-gluon fusion"



"associated production with heavy top or bottom quarks"

Main background processes (LHC@14TeV)

process	cross section	comment	
$\sigma_{tot}(pp \to X)$	110 ± 10 mb	different models	
$\sigma_{tot}(pp \to X)$	$111.5 \pm 1.2^{+4.1}_{-2.1}$ mb	COMPETE Coll.	
process	**CTEQ5L**	**CTEQ6M**	**comment**
Z-boson	48.69 nb	$50.1^{+4.19\%}_{-4.76\%}$ nb	
$Z + \text{jet}(g+q)$	13.94 nb	$12.73^{+3.16\%}_{-3.94\%}$ nb	$P_0 = 20$ GeV
$q\bar{q} \to Z\gamma$	44.21 pb	$46.7^{+3.93\%}_{-4.22\%}$ nb	$P_0 = 20$ GeV
$W^\pm$-boson	158.5 pb	$161.3^{+4.32\%}_{-4.93\%}$ nb	
$W^\pm + \text{jet}(g+q)$	41.42 nb	$37.24^{+3.34\%}_{-4.10\%}$ nb	$P_0 = 20$ GeV
$W^\pm\gamma$	56.21 pb	$56.42^{+4.11\%}_{-4.38\%}$ nb	$P_0 = 20$ GeV
W^+W^-	69.69 pb	$75.0^{+3.87\%}_{-4.03\%}$ pb	
$W^\pm Z$	26.69 pb	$28.76^{+3.93\%}_{-4.08\%}$ pb	
$q\bar{q} \to ZZ$	11.10 pb	$10.78^{+4.02\%}_{-4.21\%}$ pb	
$WQ\bar{Q}$	$m_b = 4.8$ GeV, $m_c = 1.5$ GeV, TopReX		
$W^\pm c\bar{c}$	1215 pb	$1086^{+4.12\%}_{-4.53\%}$ pb	$M_{c\bar{c}} \geq 3.0$ GeV
$W^\pm c\bar{c}$	33.5 pb	$31.3^{+4.00\%}_{-4.18\%}$ pb	$M_{c\bar{c}} \geq 50$ GeV
$W^\pm b\bar{b}$	328 pb	$297^{+4.04\%}_{-4.37\%}$ pb	$M_{b\bar{b}} \geq 9.6$ GeV
$W^\pm b\bar{b}$	34.0 pb	$31.3^{+4.00\%}_{-4.18\%}$ pb	$M_{b\bar{b}} \geq 50$ GeV
$Zb\bar{b}$, $m_b = 4.62$ GeV	789.6 ± 3.66 pb	MCFM	$M_{b\bar{b}} \geq 9.24$ GeV
dijet processes	819 μb	$583^{+4.78\%}_{-6.02\%}$ μb	$P_0 = 20$ GeV
$\gamma + \text{jet}$	182 nb	$135^{+4.92\%}_{-6.14\%}$ nb	$P_0 = 20$ GeV
$\gamma\gamma$	164 pb	$137^{+4.62\%}_{-5.65\%}$ pb	$P_0 = 20$ GeV
$b\bar{b}$, $m_b = 4.8$ GeV	479 μb	$187^{+9.7\%}_{-13.2\%}$ μb	
$t\bar{t}$, $m_t = 175$ GeV	488 pb	$493^{+3.24\%}_{-3.31\%}$ pb	
$t\bar{t}$, $m_t = 175$ GeV	830 ± 90 pb	NLO+NNLO	
$t\bar{t}\,b\bar{b}$	10 pb		AcerMC 1.2
inclusive Higgs	$m_H = 150$ GeV	23.8 pb	
inclusive Higgs	$m_H = 500$ GeV	3.8 pb	

Main channels involve a lepton (electron or muon) because the amount of jet production at the LHC is enormous.

Hence need at least one lepton in the final state of the process where we look for the Higgs boson.

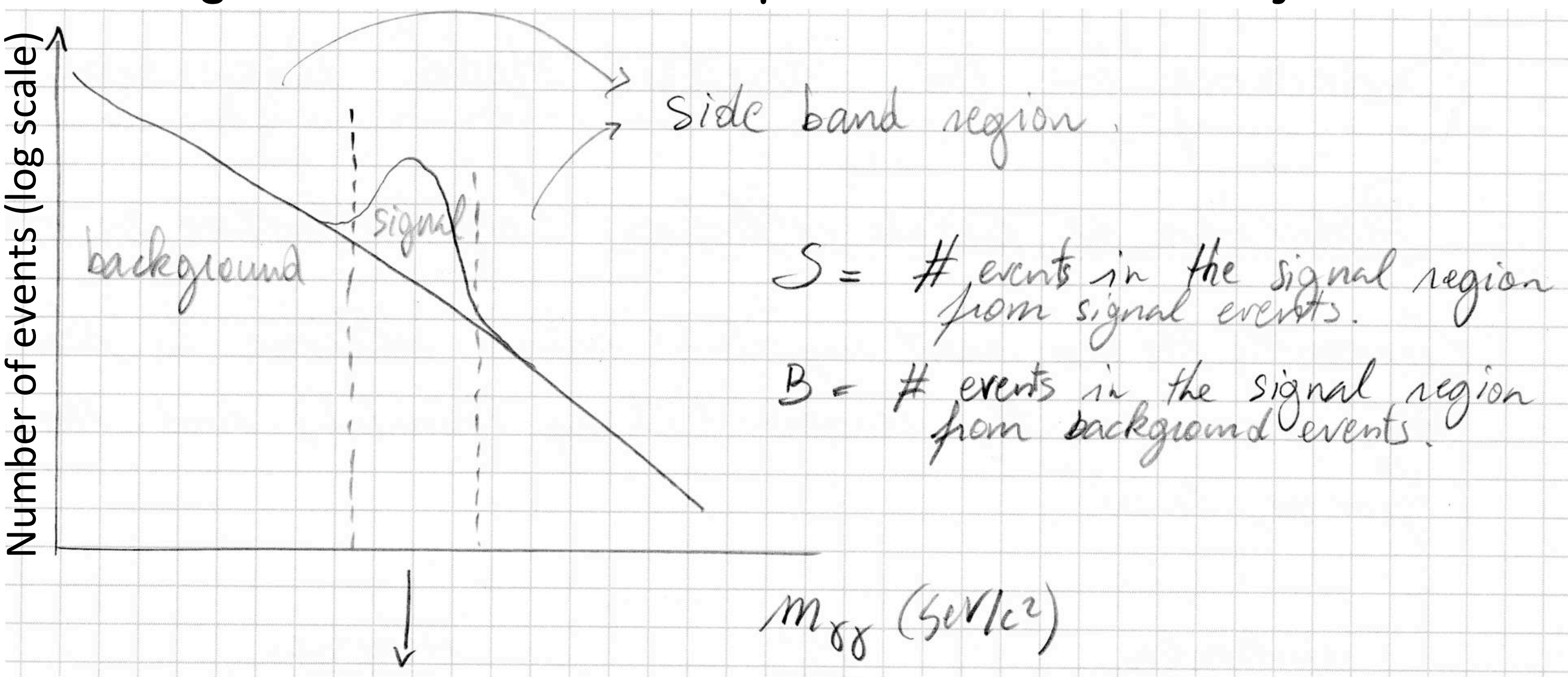
Searching for the Standard Model Higgs boson

The estimation of the amount of signal event $\hat{S}$ is obtained from the remainder after subtracting the estimated background component $\hat{B}$ from the event sample with N events after a specific event selection :

$$\hat{S} = N - \hat{B}$$

$$Significance = \frac{\hat{S}}{\sqrt{\hat{S} + \hat{B}}}$$

Background estimate for example from side-band analysis.



Cross sections for Higgs boson production

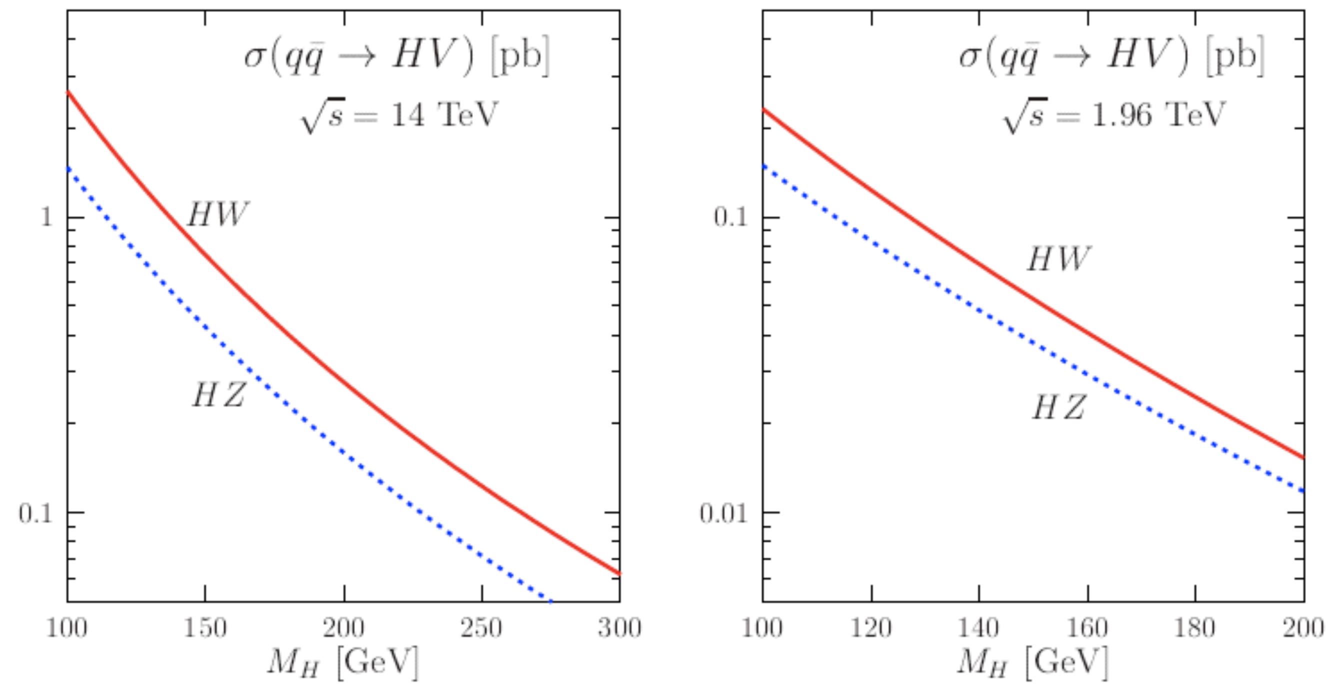


Figure 3.3: Total production cross sections of Higgs bosons in the strahlung $q\bar{q} \to H + W/Z$ processes at leading order at the LHC (left) and at the Tevatron (right). For $q\bar{q} \to HW$, the final states with both W^+ and W^- have been added. The MRST set of PDFs has been used.

Cross sections for Higgs boson production

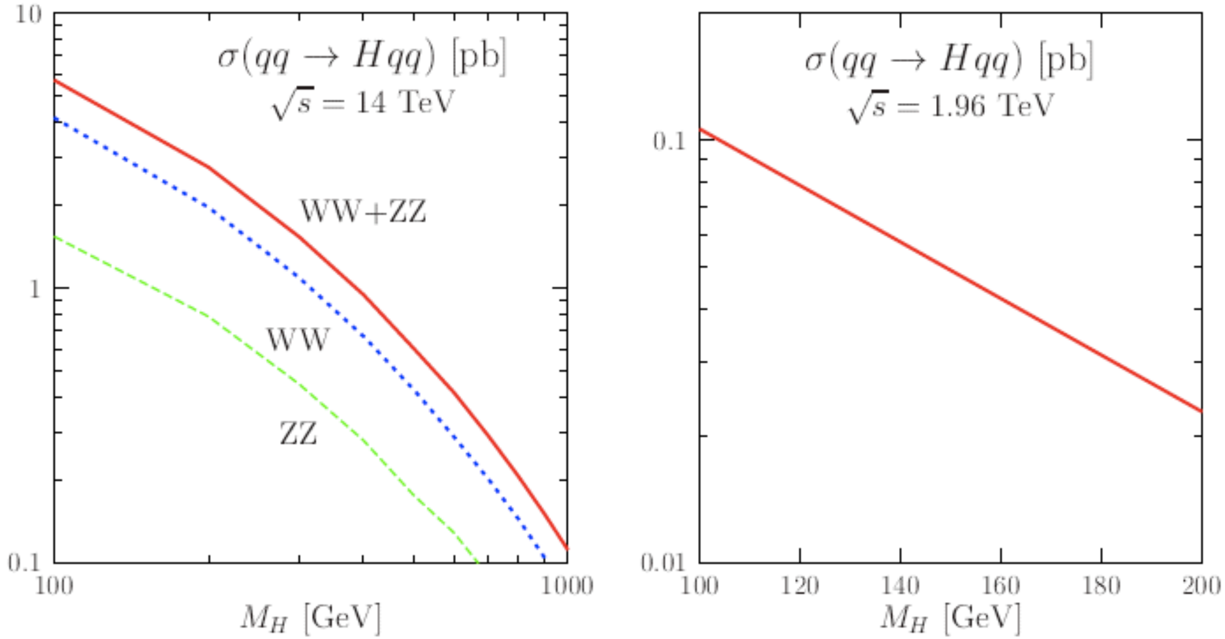


Figure 3.12: Individual and total cross sections in the vector fusion $qq \to V^*V^* \to Hqq$ processes at leading order at the LHC (left) and total cross section at the Tevatron (right).

Cross sections for Higgs boson production

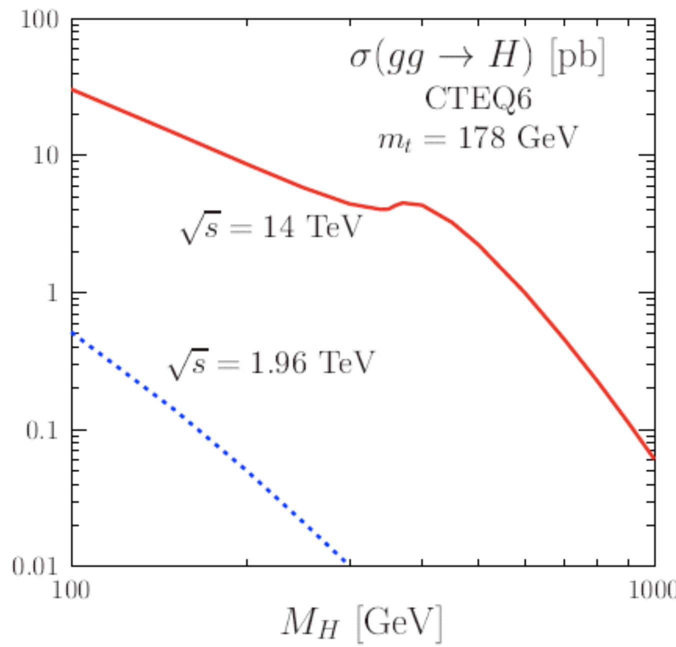


Figure 3.18: The hadronic production cross section for the gg fusion process at LO as a function of M_H at the LHC and the Tevatron. The inputs are $m_t = 178$ GeV, $m_b = 4.88$ GeV, the CTEQ set of PDFs has been used and the scales are fixed to $\mu_R = \mu_F = M_H$.

Cross sections for Higgs boson production

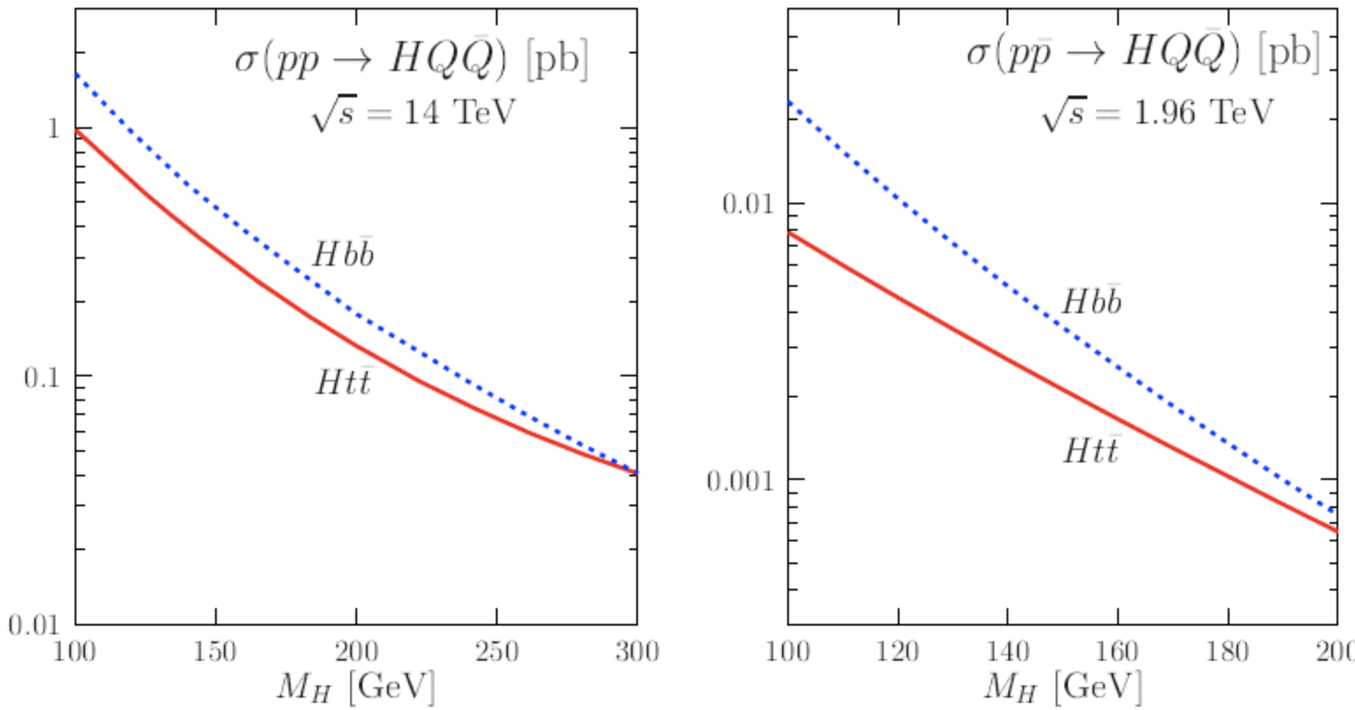


Figure 3.30: The $t\bar{t}H$ and $b\bar{b}H$ production cross sections at the LHC (left) and the Tevatron (right). The pole quark masses in the Yukawa couplings are set to $m_t = 178$ GeV and $m_b = 4.88$ GeV and the MRST PDFs are used. The renormalization and factorization scales have been set to $\mu_{R,F} = m_t + \frac{1}{2}M_H$ for $pp \to t\bar{t}H$ and $\mu_{R,F} = \frac{1}{2}m_b + \frac{1}{4}M_H$ for $pp \to b\bar{b}H$.

Cross sections for Higgs boson production

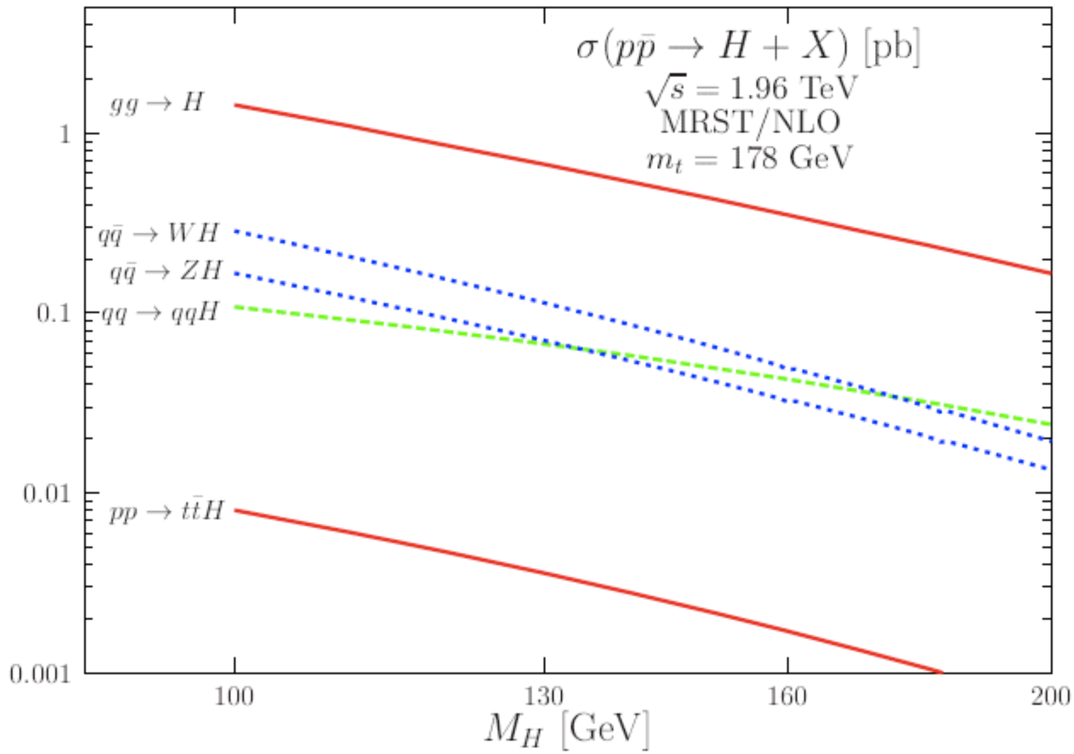


Figure 3.46: The Higgs boson production cross sections at the Tevatron in the dominant mechanisms as a function of M_H. They are (almost) at NLO with $m_t = 178$ GeV and the MRST set of PDFs has been used. The scales are as described in the text.

Cross sections for Higgs boson production

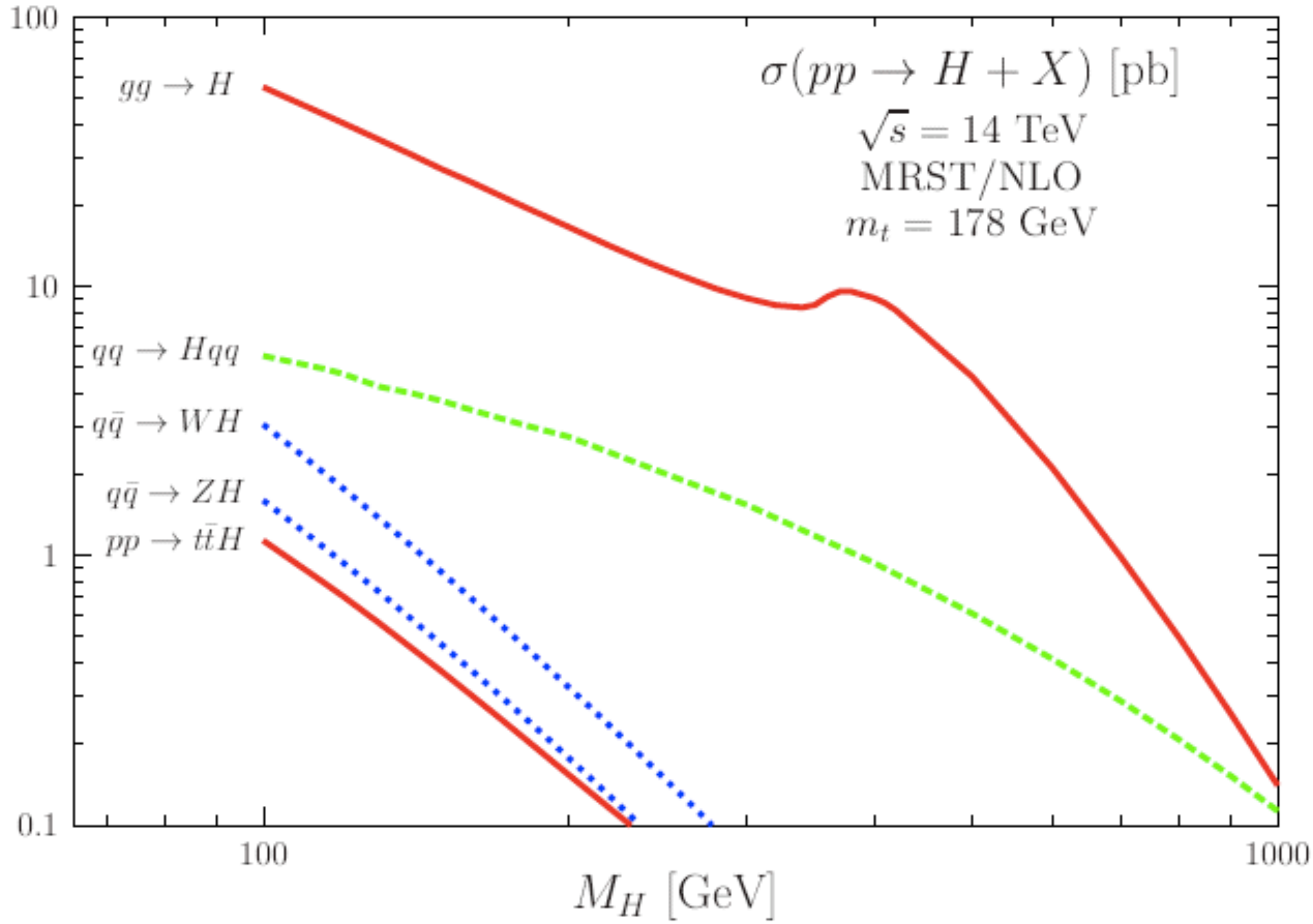
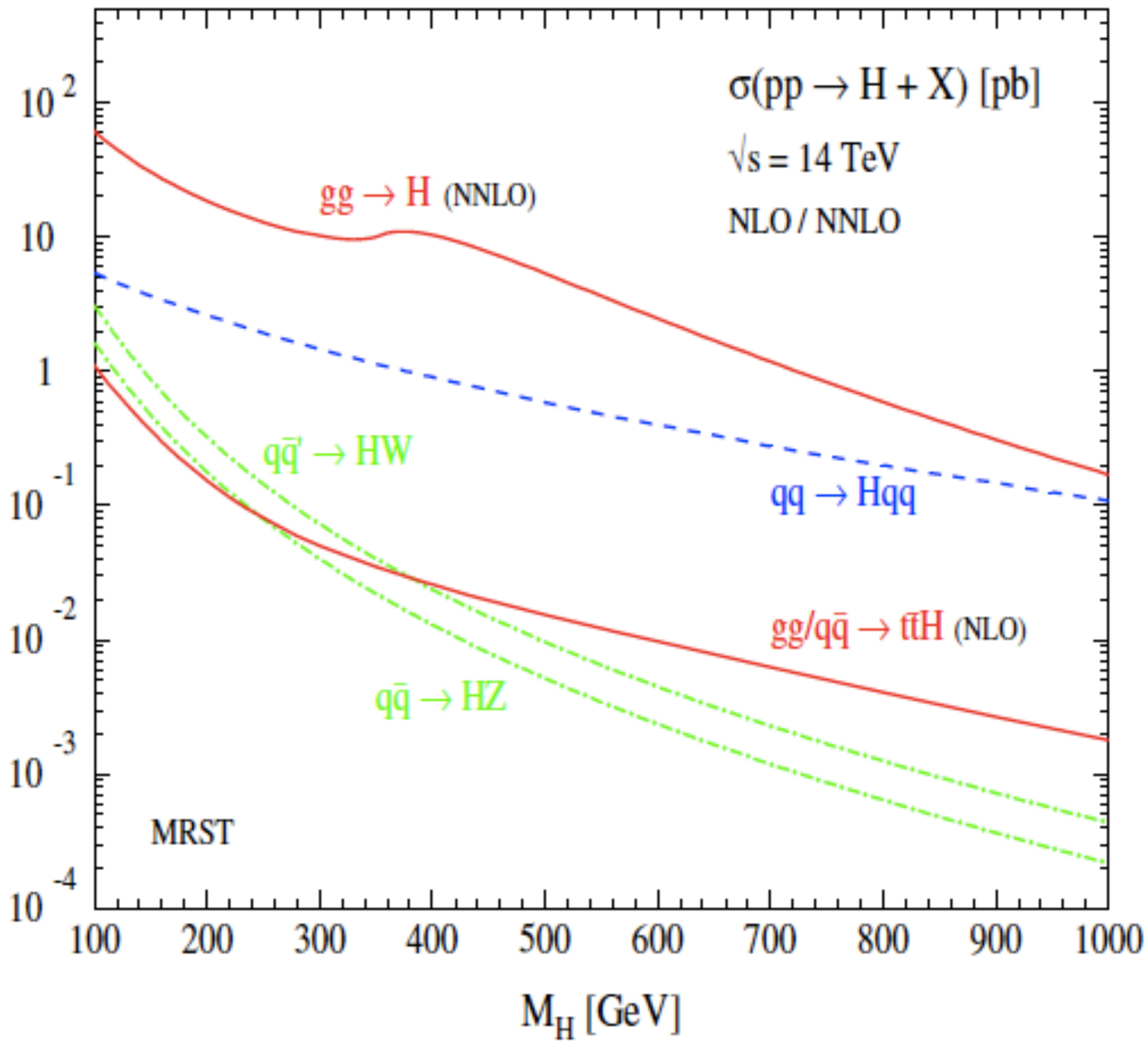


Figure 3.47: The same as Fig. 3.46 but for the LHC.

Cross sections for Higgs boson production



Reconstruction efficiencies

The reconstruction and identification of physical objects is never perfectly efficient (eg. detector acceptance). Below some benchmark numbers.

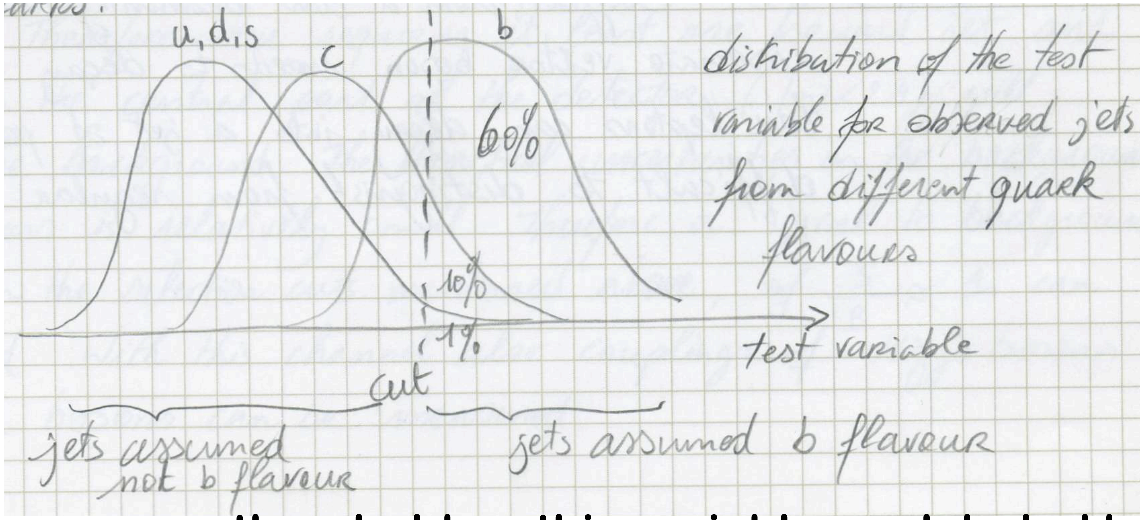
Isolated leptons (from W or Z decays) have ~80% efficiency to be reconstructed when the transverse momentum is above 20 GeV.

Jets with a transverse momentum above 30 GeV will be reconstructed in 90%, but will individually radiate in ~15% of the cases gluons, hence loosing their kinematic information for mass reconstruction.

B-flavoured jets can be identified with an efficiency of about ~50%, while c-jets will be mis-identified as b-jets in 10% of the cases and udsg-jets in 1% of the cases.

b-tagging at hadron colliders

The flavour of the quark at the origin of the jet can be determined to some extend for the heavier flavours, charm and bottom quark-jets. In the bottom and charm decay respectively B and D mesons are formed which have a significant lifetime. The meson therefore decays at some distance (~0.5-1mm) from the interaction point. A hypothesis test is performed for each reconstructed jet on the basis of a variable sensitive to this effect.



Jets above some threshold on this variable are labeled b-quark jets. This algorithm makes two types of mistakes as every hypothesis test: real b-quark jets are not labeled as b-quark jets (40% of them), and non-b-quark jets are labeled as b-quark jets (1% udsg, 10% c).

Discovery potential for SM Higgs boson

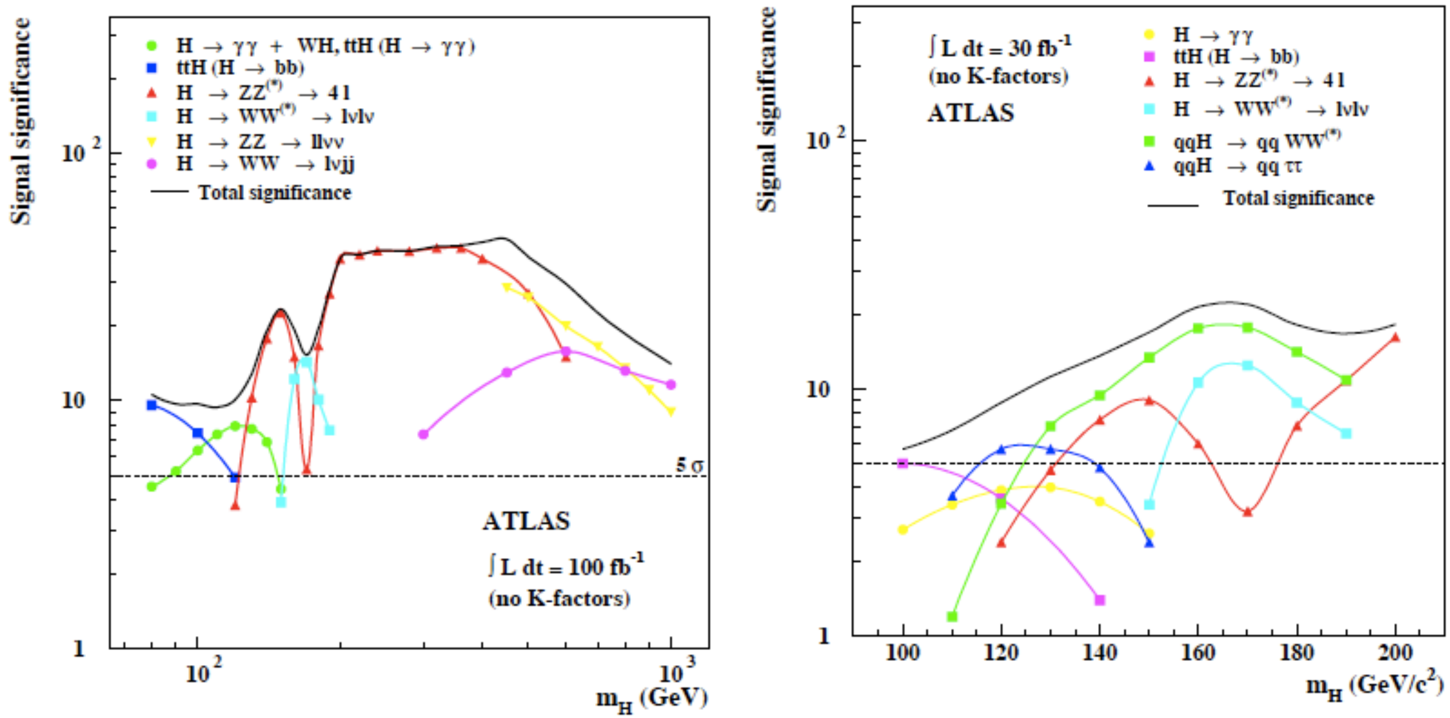


Figure 3.49: The significance for the SM Higgs boson discovery in various channels in ATLAS as a function of M_H. Left: the significance for 100 fb^{-1} data and with no vector boson fusion channel included and right: for 30 fb^{-1} data in the $M_H \leq 200$ GeV range with the $qq \rightarrow qqH$ channels included [234].

Discovery potential for SM Higgs boson

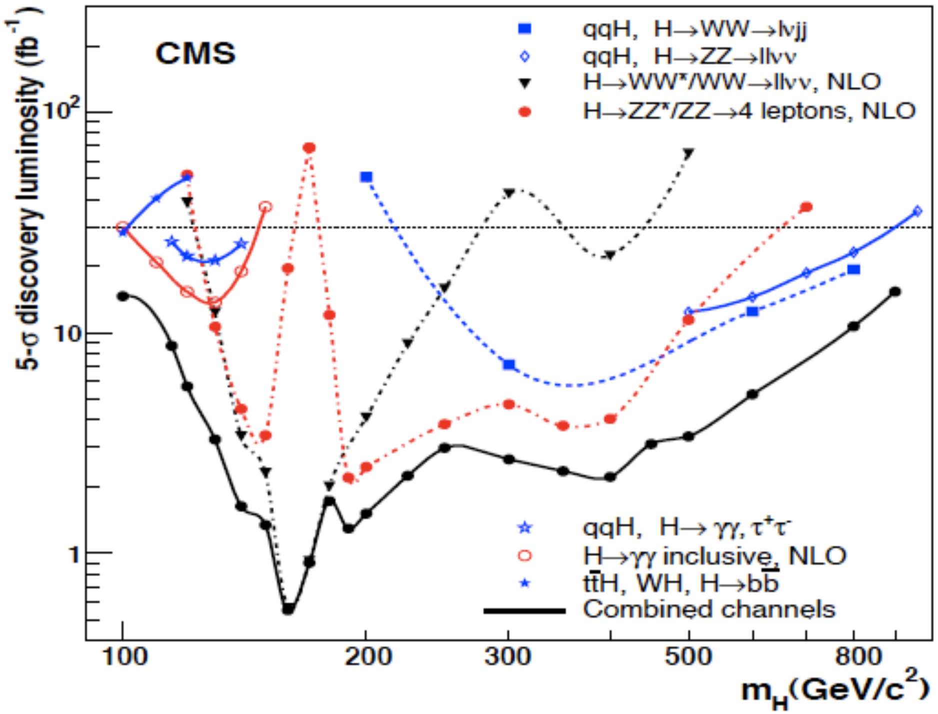
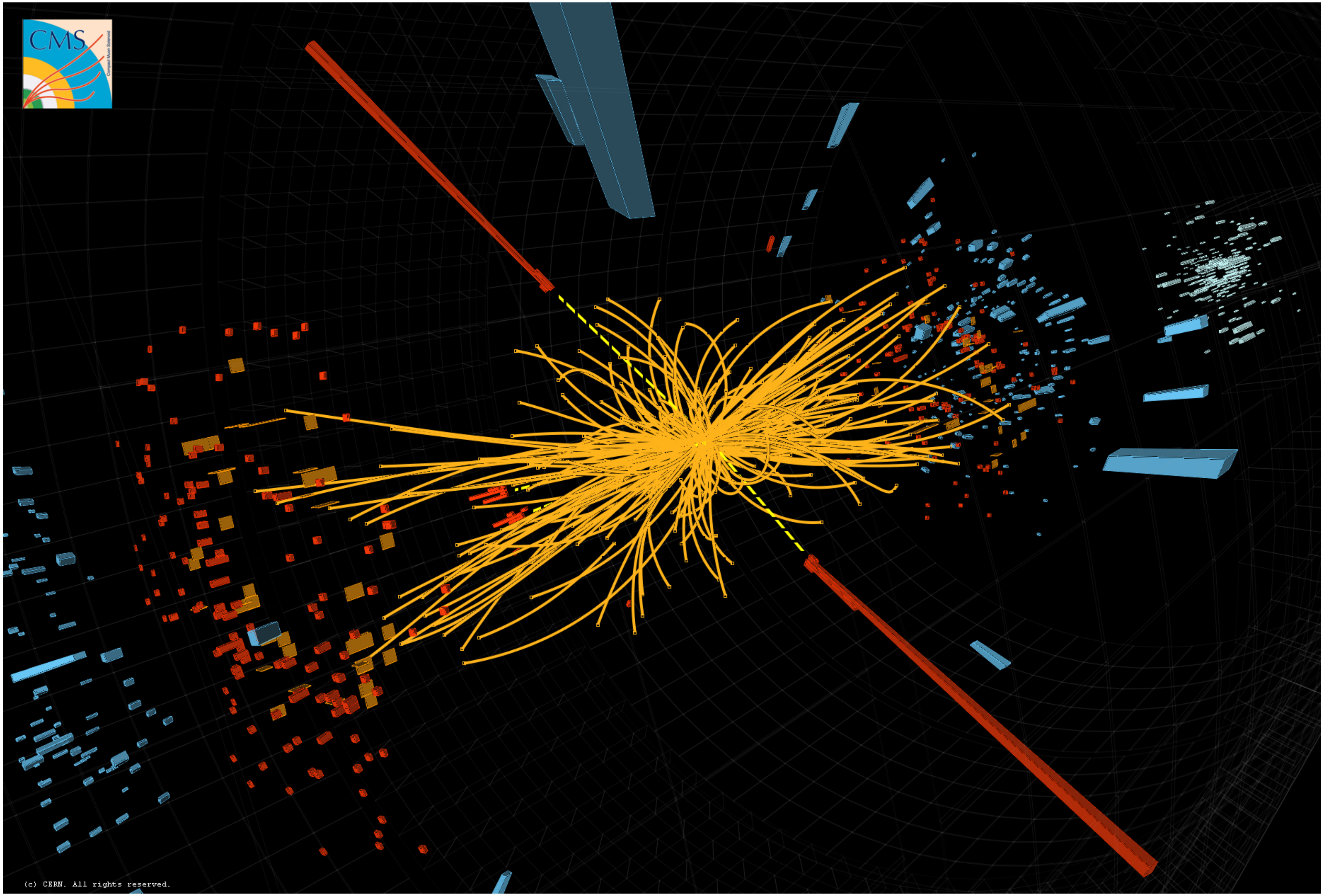


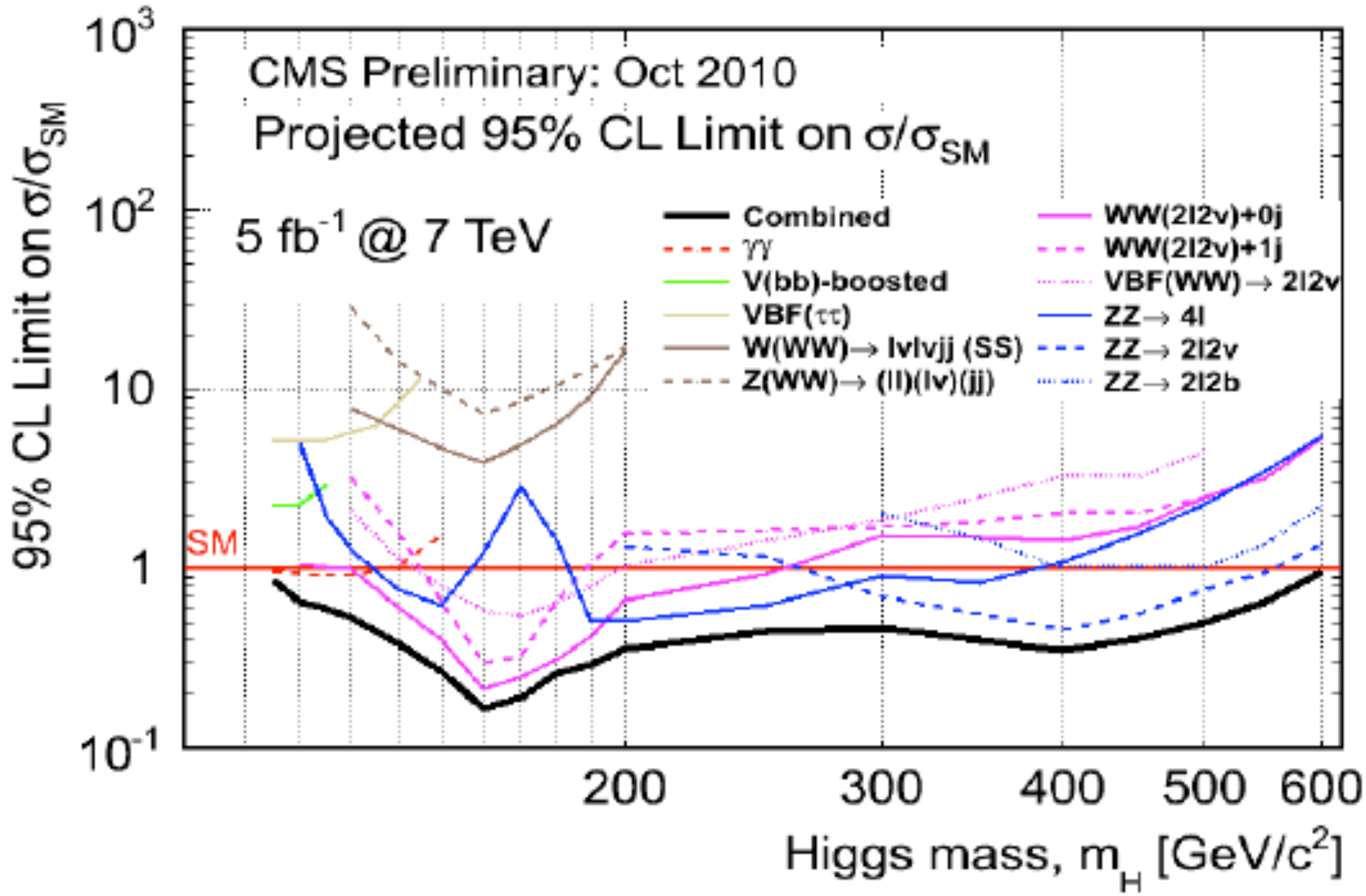
Figure 3.50: The required integrated luminosity that is needed to achieve a 5σ discovery signal in CMS using various detection channels as a function of M_H [235].

Latest results (yesterday's press release)

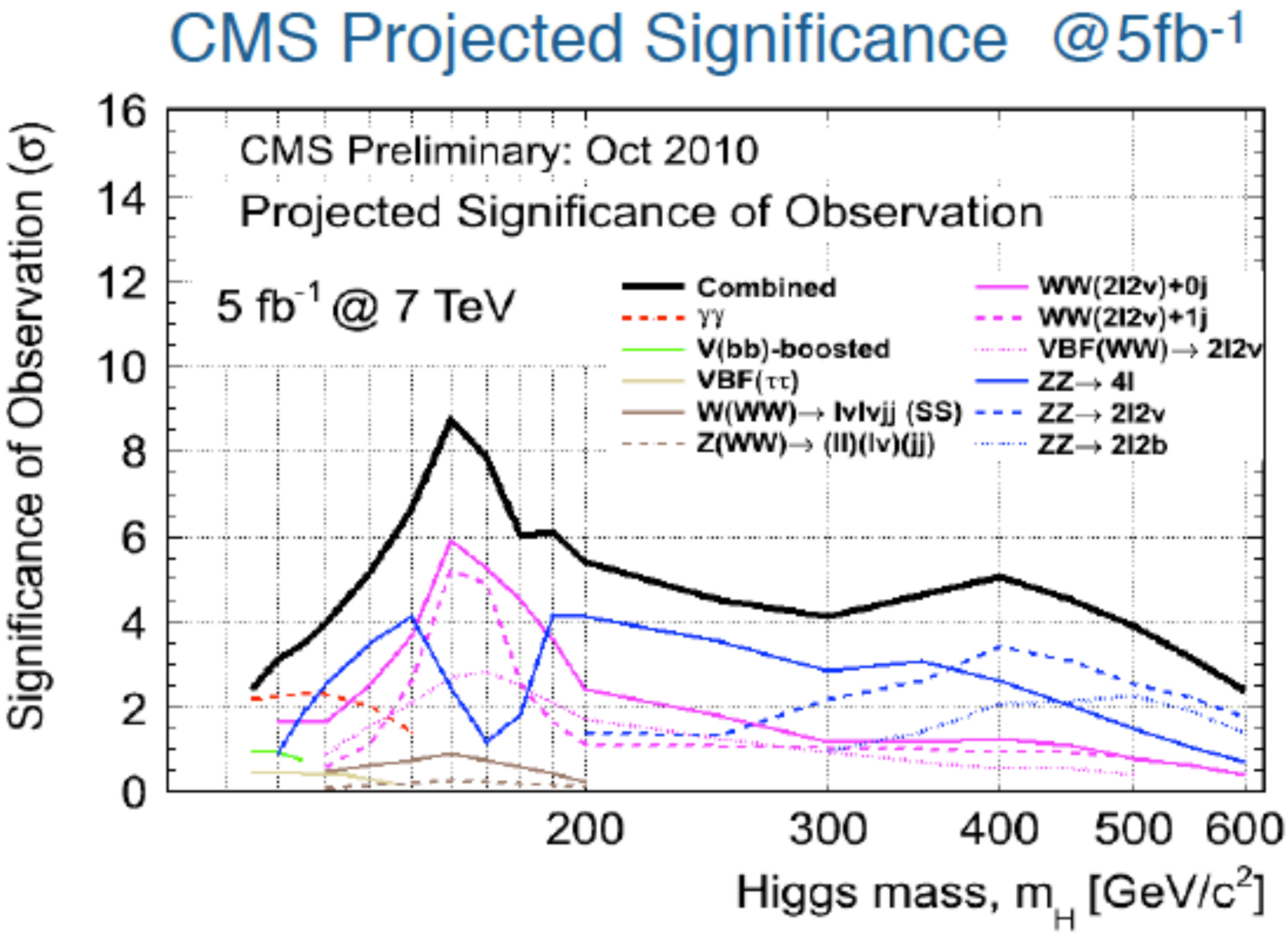


Latest results (yesterday's press release)

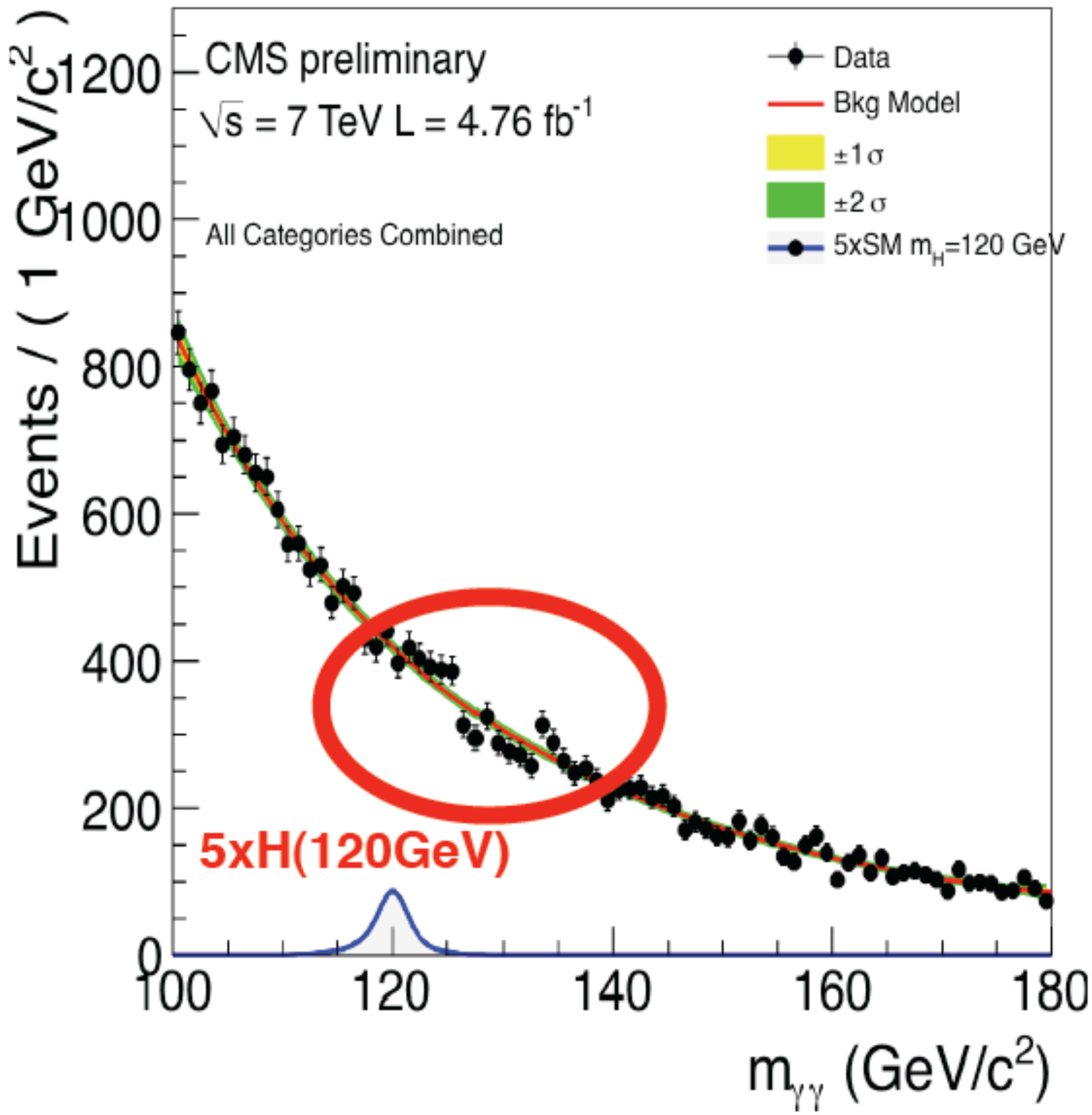
CMS Projected Sensitivity @5fb^{-1}



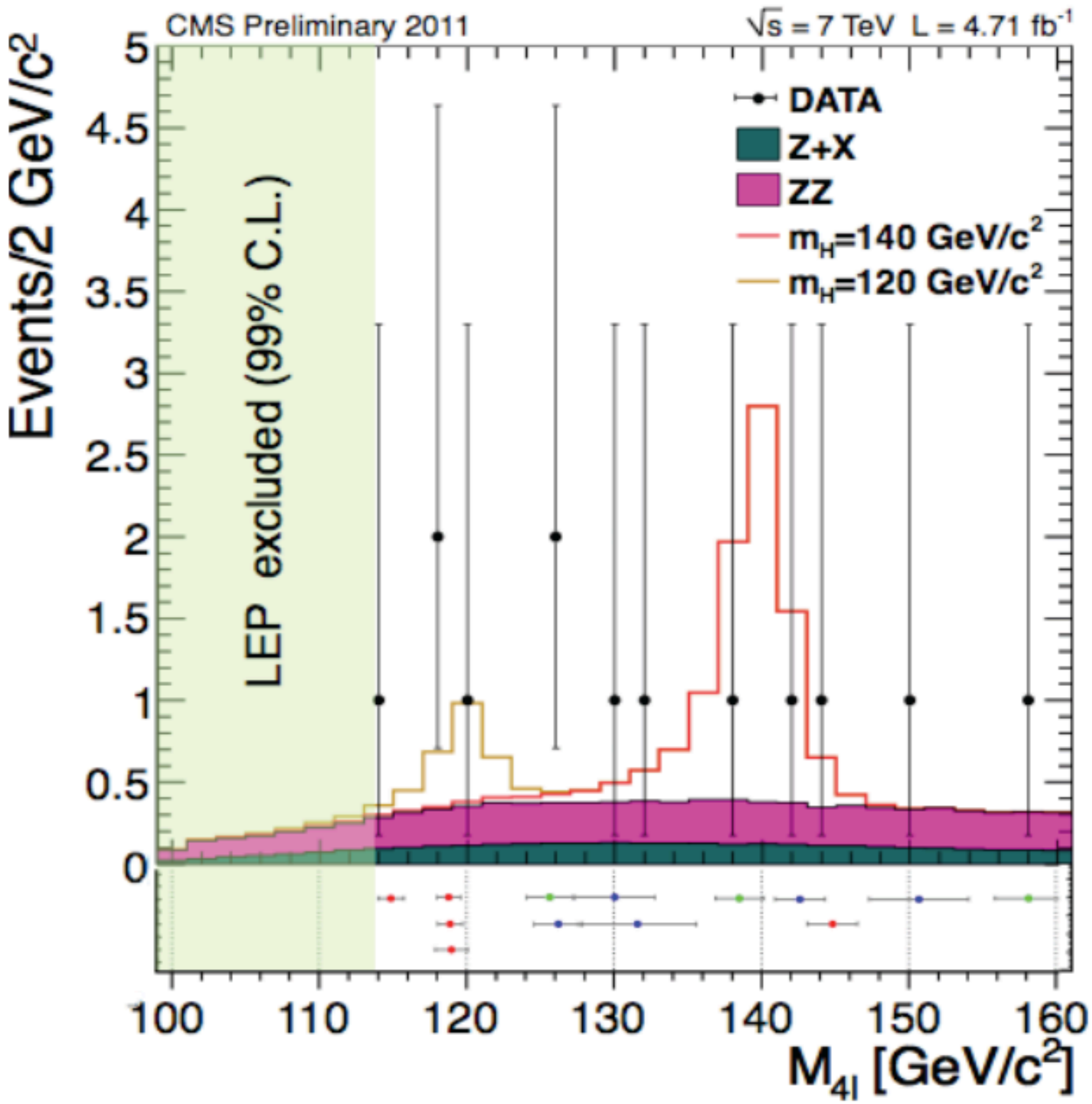
Latest results (yesterday's press release)



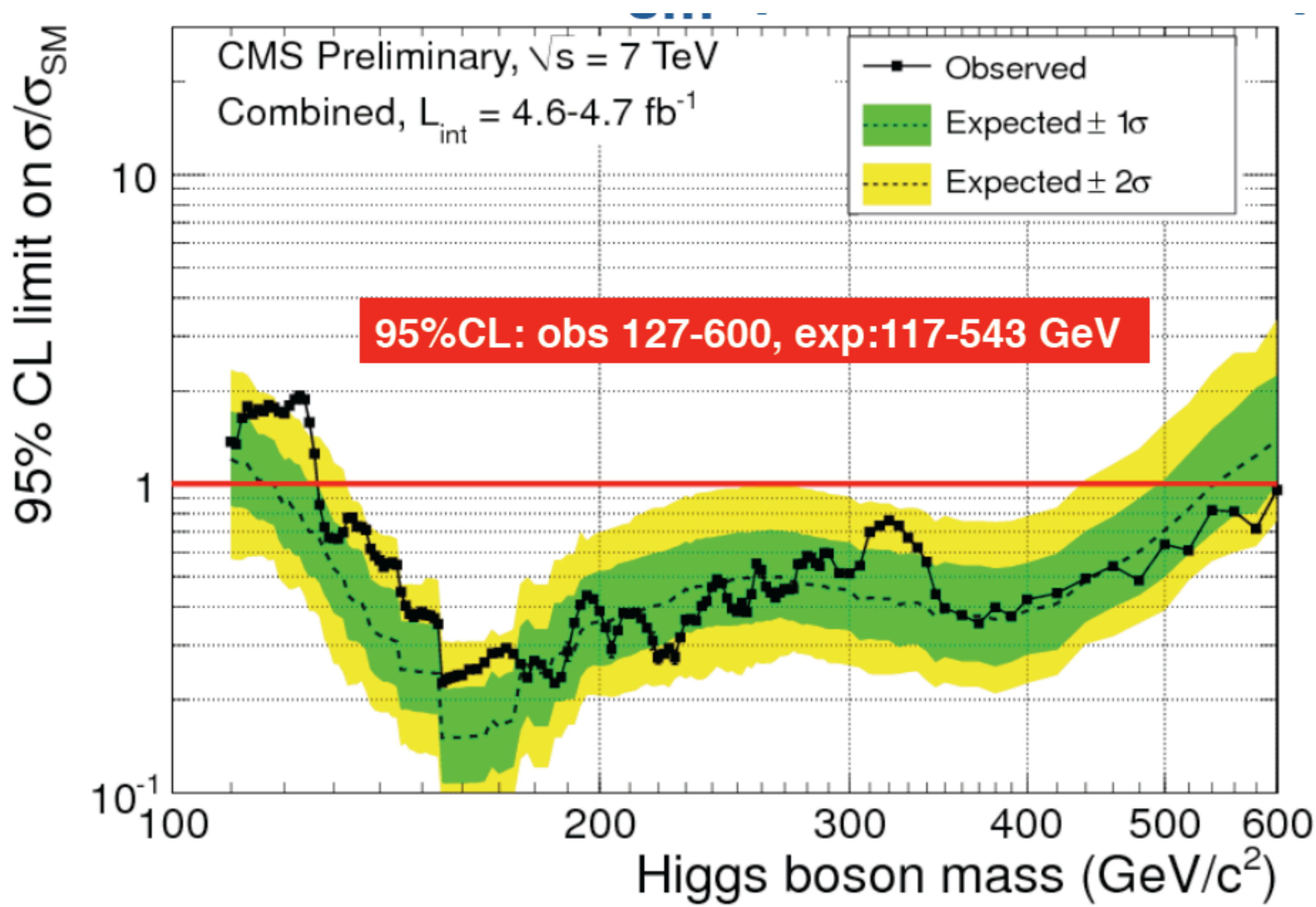
Latest results (yesterday's press release)



Latest results (yesterday's press release)



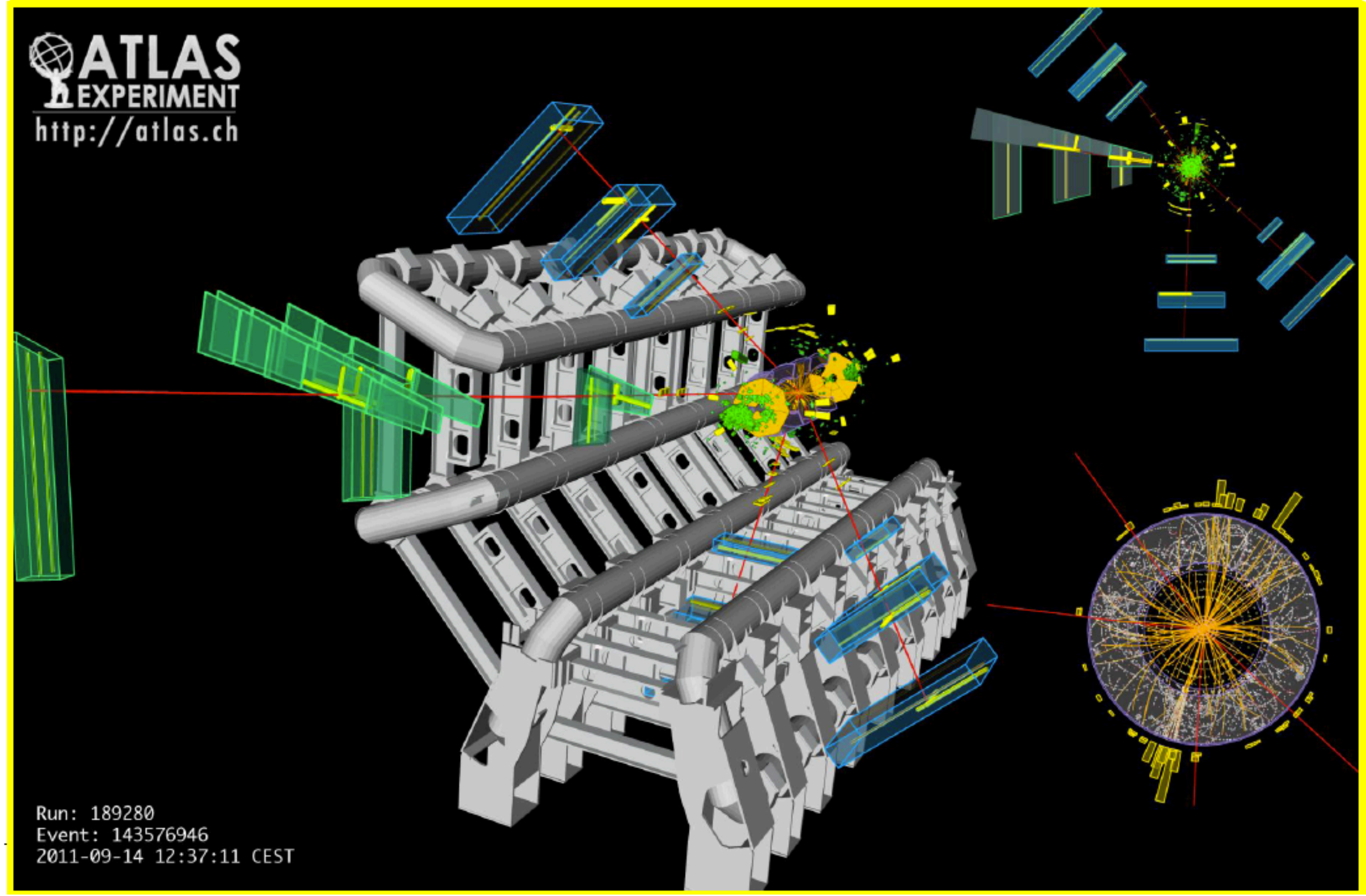
Latest results (yesterday's press release)



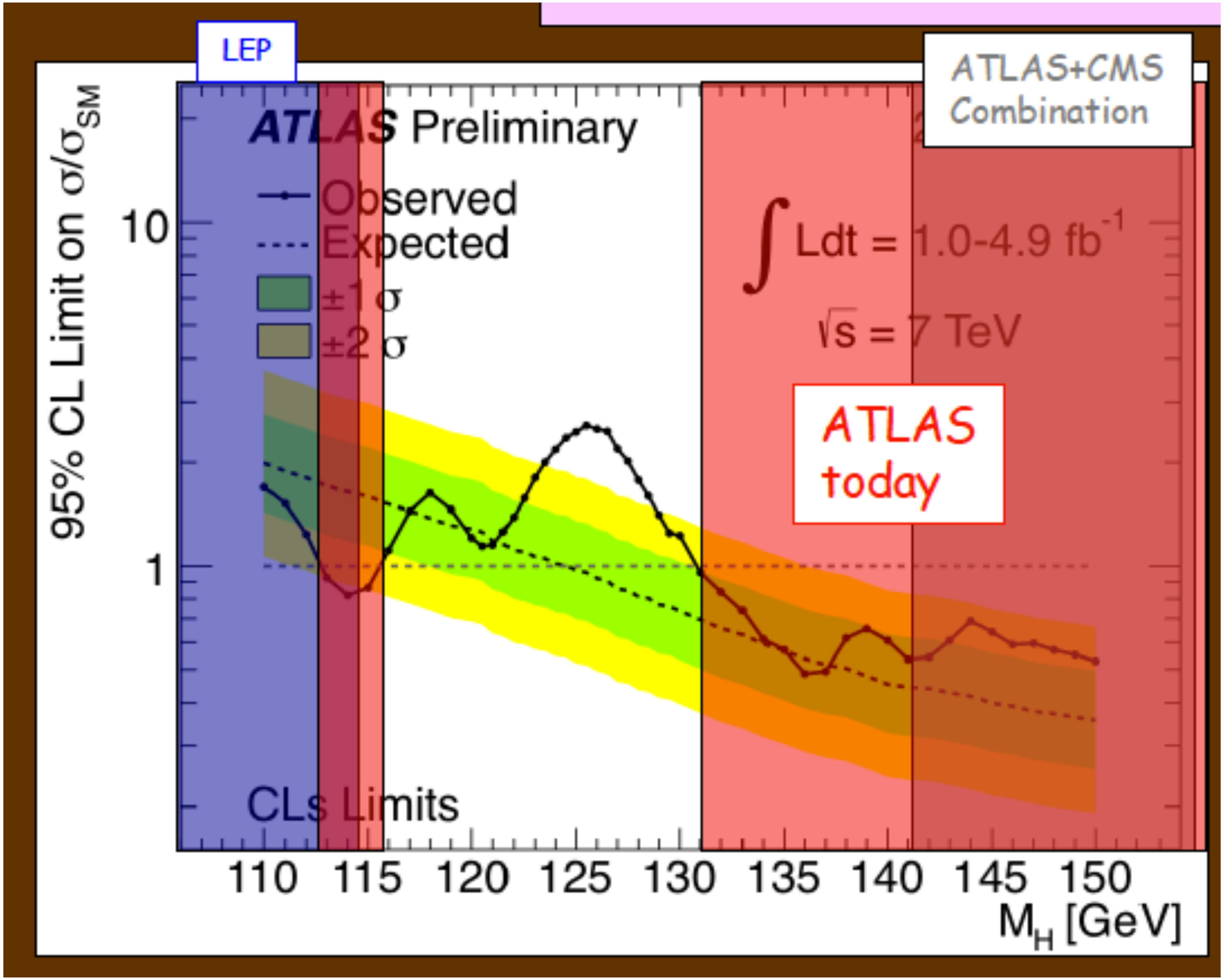
Latest results (yesterday's press release)

4μ candidate with $m_{4\mu}$= 124.6 GeV

p_T (μ^-, μ^+, μ^+, μ^-)= 61.2, 33.1, 17.8, 11.6 GeV
m_{12}= 89.7 GeV, m_{34}= 24.6 GeV



Latest results (yesterday’s press release)



Lecture summary

- Production and decay properties of the Higgs boson
- In the design of experiments one has to be able to have a phenomenological insight in the event with for example Higgs bosons
- How can we estimate quickly if certain processes can be discovered or not
- Exercise: can we observe the pp→ttH process and if yes, at which Higgs boson mass and for which integrated luminosity